ON SUBATOMIC ELECTROMAGNETIC WAVEFORMS

THE LEPTONS, PHOTONS, QUARKS, AND

THE TERNARY FORCE INTERACTION CONSTANT

A REPORT OF THE MATHEMATICAL PHYSICS RESEARCH WORK

OF

JIM FISHER

04 July 1991 to 19 September 2013

And Again Between

August 2016 and June 2019

TECHNICAL SUMMARY AND OVERVIEW

1 Executive Summary

These collected reports summarize the mathematical physics work of Jim Fisher describing the elementary electromagnetic waveforms, the leptons and the photons, and the charge of the quarks.

The mathematical approach of this work is completely outside of the standard hypothesis-first top-down procedure for doing mathematical particle physics for the last 30 years. The methodology used by Jim Fisher could variously be described as; a correlative mathematical analysis of the data, a data-first approach, a non-hypothetical approach to particle physics, or a pragmatic engineering approach to particle physics.

As a result of 12 1/2 years of intense research efforts Jim Fisher discovered mathematical-geometric equations which give exact descriptions of the masses and charge of the leptons. Before now, humans have never had any intellectual explanations for their experience of these two most basic properties of these elementary units of matter.

The Laguerre orthogonal polynomials, which are solutions to second order differential equations, have been used to step thru the lepton mass series of electron, muon, and tau. Therefore a person could say that the mathematical solutions for mass which were found are essentially quantum mechanical in nature. Additionally the geometric forms found for both the leptons and the photons could be said to consist of spinning radial-angular plane waves, which propagate into space with time. Thus a person could say that these two elementary energy forms could be ascribed as being membranes of the string-membrane school of hypothetical particle physics. Mathematically both the leptons and the photons are outlined as 3 dimensional curves in space. Further the mathematics describing the masses of the leptons and the (ML)(L/T) of the photons are almost identical. Only what are almost trivial changes need to be made in their mathematics to swap between these two basic classes of electromagnetic waveforms. Yet at the same time, these mathematical changes point towards the many strikingly different physical properties between these closed and open ended forms of energy wave patterns. Finally the equation found which exactly describes the charge of the leptons begins from vector origins and results with an invariant fixed numeric, as required.

These reports also cover two other topics. They describe the exact correlation between the ± 2/3 and ± 1/3 charge of the quarks and the differential geometries of certain vector curves in 4D space. This is by three calculational routes, a red solution, and partner blue and green solutions. Lastly these reports describe the origins of a ternary force interaction constant amongst the three basic forces gravitational, electrical, and magnetic. This interaction constant, which has here-to-fore been undiscovered, appears to underlie the wave form structures of both the leptons and the photons.

This work is important because many future avenues of particle physics research have now been opened. For example, a four column Periodic Table of the Elements of Physics (PTEP) is now conceivable, with the leptons filling the second column. The neutrinos would fill the first column and the two classes of quarks the last two columns. Brief investigations have shown that dark matter or energy might be possibly be explained by the mathematics of hypothetical upper members of a photon series analogous to the lepton series or possibly by waveforms existing only in 4 dimensional space.

2 Overview, What Is Presented In These Reports

These collected reports of the mathematical physics research work of Jim Fisher present the ultimate physical property equations which were discovered for the 5 principle physical properties of the elementary electromagnetic particles; e, m_e, m_μ, m_τ, and *h*. Additionally an equation for the universal ternary interaction force constant between the gravitational, electrical, and magnetic forces is specified. These equations form the core narrative of this body of work. And now as June 2019 the ± 2/3 and ± 1/3 charge of the quarks has been shown as matching the fixed curvature of certain vector curves in 4 dimensional space.

The equations are presented and then parsed into their various factors. These factors are thoroughly discussed to ensure their correct understanding and use so that the reader can reproduce the research findings if they so chose. The equations are set into the contexts of consensus space and time and given interpretations as to the probable meaning of their mathematical quantities in terms of the "real" world. All the equations are carried from blackboard mathematics, conceptual spaces, to the real world, physical time and space, by use of scaling constants.

These equations are simply presented as the result of many years of correlative research efforts. They are not derived in the usual step-by-step fashion of hypothetical physics presentations because they did not flow from hypotheses. Never-the-less, the numerics of all the calculus-mathematical-geometric blackboard quantities are rigorously proven to be universal, measurement system independent. That is these conceptual quantities come with meta or place holder units and can be placed into the absolute and relative scale systems of any species of inquiring beings. Many reports/chapters are devoted to the necessary proofs which flow from the analysis of measurement systems and the meanings and definitions of various scientific units.

Separately away from the formal mathematical equations several reports/chapters are devoted to speculations, implications of this work, and future work which can flow from it. Likewise a few other reports/chapters give a personal perspective on the 22+ years of the project effort at that time. A short outline is given of the methodology and criteria used to guide the correlative trial and error efforts from which ultimately the equations were pieced together. While the final resulting equations are the main story here, how and why they were discovered without resorting to any of the typical assumptions and procedures of hypothetical particle physics is of great value to science in itself.

3 Historical Effort

This research work was carried out in private by Jim Fisher from July 4 1991 until February 19 2012. This research effort was essentially continuous over the entire 20 1/2 year period.

1 The first 12 1/2 years of the research work were devoted to discovering a correlation for the masses of the three known leptons; m_e, m_μ, and m_τ. Incidentally, after about 6 years the first equation discovered for a physical property of the basic electromagnetic particles was that for the elementary charge of the leptons, e.

2 After finalizing the discovery of the equations calculating to the lepton masses, then about another 1 1/2 years was involved in discovering-developing an equation describing the fine structure constant α. This then ultimately leads to the constant for the (ML)(L/T) for the photons, the Planck constant *h*.

3 After that, about another 4 years was needed to lay out the underlying mathematical nature of the ternary force interaction constant which appears to be involved in the structures of both of the classes of electromagnetic particle.

4 Finally the last 4 1/2 years of this research project involved analysis of systems of absolute measurement units to demonstrate the universality of the mathematical-geometric results used in this work. A rigorous proof was developed for the measurement system independence of the units used with calculus-geometric quantities of this work. That is, for those absolute measurement systems based upon SI analogous relative measurement systems.

5 Additionally during the last 6 1/2 years of the project the work of others in academia was studied to better understand what their objections to this work were and why these objections even arose.

All of the research efforts to find the objective numerical values with their units of the five physical properties for which precise mathematical equations were discovered required intense mathematical-calculus-geometric trial end error searches. These searches involved pin-ball like mixes of mind numbing intellectual efforts and completely non-scientific intuitive endeavors.

This work of this project was solely that of Jim Fisher. The only outside assistance received was from a few non-scientific friends in the form of a several hypnosis sessions to help visualize some of the geometric forms which became some of the critical findings of this complete work. Occasional off-the-cuff discussions were indulged in with coworkers concerning mathematics in the realm of beginner's electrical engineering. Additionally coworkers helped in the first several attempts at formal publication of the results of this work.

Specifically this work did not involve the advice, assistance, or input of any credentialed or published academic hypothetical physicists. No outside financial sponsorship of any form was received, neither from governmental bodies, corporations, non-profits, nor from private individuals.

Additionally now as an add-on starting before August 2016 Jim Fisher investigated the possibility of matching the ± 2/3 and ± 1/3 charge of the quarks with the fixed curvature of certain vector curves in 4 dimensional space. The final correct portion of the work was done with input from Professor Jeanne Nielsen Clelland at the University of Colorado Boulder, an expert in the field of differential geometries. This effort was successful and culminated with its first solution on 20 May 2019.

4 Approach

The mathematical approach of this work was completely outside the standard top-down procedure of the last 30 years for doing mathematical particle physics. This approach, the only approved one, has been hypothesis-first or equally make assumptions first. This methodology of the old style hypothesis-first schools of thought has been; start with the unknowns (assume some grandiose pattern as dictated by a thesis advisor) and work towards the knowns (the data).

The methodology used by Jim Fisher could variously be described as; a correlative mathematical analysis of the data, a data-first approach, a non-hypothetical approach to particle physics, or a pragmatic engineering approach to particle physics. This correlative data-first approach started with the knowns and worked towards the unknowns. The knowns were the physical property data of the particles. The unknowns were the mathematical-geometric structures of the fixed waveforms, the particles, which were assumed to exist, of some unknown form, and were hoped to be found.

Initially a correlation was to be made of the masses of the three known leptons and this correlation was to arise from the data as a result of trial and error searches. Any correlation attempts and equations to be developed were required to meet a strict list of 15 common sense criteria. As examples;

1 Any equations developed must match (predict) the objective quantities to many decimals of accuracy, preferably matching that of their measurement.

2 Every mathematical factor in an equation must be identifiable as having some plausible physical-geometric origin or correspondence.

3 Of great importance, the final body of equations needed to be interlocked and display underlying patterns.

4 Likewise any equation discovered was prohibited from containing any arbitrary or unexplainable numerical constants, such as the number 42, and also could not create circular references.

5 Probably most importantly, any equations giving descriptions of physical properties for the elementary particles could not either predict or invoke the use of anomalies at odds with known science or physical common sense. Requiring 9, 10, or 26 spatial dimensions to describe sub constituents of the consensus world was prohibited.

5 Summarizing The Differences Of Jim Fisher's Mathematical Particle Physics Research Work From That Of Academia

The mathematical particle physics research work of Jim Fisher is utterly different in its approach from all other calculational particle physics work of academia in the following manners.

1 First and most importantly this work did NOT use a hypothesis-first methodology. The methodology of this research work was to use trial and error mathematical-calculus-geometric searches to initially match the shape of the curved plot of the three known lepton masses. This trial and error effort was intentionally a no holds barred, all ideas are fair game, open ended search. No preconceived molds, models, or frameworks were to be used to find the physical-mathematical-geometric nature of the leptons' waveforms. Likewise no assumptions were made as to the correct or applicable field of mathematics which would result in the correlation of the lepton masses. Ultimately the highest level of mathematics that was needed was second semester calculus and analytical geometry.

To be utterly clear, this work does NOT start by expounding a "theory", because it is NOT a "theoretical" based work. This work does not jumper off what some esteemed professor emeritus at some prestigious university thought, said, wrote, or did. This research work and the resulting equations discovered are independent and free standing!

Likewise this work does NOT use insider trade jargon as used by PhD high energy physicists. The reader does NOT need to have spent the last 11.526 years of their lives studying gauge symmetries, the Clebsch-Gordon Coefficient, the Yang-Mills Theory, the Kaluza-Klein Theory, nor any other such insider particle physics concepts and mathematics. Nor does the reader need to know anything about Hilbert space, Banach spaces, ladder operators or other such insider trade jargon used by mathematicians.

2 A second distinct difference and another key criterion of this work was that of restricting all work to a very limited scope. All the initial mathematical research work was to stay strictly focused on searching for a correlation for the masses of the three observed members of the electron family.

The use of or speculations about of the masses of the other known elementary particles, such as those of the neutrinos or quarks, were prohibited. The invoking of unproven, speculative, or other hypothetical particles was prohibited. The referencing of imaginary angle-like super symmetric partners guiding the affairs of the mere mortal particles was prohibited. The existence or lack thereof of other particles, known or unknown, was to be completely irrelevant.

In short, mathematically the specific properties of concern of the leptons were to be dependent upon the leptons themselves, alone, and upon the three a-priori force constants. The wealth of undisputable additional experimental data and the well established properties of other particles were prohibited from entering the mathematics of any equations to be found for the lepton physical properties. Further, the mathematics of an individual member within the lepton class of particles were not to have variables dependent upon or entangled with the mathematics of the other members from within that class.

Likewise uses of or discussions about collision product scattering angles, relativity, and other such off topic matters were strictly prohibited.

Discussions of the background or how the various classes of particles came about to be and other such matters were not to be involved. The age of the universe, what occurred during its first few instants of existence, the origins of the matter composition of the universe as is now found, et cetera were all to be completely irrelevant. Finally, grandiose discussions concerning nebulous religious-like guiding metaphysics such as a Grand Unified Hypotheses of Everything were to be utterly avoided.

3 A third unique feature was that of assigning parametric units to the results of mathematical-geometric equations. These assignments involved units from both the SI set of relative scales and also from absolute or "natural" sub-sub-atomic physics scales. For this work the absolute physics Squigs scales (Squigs l, t, m, and q; l_{Sgs}, t_{Sgs}, m_{Sgs}, and q_{Sgs}) are used. These Squigs scales are analogous to the measurement units put forth by George Johnstone Stoney in 1874. Except the Squigs scales have had his assumed 2 or 3 dimensional π constants removed. The scaling constants involved in this work use the three measured force constants G, ε_o, and μ_o, and the elementary charge e as a-priories. These measured properties are then used to develop absolute physics scales analogous to the Stoney units for distance-length, duration-time, mass, and static quantities of charge (L, T, M, Q). Finally this means that quantities resulting from mathematical expressions, factors within equations, etc, were not forced to be unitless ratios nor involved other such mathematical trickery which results in meaningless sterile numbers.

4 A fourth critical difference was the involvement of G, the universal gravitational constant, when and where it appeared to be an inherent and necessary factor, such as being embedded in scaling constants.

5 This work used very common low level calculus and simplistic geometries in its mathematics. A senior in high school vaguely familiar with the concepts and symbols of integral calculus can read and follow the discussions in the core research chapters in Part 1 of this work. Tensors, imaginary numbers, probabilities, etc were not used in the mathematics, nor were discrete, disjointed, or otherwise discontinuous forms. Geometries only involved 2 and 3 spatial dimensions for the work with the leptons and photons Four spatial dimensions were necessary to describe the charge of the quarks.

6 As a consequence of invoking only known consensus real world particles, using numerical outcomes having parametric units, and easy to understand geometries, the results of this work are verifiable. This is a major difference from all assumption or hypothesis driven particle physics research.

6 General Outcome

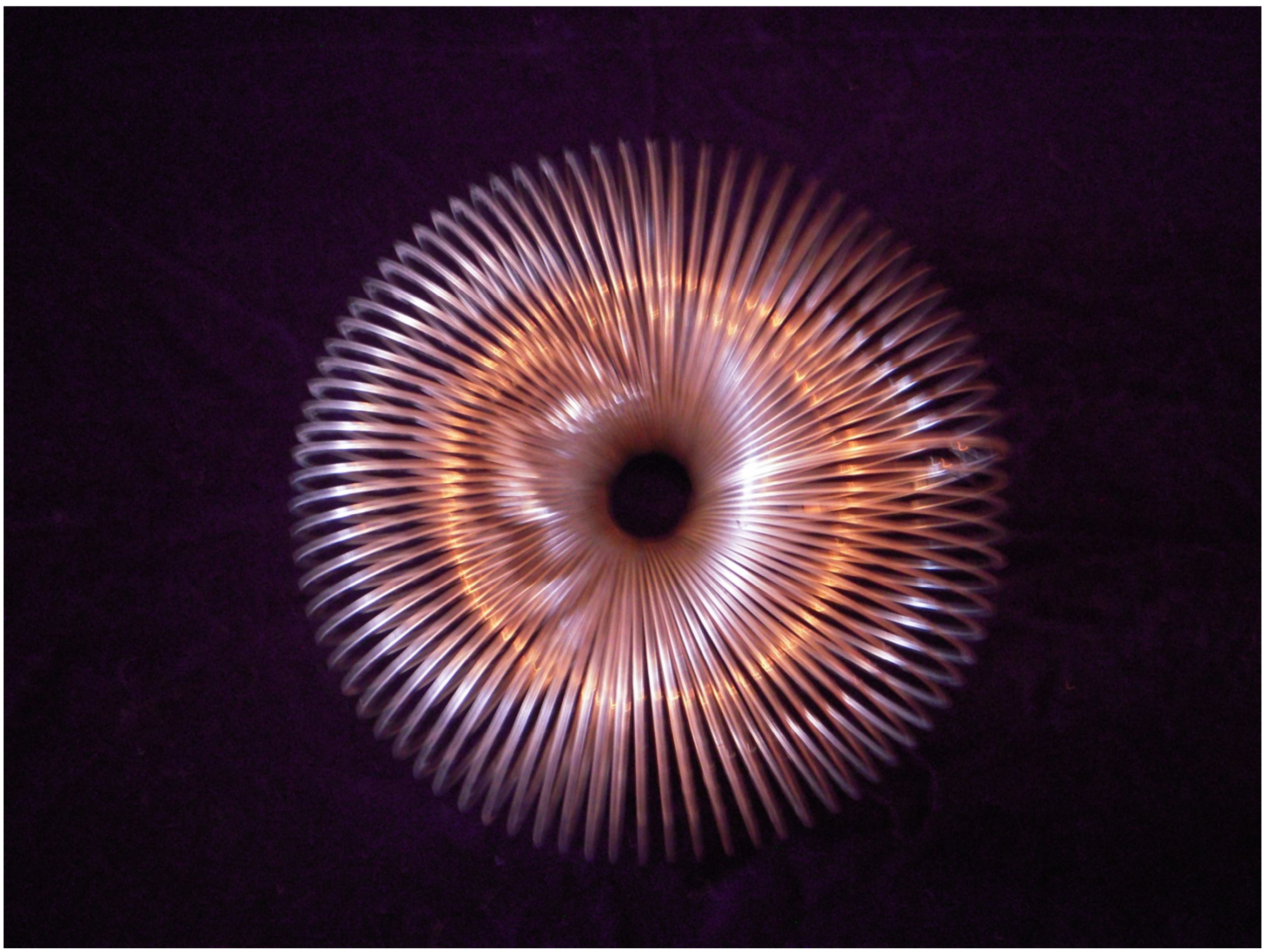
Matches for the several objective measured physical properties of the leptons and photons ultimately were discovered and placed into equation form. The correlative approach of this mathematical physics research work and the trial and error searches were successful in many other unintended ways.

What is amazing is not that structural equations can be developed which lead to the exact calculation of the masses of the leptons in kg and the (ML)(L/T) for the photons in (kgm)(m/s), but that additionally all the component factors within these equations have real world meaning. All the component factors within the equations discovered have common simplistic geometric mappings to the physical world as humans understand it. Further, analyses of the various factors and implicit variables within these equations directly lead to explanations for many of the other observed physical properties of the waveforms. That is, the equations discovered lead to much more than just their precise target or objective numerical values.

This work found that both the leptons and photons have definitive energy density structures. That is, the leptons are not mathematical points with no length, width, height, time, etc. to which somehow quantum numbers and physical properties are magically attached. Likewise the photons were shown to have a mathematical component radial to their flight path and are not mathematical lines like wires with no widths. The general form of the geometric shape found for the energy densities of the leptons and the photons alike was that of a two dimensional radial-angular plane. These planes then spin and are propelled forwards in time. The resulting already well established appearance of the photons being that of an open ended or moving cylindrical spiral progressing in a straight line. That is in the absence of massive gravitational bodies. The leptons were found to progress in the next most simple and conceptually common path in terms of which humans think, that of a circle. They form a closed loop and make the outline of a donut, mathematically a torus. In simple layman's terms the leptons go forever in tight little circles like a dog chasing its tail. In terms of simplistic static demonstrations, a stretched out kid's toy Slinky could be used to represent the path of the photon's energy structure and a Slinky which has been coiled around end-to-end to make a donut to represent the path of the electron's energy structure.

Of course the higher members of the lepton series, the muon and tau, explode or self destruct before they get too far. This is because the outlines of their radial-angular energy density planes are not truly circular in nature but are actually flower petal shaped. As they spin about their axes of progression, they tumble or flop around like badly imbalanced airplane propellers. Additionally these energy planes do not uniformly fade away as they extend outwards, but are bumpy with multiple highs and lows of their density patterns. Even more complicated and probably contributing significantly to the self destruction of these higher members of the series is the fact that the outline of their orbit or revolution about the center of their donut holes also are not circles, as with the electron, but again are flower petal shapes. These waveforms progress more like high speed drunks trying to run around large daisy wheels all the while doing cart wheels.

PICTURE 1 SNAPSHOT OF A HIGHLY MAGNIFIED FROZEN ELECTRON



All the energy structure equations found for both the leptons and the photons have the same general mathematical form. In the most broad or general terms the numerical value of the objective physical property can be recognized as a product of two factors. There is nothing unusual about decomposing or parsing the value of a physical property into two such factors. In fact such practice might be considered mandatory.

One factor is a scaling constant. The scaling constants turn what would otherwise remain as correlations into actual equations. These numeric constants with their measurement units scale the correlations from the arbitrary sized realm of blackboard conceptual math-geometry to the consensus sized physical realm of humans or more correctly to the absolute size realm of George Johnstone Stoney. A simple example of such a well known factor is G found in the force equation $F = Gm_1m_2/r^2$, where the relative SI measurement units of G are $(m/kg)(m/s)^2$. Similarly there is the premultiplier $8m(\pi/h)$, with SI units of (s/m^2), in the Schrodinger wave equations describing possible electron shells for the hydrogen atom.

The other factor of the general mathematical forms are numerically universal values which results from calculus-mathematical-geometric considerations. These are seen as the m_1m_2/r^2 part of the gravitational force equation above. Likewise the second part of the quantum mechanic wave equations, which is not shown, are purely mathematic or geometric-calculus in nature.

Approximately 4 1/2 years were devoted to proving that the numerical values found by analytical geometry and calculus at the core of the physical property equations are universal or measurement system independent. This universality does require that the relative scales, underlying the absolute measurement systems used by the scaling constants, must be SI analogous. That is, the mathematical-geometric constants can be shown to have meta or place holder units of the mixed form; particle property with relative scale units / distance with absolute scale units. These constants can be scaled into any arbitrary, but SI analogous, set of units invented by any species of conceptualizing beings.

Now adding to this work the ± 2/3 and ± 1/3 charge of the quarks have been matched with the fixed curvature of certain vector curves in 4 dimensional space. These mathematical-geometric models cover three possible solutions, a red solution and partner blue and green solutions.

7 Specific Findings, Research Results, And Strong Points

This private unsponsored non-standard mathematical particle physics research work of Jim Fisher has resulted in the following.

1 Equations have been discovered which describe, predict, or match several physical properties of the elementary electromagnetic particles to the decimal accuracy of their measurement.

Elementary charge	$1.602,177,33 \times 10^{-19}$ C	(8 decimals)
Electron mass	$9.109,389,7 \times 10^{-31}$ kg	(7 decimals)
Muon mass	$1.883,532,7 \times 10^{-28}$ kg	(7 decimals)
Tau mass	$3.167,88 \times 10^{-27}$ kg	(5 decimals)
Planck constant	$6.626,075,5 \times 10^{-34}$ (kgm)(m/s)	(7 decimals)

If the equations found are assumed to be totally coincidence or accident, then multiplying together the random chances for each decimal of each property, there are 5 chances in $10^{(8 \times 7 \times 7 \times 5 \times 7)}$ or 5 chances in 10^{13720} that this work is accidental and meaningless.

Adding to this, now the ± 2/3 and ± 1/3 charge of the quarks have been matched with the invariant curvature of certain vector curves in 4 dimensional space, by three calculational routes.

2 In addition to matching the numerical value of the objective physical parameters, the equations discovered also explain many obvious physical property differences between the leptons and photons. These are;

2.1 The leptons have mass. The photons are massless.

2.2 The leptons have a fixed electrical charge. The photons do not display a constant fixed electrical charge observable to, nor measurable by humans.

2.3 The leptons have a spin angular momentum ratio of 2:1 to that of the photons.

2.4 These equations can be used to directly examine the parity relations of both classes of particles.

Additionally, this work can explain, at least qualitatively, the shortening half lives of the higher members of the lepton series.

3 A fourth member of the lepton series, the shipa, with a net positive mass is mathematically possible, but its stability is questionable due to highly complex angular wave patterns. Additionally one of this particle's radial shells has a negative value. While mathematically possible, this brings into question as to whether such a particle can exist at all with an effective energy drain in its wave pattern. Should such a particle exist it probably would have such an extremely short life that it was never seen by older collider detection instrumentation. The mass for the shipa was found to be approximately 4.647,568 x 10^{-30} kg which lies about 2% of the distance from the electron mass towards the muon mass. This particle would stabilize about 5 times more energy than the electron as compared to the approximate 207 times greater amount stabilized by the muon. This mass cannot be predicted exactly since there are only three lower members of the mathematical series, the electron, muon, and tau. Strong patterns have not been set for several of the shipa's radial energy shells.

After the potential shipa, the mathematics of the lepton mass series produced negative values or effectively explains the termination of the real world physical particle series.

4 A universal ternary force interaction constant has been specified and mathematically described. This constant $(G/\varepsilon_o)^{0.5}/\mu_o$ = 2.184,555,091 x 10^{+6} (C/kg relative)$(L/T \text{ absolute})^2$ appears to describe how the three basic forces (gravitational, electrical, and magnetic) interact as they assemble to form the electromagnetic particles. To grasp this concept an analogy to another well understood physical property constant can be made. The gas law constant 82.0575 (atm cm^3)/(gmole K) describes the quaternary relationship of how the 4 parameters of pressure, volume, temperature, and molar quantity interact to form a body of gas.

5 This body of six equations for the elementary charge, the lepton masses, the photon (ML)(L/T), and the ternary force interaction constant is solidly cross linked. Specifically, there are many common identical mathematical factors in the equations for the electron's mass and that for the photon's (ML)(L/T). That is to say, to go from an open ended or moving photon wave pattern to the closed or bounded cyclic donut shaped form of the electron only requires a very few simple mathematical changes.

6 There are several scaling constants used in the equations for the lepton charge and mass and the photon (ML)(L/T) which scale the arbitrary sized math-geometry-calculus results to the consensus world realm size of humans or again more correctly to the absolute size realm of George Johnstone Stoney. The numerical values and meta units of the mathematical-geometric results have been proven by the use of measurement systems' analysis to be universal or measurement system independent, for SI analogous based sets of scales for L, T, M, and Q. Specifically;

For the elementary charge e, the geometrical calculus result is a constant, A =5.245,406,17 x 10^{-3} with the mixed relative per absolute physics units $C^2\ l_{Sgs}{}^{-1}$. The scaling constant is $[\mu_o\ (G\ \varepsilon_o\ )^{1/2}]$ = 3.054,438,950 x $10^{-17}\ l_{Sgs}$ / C. The final product is 1.602,177,29 x 10^{-19} C.

For the mass of the leptons, using the electron's mass m_e as an example, the geometrical calculus result is a constant1.861,432,180 x 10^5 with mixed relative per absolute physics units kg/l_{Sgs}. The scaling constant used for all the lepton masses is $e\mu_o(G\varepsilon_o\ )^{1/2}$ = 1.0 L_absolute = 1.0 l_{Sgs} = 4.893,752,96 x 10^{-36}m. The final product is 9.109,389,239 x 10^{-31} kg.

For the Planck photon constant *h*, the product of the radial and angular integrals is 68.517,994,75 (ML)(L/T) Sgs units. The scaling constant is $e^2(\mu_o/\varepsilon_o)^{1/2}$= 9.670,562,404 x 10^{-36} SI / Sgs units. The final product is 6.626,075,440 x 10^{-34} (kgm)(m/s).

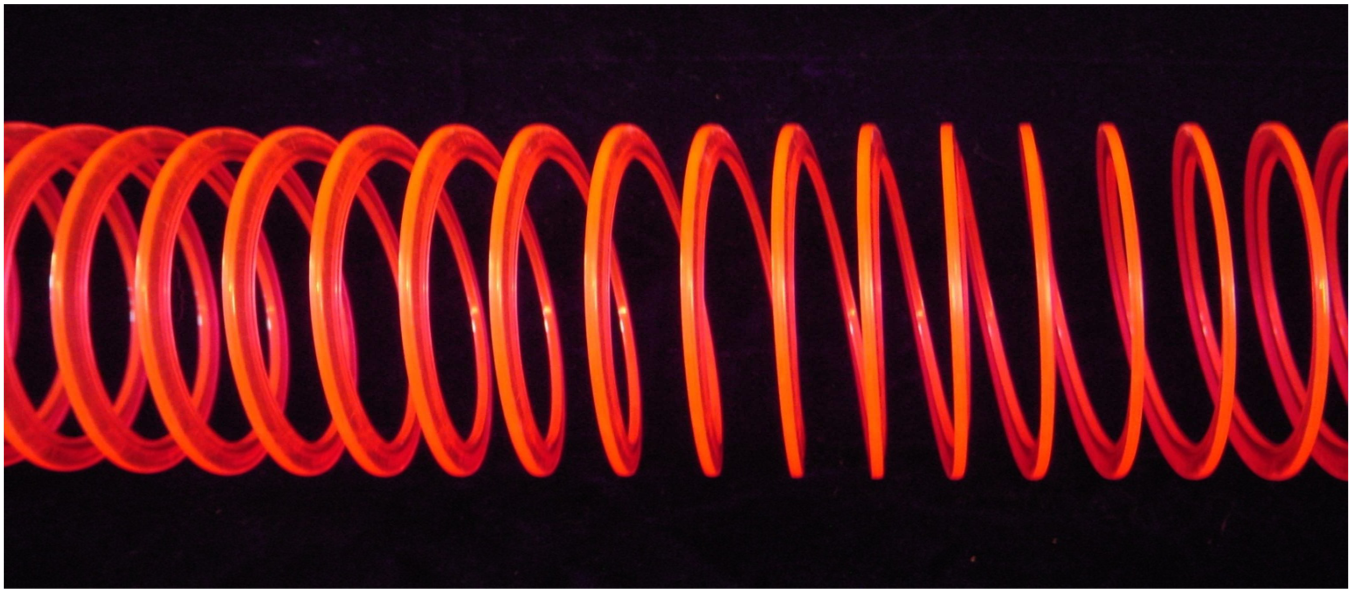
7 All the equations for the leptons and photons are placed in 2 and 3 dimensional mathematical-geometric spatial and temporal conceptual structures. These frameworks are in tern grounded in concrete references to the real world observed physical properties of the particles, the waveforms, of discussion. The resulting numerical values of the equations are in units of the common consensus real physical world, and are NOT in terms of probability space, momentum space, or other such conceptual and mathematical spaces. Further the equations, their independent and dependent variables never even start in such realms but are always confined to the quantities of L, T, M, and Q either absolute or relative.

8 This initial work with the leptons and photons does not either predict or invoke the use of anomalies at odds with known science or physical common sense such as; hidden, invisible, rolled up, or otherwise magical physical spatial dimensions.

9 Probably the most important finding of this entire body of mathematical research work for the leptons and photons is that the subcomponents of the consensus universe are simple and conceptually easy to understand and demonstrate. The mathematical-geometric aspects of this work, the waveforms found, can be explained to 6th grade students with a common kids' toy, the Slinky.

10 Finally and of great importance, the results of the add-on portion to the overall project have shown that the ± 2/3 and ± 1/3 charge of the quarks can be matched with the fixed curvature of certain vector curves in 4 dimensional space. In short, the quarks are confined all right! They are confined to 4 spatial dimensions! They are truly creatures existing only as 4 dimensional forms and live in a 4D space. To try to down grade the quarks and force them into the 3 spatial dimensions destroys the quarkiness of the quarks.

PICTURE 2 SNAPSHOT OF A HIGHLY MAGNIFIED FROZEN PHOTON



8 Why Is This Work Important And Future Work

This work is important because it provides mathematical definitions for the human experiences of mass and charge, the two most basic properties of matter. Exact mathematical definitions have long existed for the scientific concepts of velocity (d_position/dt), momentum (m x v), kinetic energy (m x v^2), potential energy (m x distance x acceleration), etc. Over 100 years ago Einstein gave a mathematical definition for the general concept of subatomic energy, $E = mc^2$. Many named equations exist for the interaction of electrical and magnetic quantities, such as Ampere's Law, Coulomb's Law, and Ohm's Law. What are highly conspicuous by their absence are any definitions for the two basic concepts which are embedded in all these mechanical, subatomic, and electromagnetic definitions. In short all these mathematical equations, laws, and such are defined in terms of two other basic unknowns; mass and charge. Defining an unknown or a new quantity in terms of another unknown allows detractors to laugh at science. This work closes that gap and elevates science to a more solid foundation.

In dealing with both macro human scale phenomena and subatomic reactions scientists hold as dear and utterly sacred the two assumptions of the conservation of matter or mass/energy and of charge. There is total irony here in that the scientific, engineering, and general technical communities can say that a quantity is conserved when there has been no actual definitions as to what these quantities are. This work closes this gaping opening thru which unscientific persons bent on discrediting science like to drive.

This work is also important because many avenues for future investigation into the elementary physics energy waveforms have now been opened. These include work across the experimental, calculational, and conceptual arenas. As examples;

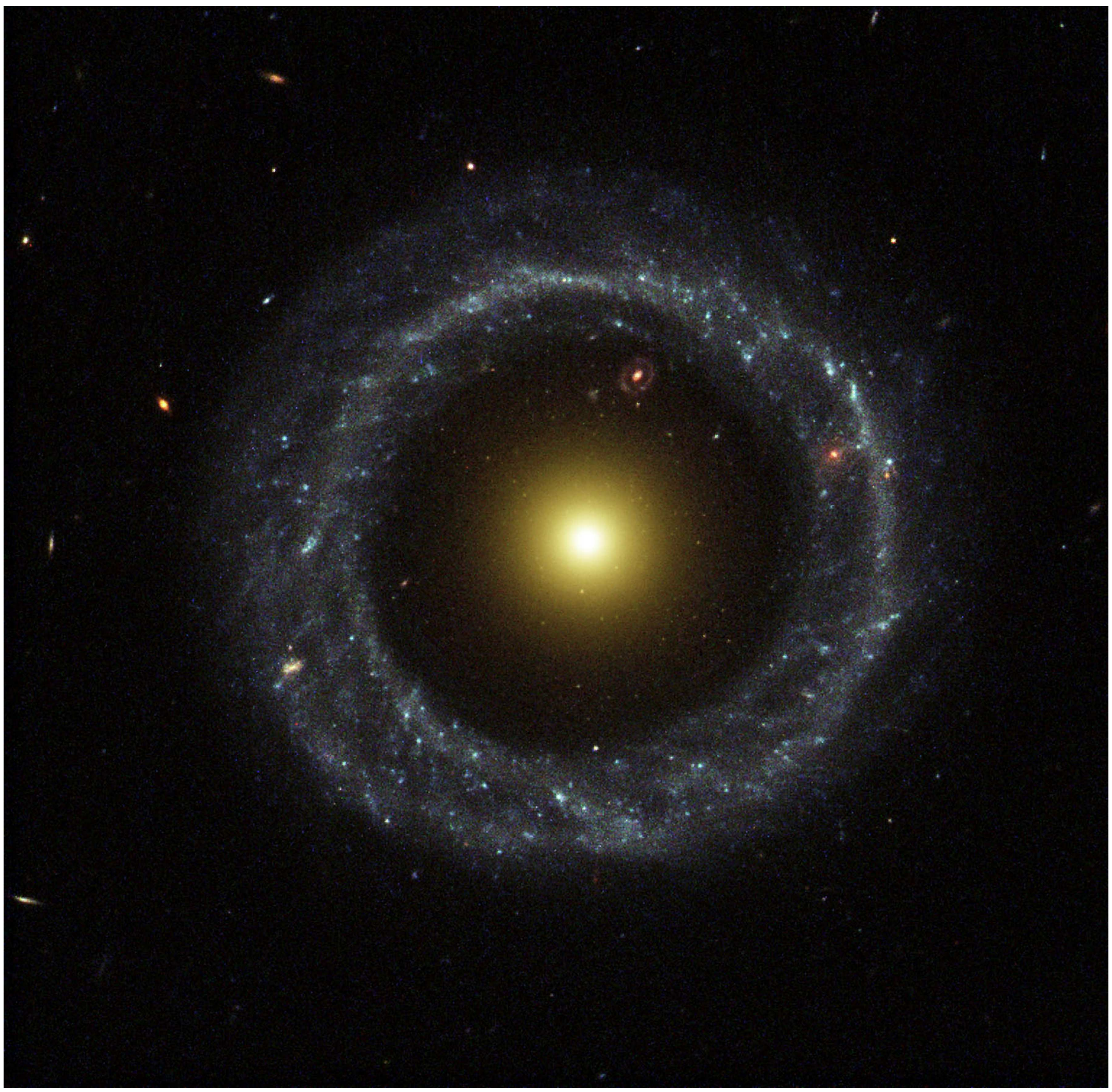
1 A four column Periodic Table of the Elements of Physics (PTEP) is now conceivable, with the leptons filling the second column. The neutrinos would fill the first column and the two classes of quarks the last two columns.

2 Brief investigations have shown that dark matter or energy could possibly be explained by the mathematics of hypothetical upper members of a photon series analogous to the lepton series or possibly by waveforms existing only in 4 dimensional space.

3 Can the donut shape of the leptons be used to help explain anomalies or "violations" found in the results of various collider experiments? For example if all the electrons, or the muons or the taus, line up

"chocolate side" of the donut facing forwards as they fly down collider tubes, would they ever smash into their anti-particle partners coming at them in a side-to-side manner? Could these rare events be what humans are calling violations?
4 Knowing the geometric structures of the leptons, can their magnetic moments now be calculated from first principles?
5 Can a mathematical proof of the uniqueness of mass equations be developed using the fact that many factors within the mass equations appears to be solutions to differential equations.

PART 1
THE RESULTS OF THE MATHEMATICAL PHYSICS RESEARCH CONCERNING THE ELEMENTARY ELECTROMAGNETIC WAVEFORMS THE LEPTONS AND THE PHOTONS



AND FOR THE CHARGE OF THE QUARKS

Scope

These reports of Part 1 contain the core findings of the mathematical physics research work of Jim Fisher which was begun on July 4 1991 and continued almost nonstop for the next 21 years. This research work focused on discovering mathematical descriptions for many of the essential measured physical properties of the elementary electromagnetic waveforms, the leptons and the photons. These elementary waveforms stabilize both gravitational and electromagnetic energy which are interpreted by humans as mass and charge respectively. In addition to the efforts focusing on these stabilized or encapsulated energy quantities, this work was also extended to investigating the interactive relationship between the three measured free space forces; gravitational, electrical, and magnetic, as they assemble to create both of these electromagnetic particle classes.

Now adding to this original 21 years of work, starting in August 2016 efforts were begun to investigate the possibility of matching the ± 2/3 and ± 1/3 charge of the quarks with the fixed curvature of certain vector curves in 4 dimensional space. This add-on to the original project continued thru 18 June 2019.

All these research efforts were successful and the equations discovered are presented in these next several reports. These next several reports are technical in nature. Never-the-less they only require a knowledge of second semester calculus to understand the equations presented and even less to follow the discussions.

Notice To Academic PhD's

As very clearly stated in the Introductions, Sections 1.2, their last paragraphs, in each of the next four chapters, this research and work was NOT "theoretical" based work. This research work and the resulting equations discovered were the result of correlative trial and error searches. That is; <u>This work is independent and free standing!</u>

To be utterly clear, this work does NOT start by expounding a "theory", because it is NOT a "theoretical" based work. This work does not jumper off what some esteemed professor emeritus at some prestigious university thought, said, wrote, or did.

Likewise this work does NOT use insider trade jargon as used by PhD high energy physicists. The reader does NOT need to have spent the last 11.526 years of their lives studying gauge symmetries, the Clebsch-Gordon Coefficient, the Yang-Mills Theory, the Kaluza-Klein Theory, nor any other such insider particle physics concepts and mathematics. Nor does the reader need to know anything about Hilbert space, Banach spaces, ladder operators or other such insider trade jargon used by mathematicians.

General Notes - Definitions

There are some general notes which apply to all the chapters / reports in this Part of the book. Further, these notes apply to and should be remembered for all Parts of this book. In this work the following definitions or meanings are used:

1 Lepton(s) -- Means the the electron family; the electron, the muon, and the tau, or their charge reversed counterparts of the positron family.

It does NOT mean the electron family plus the neutrinos in this work.

2 Unit(s) -- Means measurements units, either; the relative SI units, the absolute Squigs units, or generic, meta, place-holder, universal, or as yet unspecified parametric units. These are always static quantities, "blobs" of something, and not quantities plus motion to make dynamics.

It does NOT mean the number of decimal places or digits in a number. The author gets tired of having to repeat the word measurement before the word units.

3 Dimension(s) -- Means spatial and temporal dimensions.

It does NOT mean variables, parameters, measurement units, or the number of arguments for a mathematical function or expression. It does not mean the parameters which are often grouped together in engineering, scientific, and technical work under the heading of "Dimensional Analysis" or to make "Dimensionless Numbers". The author gets tired of having to repeat the word spatial before the word dimensions.

A general reminder is needed that the world size realm of George Johnstone Stoney and the particles, the electron family, is at a scale 36 orders of magnitude smaller in distance than humans and 44 orders of magnitude smaller than the human invented second. The electron is 33 orders of magnitude smaller in mass than a human and the quarks appear to inhabit a world of 4 spatial dimensions. Futher the little critters of investigation are really only just wave forms or energy bodies and do not really have any "solid" form. Assuming or trying to impose laws and physical property inter-relations upon them based upon the human world experience and mechanics is a seriously dubious proposition.

There are several basic physical constants that are used in this work, specifically throughout PART 1 in all of its reports.

Table 1 Basic Physical Constants Used In This Work [1]

FUNDAMENTALS, a-priori	UNITS	NUMERICAL	ERROR
G, gravitational constant	(m/kg) x $(m/s)^2$	6.672,59 x 10^{-11}	8.5 x 10^{-15}
ε_o, electrical constant	C^2/(kg m) x $(s/m)^2$	8.854,187,817 x 10^{-12}	0
μ_o, magnetic constant	(kg m)/C^2	1.256, 637,061 x 10^{-6}	0
DERIVABLE, but used as a-priori			
e, electron charge	C	1.602,177,33 x 10^{-19}	4.9 x 10^{-26}
α, fine structure scaling constant	$((ML)(L/T))^{-1}$	7.297,353,08 x 10^{-3}	3.3 x 10^{-10}
DERIVATION OBJECTIVES			
electron mass	kg	9.109,389,7 x 10^{-31}	5.4 x 10^{-37}
	MeV/c^2	0.510,999,06	1.1 x 10^{-7}
muon mass	kg	1.883,532,7 x 10^{-28}	1.1 x 10^{-34}
	MeV/c^2	105.658,389	3.4 x 10^{-3}
tau mass	kg	3.167,88 x 10^{-27}	5.2 x 10^{-31}
	MeV/c^2	1,777.05	+0.29, -0.26
DERIVABLES			
c, speed of light	m / s	2.997,924,58 x 10^{8}	0
h, Planck constant	(kg m) (m/s)	6.626,075,5 x 10^{-34}	4.0 x 10^{-40}
CALCULATED, for scaling	FORMULAS	NUMERICAL	
G, see Lepton Report, Sec 4.1	G_improved	6.672, 590, 32 x 10^{-11}	
L_absolute / Q_relative	$\mu_o (G \varepsilon_o)^{1/2}$	3.054,438,950 x 10^{-17}	
meter per L absolute, (m / l_{Sgs})	$e\mu_o(G \varepsilon_o)^{1/2}$	4.893,752,96 x 10^{-36}	
Conversion, (ML^2/T) rel / abs [kgm (m/s)] / [m_{Sgs} l_{Sgs} (l_{Sgs}/t_{Sgs})]	$e(\mu_o / \varepsilon_o)^{1/2}$	9.670,562,404 x 10^{-36}	
MeV per kg	$1/(10^6 \mu_o \varepsilon_o e)$	5.609,586,16 x 10^{29}	
l_{Sgs} x MeV/C^2	$(G/\varepsilon_o)^{1/2} / 10^6$	2.745,192,89 x 10^{-06}	
1 / (2α)	(ML)(L/T) abs	6.851,799,475 x 10^{1}	

[1] E.R. Cohen, The Physics Quick Reference Guide, AIP Press, 1996, p.54-56

There are several items to be noted in this table.

1 In these reports and in the other parts of this work, the units listed with numerical quantities intentionally have the grouping for velocity $(L/T)^{\pm n}$ or specifically $(m/s)^{\pm n}$ isolated from the other units where possible for conceptual reasons.

2 The listing of units with α are intentional. These units have been rigorously derived (proven) in Part 3, Chapter 3.4 Systems Analysis I, Section 5. As clearly demonstrated this constant is actually the result of the value of *h* with relative units of (kgm^2/s) which has been imported into the system of absolute physics Squigs scales. This constant α can be rigorously shown to have the absolute units of (ML^2/T) or specifically $(m_{Sgs} l_{Sgs}{}^2/t_{Sgs})$ for SI based humans. This is with the quantity $e^2(\mu_o/\varepsilon_o)^{1/2}$, in this specific case, being a conversion constant from the absolute to the relative with units of [kgm (m/s)] / $[m_{Sgs}\ l_{Sgs}\ (l_{Sgs}/t_{Sgs})]$. Generalizations from this specific case should not be made to other usages of the quantities e, μ_o, and ε_o.

3 As is obvious the relative SI unit of kg comes with the listed masses of the electron, muon, and tau. These units cannot be thrown away, no more so than the measurement units of any other data can be thrown away. They are part of the data.

The following unit designations are used in this and the other parts of this work.
Relative units are from the SI set of common measurement scales.

distance-length,	meters (m)
duration-time,	seconds (s)
mass,	kilograms (kg)
charge,	coulombs (C)

There are, absolute or "natural" physics scales appropriate to this sub-sub-atomic scale of distance and time. These are the Squigs scales based upon the measurement units put forth by George Johnstone Stoney in 1874. Except the Squigs scales have had his assumed 2 or 3 dimensional π constants removed.

Absolute units are from the Squigs set of "natural" scales. These scales are defined as follows.

Table 2 Definition Of Absolute Physics Measurement Units

		Input -- Exponents				Derived -- Exponents				1 Squigs or Absolute Unit	
Quantity	Symbol	of Unit Combinations				of Force Constants				= n Common or Relative Units	
		L	T	M	Q	G	ε_o	μ_o	e	n	reciprocal
Length	l_{Sgs}	**1**				**0.5**	**0.5**	**1**	**1**	4.893753×10^{-36}	$2.043422 \times 10^{+35}$
Time	t_{Sgs}		**1**			**0.5**	**1**	**1.5**	**1**	1.632380×10^{-44}	$6.126024 \times 10^{+43}$
Mass	m_{Sgs}			**1**		**-0.5**	**-0.5**		**1**	6.591572×10^{-09}	$1.517089 \times 10^{+08}$
Charge	q_{Sgs}				**1**				**1**	1.602177×10^{-19}	$6.241506 \times 10^{+18}$

Absolute distance = Squigs distance, $l_{Sgs} = G^{0.5}\ \varepsilon_o{}^{0.5}\ \mu_o{}^1\ e^1$ = 4.893,753 x 10^{-36} common meters
Absolute time = Squigs time, $t_{Sgs} = G^{0.5}\ \varepsilon_o{}^{1}\ \mu_o{}^{1.5}\ e^1$ = 1.632,380 x 10^{-44} commom seconds
Absolute mass = Squigs mass, $m_{Sgs} = G^{-0.5}\ \varepsilon_o{}^{-0.5}\ \mu_o{}^{0}\ e^1$ = 6.591,572 x 10^{-09} common kilograms
Absolute charge = Squigs charge, $c_{Sgs} = G^{0}\ \varepsilon_o{}^{0}\ \mu_o{}^{0}\ e^1$ = 1.602,177 x 10^{-19} common Coulombs

Finally generic, meta, place-holder, universal, or as yet unspecified parametric units, either relative or absolute, are designated as follows.

distance-length	L
duration-time	T
mass	M
charge	Q

These Reports Cover Material as Follows:

Chapter 1.1 A Model For Determining Physical Properties I: Properties Of Leptons
This report demonstrates a mathematical model for calculating the elementary charge of the leptons and for the masses of all three of the known leptons.

Chapter 1.2 A Model For Determining Physical Properties II: Properties Of Photons
This report demonstrates a continuation of the mathematical model for calculating physical property information for all the elementary electromagnetic waveforms in a very analogous manner, for both the leptons and the photons alike.

Chapter 1.3 A Model For Determining Physical Properties III: Charge Of The Quarks
This report demonstrates a mathematical model relating the ± 2/3 and ± 1/3 charge of the quarks to the fixed curvature of specific 4 dimensional curved vector forms.

Chapter 1.4 A Model For Determining Physical Properties IV: Ternary Force Interaction Constant
This report shows how there is a Ternary Force Interaction Constant which appears to underlie the structures discovered for the elementary electromagnetic waveforms.

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CHAPTER 1.1 A MODEL FOR DETERMINING PHYSICAL PROPERTIES I: PROPERTIES OF LEPTONS

1 Introduction

1.1 Abstract

Equations have been found which explain some of the physical properties of the leptons, to a decimal accuracy matching that of their measurement. These equations set logical patterns and can be explained in terms of easy to understand three dimensional geometric pictures. This mathematical model is not derived from any hypothesis but was developed from a data correlative approach. It contains several new mathematical constants. Anyone familiar with the quantum mechanical description of the hydrogen electron shells can easily use these analogous equations to derive the masses of the three leptons; the electron e with m_e = 9.109,389,7 x 10^{-31} kg, the muon μ with m_μ= 1.883,532,7 x 10^{-28} kg, and the tau τ with m_τ = 3.167,88 x 10^{-27} kg. Additionally, the charge of the leptons, e = 1.602,177,33 x 10^{-19} C, can now be understood as arising from the curvature of certain energy structures for these particles, when formulated in terms of vectors. The model also predicts that an overlooked fourth member of the lepton family, the shipa, with a net positive mass is mathematically possible.

1.2 Objective & Scope

The general objective of this report is to show observed mathematical organizing principles and patterns for the basic subatomic particles. The specific objective of this work is to demonstrate a mathematical-geometric model that allows the determination of the physical properties of the three known leptons: electron e, muon μ, and tau τ.

The formulas discovered are simply presented here. Their parts (factors) are described and analyzed to ensure understanding of their correct calculation and use. This work is best described as a mathematical analysis of some very limited and exact data, which has resulted in an applied mathematical model. This presentation should be thought of as only a demonstration. The descriptive equations found are not proven nor derived in the formal or rigorous mathematical sense of those words.

The model discovered was not derived from a hypothesis but was the result of a correlative approach. The model presented here does not start with a hypothetical platform, does not specifically support any particular hypothesis, has not been embedded into any pre-existing hypothesis, nor has a hypothesis been constructed around it. While there may be implications concerning particle hypotheses indirectly supported by this work, only those conclusions directly supported by the mathematics of the equations that were found are reported. Implications of this work for the multitude of current grand particle hypotheses are beyond the scope of this work.

1.3 Nomenclature

The following nomenclature is used in this report. It is included here for ease of reference. Also see Table 1 in the introductory pages at the beginning of Part 1.

A = Charge correlation constant, Q^2 relative / L radial_absolute, specifically C^2/l_{Sgs}
$A_i(t_\theta)$ = Angular inner or implicit function within the angular mass density equation
A_{osf} = Angular outer function within the angular mass density equation
C_g = General mass correlation constant, generic absolute distance units, specifically l_{Sgs}
C_p = Individual particle constant, unitless
C_{rk} = Initial radial constant for shell k
$C_{\theta k}$ = Initial angular constant for shell k
D_p = Mass density of particle, M relative / L radial_absolute, specifically kg/l_{Sgs}
$D_k(r)$ = Radial mass density of particle shell k

$D_k(\theta)$ = Angular mass density of particle shell k
F_c = Constant factor for particle
FHDif[F(r)]= The Fraunhofer Diffraction Function, of F(r)
F_m = Series member factor for particle
F_s = Shielding or mass defect factor for particle
I(r) = Initial radial condition
I(θ) = Initial angular condition
Integ$D_{pk}(r)$ = Integral of $D_{pk}(r)$ w/o C_{rpk}
Integ$D_{pk}(\theta)$ = Integral of $D_{pk}(\theta)$ w/o $C_{\theta pk}$.
$L_n^d[F(r)]$ = the d^{th} derivative of the n^{th} Laguerre Orthogonal Polynomial, of F(r)
M_p = Mass of particle, kg
R_{csf} = Contractive factor within the radial mass density equation
R_{esf} = Expansive factor within the radial mass density equation
$R_c(t_r)$ = Contractive implicit function within the radial mass density equation
$R_e(t_r)$ = Expansive implicit function within the radial mass density equation
R(t) = Vector expression for a generalized cylindrical function or figure
S_k = Shell correlation constant, unitless
t = Generic temporal variable of vector charge expressions
t_r = Radial parameter of mass density expressions, radial time
t_θ = Angular parameter of mass density expressions, angular time
$T_n^\dagger[F(\theta)]$ = n^{th} Chebyshev T Orthogonal Polynomial, shifted, of F(θ)
α = Fine structure constant, unitless or with units of (ML)(M/T), specifically $(m_{Sgs} l_{Sgs})(l_{Sgs}/t_{Sgs})$ as is appropriate to the expression involved
κ = Curvature of a generalized cylindrical figure
ρ = Radial parameter of vector charge expressions
τ = Torsion of a generalized cylindrical figure

1.4 Background

The Standard Model of particle physics has been overwhelmingly verified and yet there are still major outstanding questions. The Standard Model has not and due to its inherent nature can never explain the masses of the elementary particles such as the neutrinos, leptons, and quarks [1,2]. Likewise, the Standard Model cannot explain the occurrence of multiple generations of particles, whether this number is 2, 3, 4, or anything greater than one [1]. Tens of thousands of high energy particle physicists have worked on these problems and searched for ways to explain them by extending the Standard Model. This is by including it as a subset of broader schools of thought such as those of super symmetry [2], super string-membrane, or super gravity. Unfortunately, despite these efforts involving tens of millions of research hours and hundreds of millions of dollars, this most basic and first measured property of all subatomic particles, mass, still remains unexplained. In short, there is no generally accepted method of calculating the masses of the elementary particles, much less one which has the required accuracy of 7 and 8 decimals.

2 Outline Of Work

2.1 Fundamental Approach & Assumptions

The primary unique feature of this work, to explain the masses of the leptons, was to start with the data and work upward toward any generalizing principles which may have become obvious. First mathematical-geometric correlations for the masses of the leptons were to be found. Correlation constants were to be added to make actual predictive equations. Then last these equations were to be placed within any broader body of mathematics of which they may be a part. By following this path then maybe those hypotheses which are applicable would become self evident. Additionally this approach guaranteed that any applicable hypotheses would be linked to the data and that the exact equation path of this linkage would be defined.

Regardless of whether a hypothesis-first or a data-first correlative approach is used, some assumptions are necessary to set a context or framework for the work. For this work an assumption was made that the subatomic particles are wave structures, and that these structures are responsible for their many observed and measured properties. Stated as the following hypothesis;

1 All objects in the consensus physical world have a form or structure. This includes the elementary particles which are the objects of discussion of physics.

2 There are no formless particles nor any particles that are mathematical points.

3 The form or structures of the basic objects of the physical universe, subatomic particles, can be described by appropriate mathematical-geometric equations.

4 Further these wave structures or "objects" can be described via mathematical-geometric equations not just in general, but precisely, to as many decimals as necessary.

The three forces applicable to the leptons were assumed to be a-priori to all else. That is; the values of G, ε_o, and μ_o were to be the only basic starting values. The values used are listed in Table 1 at the beginning of Part 1. These ultimately would be used to scale or bridge from the world of pure mathematical equations and geometry to the scale of the consensus world of physics and humans. The remaining basic values found in physics reference books such as e, h, α were felt to be derivables, and would only be used in the scaling of the lepton masses, should they become necessary and not create circular references or circular derivations. As the mathematics became exposed e and α were found to be necessary.

Stated differently, the six basic forces; the unary set gravity, the binary set electro-magnetism, and the ternary set blue-green-red do not depend on the particles for their values, but the particles definitely require these forces for their geometric structures. That is; gravity does not require an electron, but an electron depends on gravity for its existence. The implications of this decision are seen in the discussion of the accuracy of the equation describing the charge of the leptons.

2.2 Criteria

As detailed in other reports in this book, the equations developed-discovered in this work were the result of 12 1/2 years of mathematical trial and error research efforts. As clearly stated in the Objective and Scope, Section 1.2, this is not a hypothetically based work. The equations which follow in Section 4 in all its subsections and in Section 5 do not flow from a hypothesis and are not derived nor proven in the formal sense of those words.

Never-the-less the formulas are constrained by a rigid set of criteria also detailed in other reports. Probably the six most important criteria for the mass density and charge equations which need to be repeated here are.

1 The equations must calculate their objective quantities to many decimals of accuracy, preferably matching that of their measurement.

2 The equations must set or form interlinked patterns.

3 The equations must be explainable. Every part of the equations, factors in this case, must be identified as having some plausible origin which relates them to already well understood and accepted analogous mathematical-physical phenomena.

4 The inherent nature of such equations is that they must be particle centric. That is, the equations only refer to the waveforms of concern, their intrinsic nature, and do not refer to the background, machine parameters, reaction products, scattering matrixes, etc. Further, the mathematics of an individual member within the lepton class of particles must not have variables dependent upon or entangled with the mathematics of the other members from within that class.

5 The equations must not either predict or invoke the use of anomalies at odds with known science or physical common sense, such as the use of 11 or 26 spatial dimensions.
6 The equations must not contain any arbitrary constants.

2.3 Key Mathematics Used

To explain the objective particle properties of charge and mass two bodies of mathematics are needed. Vector mathematics in rectilinear coordinates is needed to describe what could be called the encapsulated electromagnetic force or the entrapped energy of the particle as charge in coulombs. Regular or scalar mathematics in polar coordinates is needed to describe what could be called the stabilized gravitational force or the contained energy of the particle as mass in kilograms. Since these two mathematical descriptions are just two different conceptual views of the same objects, as expected there are many features in common between the two descriptions.

The last time scientists had thoroughly inundated themselves with the discovery of a zoo of basic particles, was with the elements of the periodic chart. Then humans were forced once again to conceptualize about things that they could not directly physically examine. A sense of order was restored by the development of quantum mechanics. In that application of mathematics to explain a wealth of physical phenomena, second order differential equations and their solutions as the Laguerre and Legendre orthogonal polynomials became the working tools. These two series explained the patterns which described the repeating rows and columns of the periodic chart. Here in this work, again orthogonal polynomials, the Laguerre series, are found to bring order or mathematical sense to a repeating series of elementary object masses.

Since mathematically the Laguerre polynomials are an open ended series, higher members past where the pattern for the electron, muon, and tau stop were examined. A low energy fourth member of the lepton family, the shipa, was found to be mathematically possible, although it probably has an extremely short half life due to angular instabilities. Also while this potential particle has a net positive mass, one of its radial shells results in a negative value. Scientists can question whether a stable particle is possible at all. After the shipa the mathematics indicated that further members of the series resulted in negative masses or unstable energy patterns.

2.4 General Features Of Mathematical Model Of Mass Density Equations Of Leptons

The neutrinos, leptons, and quarks of physics are of a scale so small that they are completely out of touch with not only the human senses, but also with all of the machine extensions of our senses. These "objects" are so out of scale with the human ability to sense and measure that scientists held for many years that these particles are mathematical points, dimensionless, formless, and structureless. Unfortunately this view eliminated all of the common practical mathematical avenues for analyzing these particles. Here by assuming the particles are wave patterns or forms, relatively simple mathematics can be used to describe them. In fact the mathematical model found is not only conceptually simpler than the quantum mechanical model of the periodic chart but at most requires only a knowledge of second semester calculus to follow.

The best known mathematical series describing components of the physical world is the Periodic Table of the Elements of Chemistry (PTEC). In this setting quantum mechanics offered a conceptual tool to describe the existence of things that were many orders of magnitude out of scale with the human form. This mathematical field gave descriptions not just in general, but gave very exact predictions of how forms behaved at the scale of concern. Here mathematics strikingly similar to quantum mechanics have been found which explain an even smaller scale of forms, the leptons. The major elements of the mathematical model which was found for the leptons are as follows;

1 Radial mass density equations (2 dimensional, planar), based on the Laguerre orthogonal polynomials.

2 Angular mass density equations (a single angle), based on the Chebyshev $T^{\dagger}$ orthogonal polynomials.
3 A series of shells for the higher members of the series, based on Laguerre polynomial derivatives.
4 Embedded or implicit parameters in both the radial and angular equations. In this work these implicit parameters are viewed as both being temporal representations, and further that they are independent of each other.
5 Initial temporal conditions for both the radial and angular equations, which lead to initial multiplying factors or constants.
6 A general scale factor or correlation constant for all the particles, composed of basic a-priori measured physical constants.
7 Several specific scale factors for each member of the series. These factors set definite patterns or form series themselves.
8 An overall equation combining the radial equations, angular equations, and the final scale factors as multipliers.

2.5 Geometric Appearance Of The Leptons

The general geometric appearance found for the leptons in this work is that of a toroidal coil; mathematically a cylindrical helix, which is wrapped around into a circle to form the outline of a donut, mathematically a torus. This appearance is the same as that of a photon which instead of propagating linearly through space-time, has its head wrapped around and connected to its tail, and goes forever in a tight little circle, like a dog chasing its tail.

Viewing this geometric picture more rigorously, the mass density has two spatial dimensions, one radial-planar, and one angular. Likewise it has two temporal dimensions, one radial-planar, and one angular. This is to say that for each spatial dimension there was found to be an independent embedded or implicit temporal dimension underlying it. This radial-planar structure then proceeds into the third dimension with yet a third independent variable of consensus time to make the net space-time figure a three dimensional toroidal coil as described.

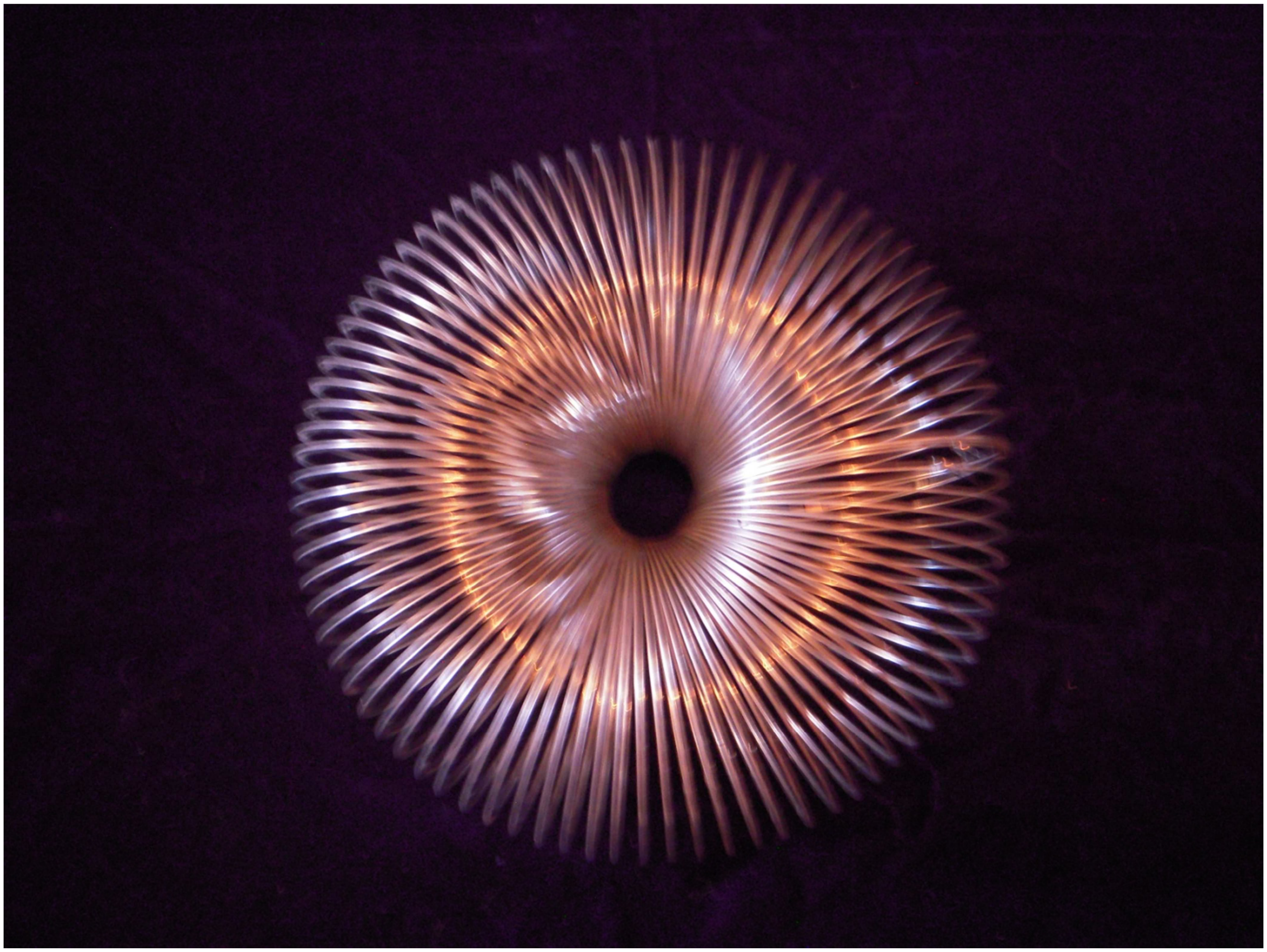
The general toroidal coil description of the mass density appearance of the leptons can be made mathematically specific, using 3 dimensional vector notation of an implicit function in time, as follows. Consider a generalized cylindrical figure:

$$\mathbf{R}(t) = a\cos[F(t)]\,\mathbf{i} + a\sin[F(t)]\,\mathbf{j} + bF(t)\,\mathbf{k} \qquad (01)$$

If $F(t) = \lambda t$, then this form is simply the cylindrical helix; the outline of an open ended, unlimited, or moving figure. This is the energy pattern of the photon.

If $F(t) = d_1\cos(e_1 t)$, then this form is the outline of a cyclic, bounded, or stationary figure, the torus or a toroidal coil. Viewing $d_1\cos(e_1 t)$ as a trigonometrically substituted Chebyshev $T_1^{\dagger}$polynomial, this form outlines the energy pattern of the electron. If $F(t) = T_3^{\dagger}[d_3\cos(e_3 t)]$ and $T_5^{\dagger}$ $[d_5\cos(e_5 t)]$, then this form outlines the general energy patterns of the muon and tau respectively.

PICTURE 1 SNAPSHOT OF A HIGHLY MAGNIFIED FROZEN ELECTRON



2.6 Brief Discussion Of Units Involved In The Equations

As discussed in Section 2.4 the general mathematical correlative form used here for the the mass of the leptons is to decompose their accepted numerical values with their units into two parts. One part is mathematical-geometric. This part may have universal, meta, or place-holder units attached. These could be measures of distance-length, duration-time, mass, and/or charge (L, T, M, Q) which can then be placed on the absolute and relative systems of physics scales created by humans or of any other species of inquiring beings. The second part of the equations is a scaling constant with its units from some actual absolute system of scales. For this work the absolute physics Squigs scales (Squigs l, t, m, and q) are used. These Squigs scales are based upon the measurement units put forth by George Johnstone Stoney in 1874. Except the Squigs scales have had his assumed 2 or 3 dimensional π constants removed. These scales are defined, derived, discussed, etc in detail in the reports Measurement Systems Bases and Analyses of Measurement Systems I, II & III. This same two part decomposition approach is also used with the correlative equation for the charge of the leptons.

The equations describing physical properties found throughout this report, in the next two reports concerning the photons and concerning the ternary force interaction constant can only be called equations because of the inclusion of these and similar scaling constants. Without these scaling constants the mathematics discovered would remain just as correlations. While the nature of the correlative part of these equations, the math-geometry, would be interesting by itself it would remain in the nebulous and arbitrarily sized realm of conceptual mathematics. These scaling constants containing or converting universal measurement parameters into relative and absolute systems of units are absolutely necessary to bring these equations into the size of the consensus real physical world realm.

This conceptual point needs to be emphasized. The quantity A = $5.245,406,17 \mathrm{x} 10^{-03} Q^2 L^{-1}$, specifically C^2/l_{Sgs}, found below in describing the charge of the leptons, and the quantity of (M relative/L radial_absolute), specifically kg/l_{Sgs} used in the several sections describing the lepton masses are both referring to meta-units, place holders for mixed relative per absolute systems of scales. In this work these are made specific by use of the arbitrary, relative, or common SI scales ratioed by the absolute physics Squigs units.

In Analysis of Measurement Systems I, II, & III the numerical values of both the electron mass per radial distance used in this work and the electron charge squared per radial distance, A referred to above, are rigorously proven to be measurement system independent or universal, for SI analogous sets of units. The analysis done in these reports is completely independent from the mathematical calculus geometric approach found in this report. In Analyses of Measurement Systems I, II & III the mathematical implications inherent in any set or system of absolute, self consistent, comprehensive or all-inclusive, and interlocked physics scales describing the consensus universe are examined. The objectives of the derivations here, the equations and their resulting numerics, have been independently verified and are not just merely four coincidences of the creation of the relative SI set of units. Coincidences incidentally each having accuracy to the required number of decimals, or a collective accidental chance of 4 in $10^{8 \mathrm{x} 7 \mathrm{x} 7 \mathrm{x} 5}$ or 4 chances in 10^{1960}.

3 Mathematical Preliminaries

The generalized cylindrical figure

$$\mathbf{R}(t) = a\cos[F(t)]\,\mathbf{i} + a\sin[F(t)]\,\mathbf{j} + bG(t)\,\mathbf{k} \tag{02}$$

is important to this work. For such a vector the curvature and torsion are rigorously calculated as

$$\text{curvature } \kappa = \frac{|\mathbf{R}'(t)\times\mathbf{R}''(t)|}{|\mathrm{R}'(t)|^3} \tag{03}$$

$$\text{torsion } \tau = \frac{[\mathbf{R}'(t)\times\mathbf{R}''(t)\bullet\mathrm{R}'''(t)]}{|\mathbf{R}'(t)\times\mathbf{R}''(t)|^2} \tag{04}$$

and as such both are scalar quantities.

Calculating the quantities $R'(t)$, $R''(t)$, $R'''(t)$, κ , and τ of this most general form of R(t) results in messy and irreducible expressions in both G(t) and F(t) for both κ and τ. If the simplifying assumption is made that G'(t) = F'(t), then the results are the simple expressions below. See Appendix 4 for details.

$$\kappa = \frac{a}{a^2+b^2} \tag{05}$$

$$\tau = \frac{b}{a^2+b^2} \tag{06}$$

With this one restriction, that the implicit function G'(t) = F'(t), then the curvature and the torsion of this generalized cylindrical figure are found to be numerical constants, independent of the implicit variable t and all functions of F(t). Of course there are the implicit unstated assumptions that R′(t), R″(t), R‴(t) must exist.

In formalized mathematical or quantum mechanic jargon, this Curvature Operator would be said to be invariant under rotation, translation, substitution of F(t), et cetera. This invariance is of prime importance in the equation found which describes the charge of the leptons. The generic mathematical form $a / (a^2+b^2)$ also arises several times in the equations found for calculating the mass densities of the leptons, as well as in that of their charge.

Equally important, there appears to be no other mathematical quantity which remains constant as F(t) changes. The unit tangent **T**, principal normal **N**, and binormal **B** vectors all vary for the circular helix. None of the classical differential operators are satisfactory, regardless of whether they operate on scalars or on vectors. The gradient ∇, both the scalar and vector Laplacians ∇^2, the divergence ∇ •, and the curl ∇ × all remain functions of t or F(t), and most still have unit vectors **i**, **j**, **k** embedded in their formulas. Additionally the meaning of any candidate quantities needs to be considered. The curvature κ and the torsion τ both refer to the curve or curved surface itself. The unit vectors **T**, **N**, and **B** refer to something which is perpendicular to the surface. The elementary differential operators refer to fields perpendicular to the surface, fluxes through the surface, circulations in the surface, et cetera.

Equally of interest there can be 4 combinations of κ and τ if the signs of a and b are varied. If F(t) is a trigonometric cos() or sin(), then there can be 2 directions of travel around the toroidal coil appearance or 2 means of "revolving" about the center of the donut, according to whether b is positive or negative. Likewise, there can be 2 directions of rotation or spins about the centerline of this axis of revolution, according to whether a is positive or negative. If curvature κ is matched to charge, as is done in detail in the next section, and torsion τ to hand, then the following combinations are found.

spin a	revolve b	hand κ	charge τ
+	+	+	+
+	-	+	-
-	+	-	+
-	-	-	-

Clockwise and + can always be linked, likewise counterclockwise and -. These 4 combinations, possibly applicable to particles and their anti-particles, need to be reduced to 2 if space is allowed to freely rotate about the "objects" described by the vector **R**(t). By varying the signs of a and b there is now a means of directly relating observed properties such as charge and hand to mathematical features of proposed wave structures for these particles.

For the purposes of this work the equation for the cylindrical figure needs to be generalized one step further, by specifying the explicit or external functions as follows:

$$\mathbf{R}(t) = a\, T_n^{\dagger}(\cos[F(t)])\, \mathbf{i} + a\, T_n^{\dagger}(\sin[F(t)])\, \mathbf{j} + bF(t)\, \mathbf{k} \qquad (07)$$

For this form the curvature for odd n is:

$$\kappa = \frac{n^2 a}{n^2 a + b^2} = \frac{a}{a^2 + b^2/n^2} \qquad (08)$$

Likewise τ can be reduced back to an analogous simple form.

4 Applications To Physical Property Determinations

4.1 Charge Of The Leptons

The final form of the equation discovered which describes the elementary charge of the leptons is the starting point. Again the reader is reminded that this equation is the result of years of mathematical correlative developmental effort and did not derive from a hypothetical approach. No hypothesis is expounded upon here before the equation is presented. This equation is then parsed and the factors of which it is composed are discussed.

$$e = [\ \mu_o (G\ \varepsilon_o)^{1/2}\ ]\ A, \text{ units of Q relative, C} \tag{09}$$

Reviewing the definitions of the absolute units in the Introduction to Part 1, $[\ \mu_o (G\ \varepsilon_o)^{1/2}\ ]$ is easily seen to have the numerical value of 3.054,438,950 x 10^{-17} and the units of L_absolute / Q_relative or specifically l_{Sgs}/C. Here A is a geometric constant, with the mixed relative per absolute physics units $C^2\ l_{Sgs}^{-1}$. This equation is in terms of consensus real world physical units, and is not in terms of probability space, momentum space, or other such conceptual or mathematical spaces.

A is an interesting constant that can be calculated or decomposed as follows:

$$A = (2\pi)^{-3/2} \times (\pi\ \rho^2) = 1/2(2\pi)^{-1/2}\ \rho^2 \tag{10}$$

Here numerically ρ is a constant which was found to be:

$$\rho = 6/(6^2 + 1^2) \tag{11}$$

which has the generic geometric form of $a / (a^2 + b^2)$ which becomes significant shortly.

Using this numerical decomposition of A results in, A = 5.245,406,17 x 10^{-03}. Using this value of A and the values of the three force constants, listed in the Introduction to Part 1, the value of e can be calculated by using Equation (09). The calculated value of e_{calc}= 1.602,177,29 x 10^{-19} C as compared with $e_{measured}$ = 1.602,177,33 x 10^{-19} ± 4.9 x 10^{-26} C and finally the ratio of the measured to calculated is

$$\frac{e_{measured}}{e_{calc}} = 1.000,000,024 \tag{12}$$

This ratio between the measured and calculated value of e is about 24 parts in 1 billion and well within the experimental error.

The derivation or origins of A are believed to be as follows. The constant ρ in the factor A represents the curvature κ, or the torsion τ of the cylindrical figure

$$\mathbf{R}(t) = a\ T_n^{\dagger}(\cos[F(t)])\ \mathbf{i} + a\ T_n^{\dagger}(\sin[F(t)])\ \mathbf{j} + bF(t)\ \mathbf{k}, \text{ for odd n} \tag{13}$$

Here a = 6, b/n = 1, and F(t) is the trigonometrically substituted, cosine, Chebyshev $T_1^{\dagger}$, $T_3^{\dagger}$, $T_5^{\dagger}$ polynomials for the electron, muon, and tau respectively. The 6 which occurs here as the amplitude coefficient of the planar **i** and **j** vectors is also found to occur in the planar radial equation that is part of the mass density or gravitational picture of the leptons.

Reviewing physics and engineering texts Fourier decompositions-transforms-integrals are found to be used very frequently when formalizing the discussions of wave patterns [3-4]. In quantum mechanics texts in particular, such discussions center around probability densities in some non-consensus space, such as momentum space. A classical example is the Fourier transform of Schrodinger's time dependent wave equation in momentum space, in which; $\Psi(r,t) = 1/(2\pi)^{3/2} \int$ (a bunch of stuff). There are a multitude of such examples in both theoretical and applied mathematical and engineering texts [3-4].

The origin of the $(2\pi)^{-n/2}$ for n = 1 or 3, can probably be assigned to either a 1 or 3 dimensional Fourier manipulation of **R**(t) in charge space.

A scientist can ask what does the implicit variable of time in the vector **R**(t) represent physically. Returning to an analogy with the photon, this variable is found to relate a measure of duration of events to position along the flight path of the particle. This same mathematical feature applied to the lepton would relate to events involving the circulation of the energy pattern around the center of the toroidal coil, the donut hole. This use of time or duration of events becomes important, for its absence, in the discussion of the mass density equations describing the leptons.

While these assignments, of this $(2\pi)^{-3/2}$ and the $(\pi\rho^2)$ where $\rho = a/(a^2 + b^2)$, a = 6 and b/n = 1 are not definitive, they are highly suggestive. While humans can physically experience velocity and acceleration, and can have an intellectual understanding of kinetic and potential energy as mathematical derivatives, they have not known the mathematical origin of their experience of charge. Here charge appears to be related to the square of the curvature or torsion of a generalized cylindrical wave expression which represents the electromagnetic structure of a particle. Curvature and torsion both involve first and second derivative expressions, the same as with the other mathematical expressions for energy. These expressions are invariant, as required. Finally, curvature and torsion, while although they are scalar quantities, are derived from vector expressions in space. This vector nature is in agreement with physics understanding of charge, and the electromagnetic fields and forces.

The equations used to derive the charge of the leptons involve exact analytical expressions. Uncertainty or the limits of accuracy is introduced through the physical constants used to scale these analytical expressions, from the world of mathematics to the consensus world of physics. Here the limiting constant is G, with certainty of only 3 decimals. This is not very satisfactory since e is measured to 6 decimals of certainty. There is the immediate impulse to rearrange the final equation,

$e = F_1(G, \varepsilon_o, \mu_o, \text{and geometry})$ to become $G = F_2(e, \varepsilon_o, \mu_o, \text{and geometry})$.

While the logic of this work requires that G not depend on the leptons for its existence, this rearrangement is desirable in that G can now be calculated to much greater accuracy than that to which it can be measured. In the next immediate derivations for the masses of the leptons, a more accurate value for G is highly desirable. This is the origin of the calculated values of

$G = 6.672{,}590{,}32 \times 10^{-11}$ m/kg $(m/s)^2$ and the conversion $m / l_{Sgs} = e\mu_o(G\varepsilon_o)^{1/2} = 4.893{,}752{,}96 \times 10^{-36}$

shown in the Introduction to Part 1. Accepting this improper but necessary logic, this calculated value of G is then used in the equations for the lepton masses through the overall scale factor of distance absolute.

4.2 Masses Of The Leptons

4.2.1 General Features Of Equations

The general or generic form of the mass density equations developed for the leptons are shown below. The specific detailed applications of these equations for the electron, muon, and tau are given in Subsections 4.3, 4.4, and 4.5, respectively. The numerical results of using these equations are shown in Tables 1 through 6. Since this work is a mathematical endeavor, these results are presented so that the reader can reproduce, verify, and validate the "experimental" findings, if they so choose, before beginning any discussions as to their meaning. Likewise in the tables nine decimals are intentionally carried so that questions of computer calculation abilities and programming techniques can be settled. Table 6 compares the ultimate calculated masses of the leptons with their empirically measured values.

Again the reader is reminded that the equations here and throughout the remainder of Sections 4 & 5 are the result of 12 1/2 years of mathematical correlative developmental effort and do not derive from a hypothetical approach. Again no hypothesis is expounded upon here before the equation is presented.

All the equations in Sections 4 & 5 are in terms of consensus real world physical units, and are not in terms of probability space, momentum space, or other such conceptual or mathematical spaces.

Beginning with the overall or final equation for calculating the mass of a lepton particle, m_p:

$$m_p = C_g C_p D_p, \text{ units of M relative, kg} \tag{14}$$

where C_g is a general correlation constant or an absolute scaling constant.

$$C_g = e\mu_o(G\varepsilon_o)^{1/2} = 1.0 \text{ L\_absolute} = 1.0\ l_{Sgs} = 4.893{,}752{,}96 \times 10^{-36}\text{m} \tag{15}$$

C_p is the unitless individual particle constant; and D_p is the mass density function for the particle. The units of the relative measure of mass, kg, per absolute length, l_{Sgs}, have been ascribed to the ultimate result of the Dp factor of this overall equation.

For the electron C_p is simply 1.0. For the higher members of the lepton series C_p is composed of three factors as follows:

$$C_p = F_c F_{mp} F_{sp} \tag{16}$$

F_c is a constant factor. F_{mp} is the series member factor for the particle. F_{sp} is a shielding or mass defect factor for the particle. The rational for these last two factors is discussed in Section 6.4.

$$F_c = \frac{1}{2\alpha} \tag{17}$$

$$F_{mp} = \left[\frac{a}{a^2+b^2}\right]^2 \tag{18}$$

where a = 6 and b = $(n-1)^{1/2}$, and n is the number of the particle in the series. F_{sp} is best illustrated by the specific examples of the particles themselves and the discussions in Section 6.4.

Now for the important and crucial findings of this work, the mass density functions. The mass density function for the particles D_p can be determined by summing across the radial and angular mass density functions for each shell that a particle may have.

$$D_p = \sum_{k=1}^{n} S_{pk}\, D_{pk}(r) D_{pk}(\theta) \tag{19}$$

where all three factors S_{pk}, $D_{pk}(r)$, and $D_{pk}(\theta)$ are specific to the particle, p, and the shell, k, that is being calculated. S_{pk} is a unitless shell correlation constant illustrated with the specific particles and discussed in Section 6.4. $D_{pk}(r)$ is the radial mass density function and $D_{pk}(\theta)$ is the angular mass density function. As is soon seens in the specific examples of the leptons all three of these factors form unique patterns or mathematical series. The specific features of the radial and angular functions are discussed in detail in Section 6. Graphical presentations of both the radial and angular functions for each lepton can be seen between Sections 5 and 6.

One of the characteristic features of this work is the embedded or implicit parameters. Initially a person might think that these could be viewed equally well as representing either spatial or temporal parameters. For this work the temporal interpretation is used. The reasons for this are discussed in the

Discussions, Section 6. These temporal parameters tend to stand out as different from the strictly spatial parameters found with the wave equations describing the electron shells of the hydrogen atom. Here embedded within both the radial and angular spatial mass density functions there are radial and angular implicit functions. With these implicit variables representing temporal parameters then there are some profound implications for the human concept of time, as is discussed later.

4.2.2 Radial Features Of Equations

Analysis of the radial equations shows that ultimately all of the several factors making up the radial mass density function $D_{pk}(r)$ are dependent variables originating with independent or implicit variables of time, t_r. Within the overall radial mass density function there are two different implicit functions, $R_1(t)$ and $R_2(t)$, both referring to the same ultimate variable which is called t_r, radial time, in this report. More realistically these functions should be labeled for the parts which they play in the overall radial equation. In this work $R_1(t_r)$ as used always helps create a contractive factor within the overall radial mass density function. That is, $R_1(t_r)$ is the simple monomial $R_1(t_r) = -6t_r^2$, and is always used as the argument of an exponential factor, R_{csf}. This implicit function is labeled as $R_c(t_r)$, for R_contractive, representing its part in a contractive wave pattern or force. This contractive factor within the radial mass density equations for all the leptons is identical:

$$R_{csf} = F(R_c(t_r)) = e^{(R_c(t_r))} = e^{(-6t_r^2)} \tag{20}$$

The other implicit radial function $R_2(t_r)$ is truly what is typically thought of as a function and helps create an expansive factor within the overall radial mass density function. This function is labeled as $R_e(t_r)$, for R_expansive, representing its part in an expansive wave pattern or force. In the overall radial mass density functions there are two factors using this same implicit variable $R_e(t_r)$ as their argument, an exponential function R_{esf} and a polynomial function. This $R_e(t_r)$ argument is the same for all the leptons. It represents the distance along the parabolic curve $\left(2\pi t_r^2/k^{1/2}\right)$. This curve describes the area as a radius progresses outward across a uniform "gray" disk, or can also be thought of as the area of an expanding circle in time weighted by $2/k^{1/2}$. This is the geometry of the mature radial energy condition in time. Note, this is the instantaneous distance, not cumulative, along the curve, not to the curve. Specifically:

$$R_e(t_r) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} ds\left(\frac{2\pi t_r^2}{k^{1/2}}\right) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}}\left[1 + \left(\frac{4\pi t_r^1}{k^{1/2}}\right)^2\right]^{\frac{1}{2}} dt_r \tag{21}$$

where k is the Fraunhofer Diffraction Constant as defined immediately below.

The expansive polynomial functions are the Laguerre orthogonal polynomials: $L_0^0(R_e(t_r))$ for the electron; $L_2^0(R_e(t_r))$ and $L_2^2(R_e(t_r))$ for the two shells of the muon; $L_4^0(R_e(t_r))$, $L_4^2(R_e(t_r))$, and $L_4^4(R_e(t_r))$ for the three shells of the tau.

Another unique feature of this work, in the realm of particle physics, is the occurrence of initial conditions for both the radial and angular mass density equations. In applied engineering texts initial conditions are a common feature used to make the solutions to second order differential equations specific to the application being discussed. Initial conditions might be expected here where both the radial and angular mass density equations appear to be the solutions to second order differential equations, which could be both space and/or time dependent.

In the radial expression for all the leptons, the initial condition in radial space or time is the same:

$$I(r) = \text{FHDif}[F(r)] = \left[\frac{2J_1[F(r)]}{F(r)}\right]^2 \tag{22}$$

where FHDif[F(r)] is the Fraunhofer Diffraction Function and J_1 being the Bessel Function of the First Kind, Order 1.

Specifically $F(r) = kr^1$, where $k = 1.697{,}525{,}53\ldots = \int_0^\infty \text{FHDif}\,[1.000{,}000, \ldots r^1]dr$ (23)

Note that $\int_0^\infty \text{FHDif}\,[kr^1]dr = 1.000{,}000, \ldots$ and as such is a self normalizing initial distribution.

See Born and Wolf [5] for the origins of the Fraunhofer Diffraction Function in its classical historic setting. There in Chapter 8, the details of this mathematical form are rigorously derived in terms of the parameters; k - the wave number, a - the aperture radius, and w - the radius of discussion across the pattern. For this work these parameters have been rolled together to become the monomial F(r).

From this initial condition, a boundary, initial, or normalizing constant needed to be built for the radial equations.

$$C_{rpk} = \int_0^\infty I(r)D_{pk}(r)dt_r \tag{24}$$

A useful reminder is that I(r), $D_{pk}(r)$ are ultimately functions of an implicit variable t_r, and this is the implicit variable which actually gets integrated here. The Fraunhofer Diffraction Function was shown in Equations (22) and (23) as a variable in r just to keep the similarity with its historical origin and the Born and Wolf text.

When looking at the overall numerical value contributed by the radial equation $D_{pk}(r)$ as a factor in the ultimate mass density function of the particles D_p, what would be nice is to further partition this quantity. Insightful information might be to find what portion of the radial contribution is derived from the initial constant factor C_{rpk}, and what portion is due to just the integration of the radial equations $D_{pk}(r)$ by themselves, where these equations are described with the individual particles in Sections 4.3, 4.4, 4.5, and Section 5. This second integral needs a name, as if it occurred in isolation, just as the initial condition has its own name C_{rpk}. This numerical portion of the overall $D_{pk}(r)$, is called Integ$D_{pk}(r)$ for integral, that is $D_{pk}(r)$ w/o C_{rpk}.

4.2.3 Angular Features Of Equations

The angular mass density functions $D_{pk}(\theta)$ need further minor explanation before going on to the specific usages with the leptons themselves. For the leptons there is only one angular spatial dimension, unlike the two angular spatial dimensions found with the hydrogenic electron shells. The integrated expression for each shell of each particle is multiplied by a common angular multiplying factor, the number 4, which is a composite. It is a product of a multiplier of 1/2 outside the Chebyshev polynomial, of 2 for angular symmetry of the integral about zero, and of 2 orthogonal forms being simultaneously applicable. This angular function is then repeated within the integral of the initial angular condition x the angular function, giving an overall common multiplier of 4.

The two orthogonal angular forms have the general form:

$$A_{osf} = F(\theta) = T_n^{\dagger}(\sin[\pi/2\ A_i(t_\theta)]) \tag{25}$$

where A_{osf} stands for Angular Outer Spatial Function and A_i stands for Angular Inner or Implicit function

$A_i(t_\theta) = T_n^\dagger(\sin[n^{-1}t_\theta])$ or $= T_n^\dagger(\cos[n^{-1}t_\theta])$ (26)

where t_θ means angular time.

In the angular expression for all the leptons, the initial condition in the angular space or time is the same.

$I(\theta) = \cos(\theta)$ (27)

Note that $\int_0^{\pi/2} \cos(\theta) d\theta = 1$ and as such is a self normalized initial distribution. This function is also a self normalized distribution as used as the argument for the interior or implicit $T_n^\dagger$ polynomial. The initial, boundary, or normalizing constants in the angular equations are calculated as:

$$C_{\theta pk} = \int_0^{\pi/2} I(\theta)\, D_{pk}(\theta) dt_\theta \quad (28)$$

A useful reminder is that $I(\theta)$, $D_{pk}(\theta)$ are ultimately functions of an implicit variable t_θ, and this is the implicit variable which actually gets integrated here. As was done with the Fraunhofer Diffraction Function above, the descriptions of the angular initial condition in Equation (27) were shown as spatial just to keep familiarity with typical trigonometric presentations.

When looking at the overall numerical value contributed by the angular equation $D_{pk}(\theta)$ as a factor in the ultimate mass density function of the particles D_p, what would be nice is to further partition this quantity. Insightful information might be to find what portion of the angular contribution is derived from the initial constant factor $C_{\theta pk}$, and what portion is due to just the integration of the angular equations $D_{pk}(\theta)$ by themselves, where these equations are described with the individual particles in sections 4.3, 4.4, 4.5, and Section 5. This second integral needs a name, as if it occurred in isolation, just as the initial condition has its own name $C_{\theta pk}$. This numerical portion of the overall $D_{pk}(\theta)$, is called $\text{IntegD}_{pk}(\theta)$ for integral, that is $D_{pk}(\theta)$ w/o $C_{\theta pk}$.

Finally, the appropriate normalizing factors for the Laguerre and Chebyshev T orthogonal polynomials are always used, except in the calculation of the initial radial and angular constants. These need to be remembered since their appearance has have been suppressed in all the equations above and those which follow. This was done so as to maintain the clarity and focus of the primary form and appearances of these equations. The effects of these normalizing factors are included in the tables.

4.3 Electron Specific Equations And Calculations

Finally the pay off of the equations of ultimate interest is reached. The following equations for the electron are obtained with (p = 1 or e) in the equations above. The electron consists of only one shell since the L_0 polynomial has no derivatives. Due to the electron being the first member of the series the equations for its mass density are simple:

To set a clarifying pattern for all the leptons the definitions of $\text{IntegD}_{pk}(r)$, C_{rpk}, $\text{IntegD}_{pk}(\theta)$, and $C_{\theta pk}$ are repeated here for the electron in their complete long form. To maintain focus, though, the repeating of these long form mathematics are suppressed with the remainder of the leptons. Again, the normalizing constants for the L_n^d and the $T_n^\dagger$ orthogonal polynomials need to be remembered since they have been suppressed in the appearances of all the equations, but have been included in the numerical listing in the tables.

Radially:

$$\text{IntegD}_{11}(r) = \int_0^{\infty} e^{R_c(t_r)} e^{R_e(t_r)} L_0^0(R_e(t_r)) dt_r \tag{29}$$

$$C_{r11} = \int_0^{\infty} \text{FHDif}[F(t_r)]\, e^{R_c(t_r)} e^{R_e(t_r)} L_0^0(R_e(t_r)) dt_r \tag{30}$$

$$D_{11}(r) = C_{r11} \int_0^{\infty} e^{R_c(t_r)} e^{R_e(t_r)} L_0^0(R_e(t_r)) dt_r \tag{31}$$

$\text{IntegD}_{11}(r)$ and $D_{11}(r)$ include normalization for L_0^0 but C_{r11} does not.

Angularly:

$$\text{IntegD}_{11}(\theta) = \int_0^{\pi/2} T_1^{\dagger}(\sin[\pi/2\, A_i(t_\theta)]) dt_\theta \tag{32}$$

$$C_{\theta 11} = \int_0^{\pi/2} \cos(t_\theta)\, T_1^{\dagger}(\sin[\pi/2\, A_i(t_\theta)]) dt_\theta \tag{33}$$

$$D_{11}(\theta) = 4C_{\theta 11} \int_0^{\pi/2} T_1^{\dagger}(\sin[\pi/2\, A_i(t_\theta)]) dt_\theta \tag{34}$$

$\text{IntegD}_{11}(\theta)$ and $D_{11}(\theta)$ include normalization for L_0^0 but C_{r11} does not.
Likewise $\text{IntegD}_{11}(\theta)$ and $D_{11}(\theta)$ include normalization for $T_1^{\dagger}$ but $C_{\theta 11}$ does not.

For the electron $C_e = 1$ and $S_1 = 1$. The combined radial-angular mass density function for the electron becomes:

$$m_e = C_g C_e \sum_{k=1}^{1} S_k\, D_{1k}(r) D_{1k}(\theta) \tag{35}$$

These equations were used to calculate the values shown for the electron in Tables 1 through 6. As seen the results show that the ultimate calculated mass is within 5 parts in 10 million to the experimentally determined value.

4.4 Muon Specific Equations And Calculations

The following equations for the muon are obtained with ($p = 2$ or μ) in the equations above. The muon has two shells, a primary represented by the L_2^0 polynomial and a secondary by the only even derivative of L_2, the L_2^2 polynomial.

$$D_{21}(r) = C_{r21} \int_0^{\infty} e^{R_c(t_r)} e^{R_e(t_r)} L_2^0(R_e(t_r)) dt_r \tag{36}$$

$$D_{22}(r) = C_{r22} \int_0^{\infty} e^{R_c(t_r)} e^{R_e(t_r)} L_2^2(R_e(t_r)) dt_r \tag{37}$$

$$D_{21}(\theta) = 4C_{\theta 21} \int_0^{\pi/2} T_3^{\dagger}(\sin[\pi/2\, A_i(t_\theta)]) dt_\theta \tag{38}$$

$$D_{22}(\theta) = 1/3 D_{11}(\theta) \tag{39}$$

The overall particle constant C_μ can be determined as

$$C_\mu = F_c F_{m\mu} F_{s\mu} \tag{40}$$

according to Equation (16) above. From the simple algebra of Equations (17) and (18) F_c= 6.851,799,475 x 10^{+01} and $F_{m\mu}$ = 2.629,656,683 x 10^{-02} respectively. $F_{s\mu}$ the shielding or mass defect factor is:

$$F_{s\mu} = \frac{1}{2}\left[\frac{1}{1-1/3!}\right] \tag{41}$$

The individualizing shell factors are S_1 = 1 and S_2 = (1 + 1/2 x 1/37). Putting all the pieces together the mass of muon can be calculated as

$$m_\mu = C_g C_\mu \sum_{k=1}^{2} S_k D_{2k}(r) D_{2k}(\theta) \tag{42}$$

These equations were used to calculate the values shown for the muon in Tables 1 through 6. As seen the results show that the ultimate calculated mass is within 1 part in 10 million to the experimentally determined value.

4.5 Tau Specific Equations And Calculations

Similarly, the equations for the tau can be determined with (p = 3 or τ). As seen the tau has three shells since the L_4 polynomial has a base L_4^0 and two even derivatives, L_4^2 and L_4^4.

$$D_{31}(r) = C_{r31} \int_0^\infty e^{R_c(t_r)} e^{R_e(t_r)} L_4^0(R_e(t_r)) dt_r \tag{43}$$

$$D_{32}(r) = C_{r32} \int_0^\infty e^{R_c(t_r)} e^{R_e(t_r)} L_4^2(R_e(t_r)) dt_r \tag{44}$$

$$D_{33}(r) = C_{r33} \int_0^\infty e^{R_c(t_r)} e^{R_e(t_r)} L_4^4(R_e(t_r)) dt_r \tag{45}$$

$$D_{31}(\theta) = 4C_{\theta 31} \int_0^{\pi/2} T_5^\dagger\,(\sin[\pi/2\,A_i(t_\theta)]) dt_\theta \tag{46}$$

$$D_{32}(\theta) = 1/5 D_{21}(\theta) \tag{47}$$

$$D_{33}(\theta) = 1/5 D_{11}(\theta) \tag{48}$$

$$C_\tau = F_c F_{m\tau} F_{s\tau} \tag{49}$$

where F_c is unchanged and equal 6.851,799,475 x 10^{+01}. Again using the simple algebra of Equation (18) $F_{m\tau}$ = 2.493,074,792 x 10^{-02}. $F_{s\tau}$ the shielding or mass defect factor is:

$$F_{st} = \frac{1}{4}\left[\frac{1}{1 - {}^{1}/_{3!} - 2^{9/4}/5!}\right] \tag{50}$$

The shell factors S_1 and S_2 are unchanged and S_3 = (1 + 1/4 x 1/37). Putting all the pieces together the mass of tau can be calculated as

$$m_\tau = C_g C_\tau \sum_{k=1}^{3} S_k D_{3k}(r) D_{3k}(\theta) \tag{51}$$

These equations were used to calculate the values shown for the tau in Tables 1 through 6. As seen the results show that the ultimate calculated mass is within 1 part in 1 thousand to the experimentally determined value.

5 A Possible 4thMember Of The Lepton Family, the Shipa?

On seeing the general pattern of these mass density equations, one should ask what happens for the higher even member L_n polynomials, those above L_4 of the τ? On checking these, the curve of increasing mass density with the row number of the L_n polynomials is found to curl over and rapidly goes negative. The shipa, the 4th member of this series, the row L_6 and its derivatives, was calculated using the equations below and is mathematically possible.

Following the pattern of the general equations, the specific equations for the shipa are as follows:

$$D_{41}(r) = C_{r41}\int_0^\infty e^{R_c(t_r)}e^{R_e(t_r)}L_6^0(R_e(t_r))dt_r \tag{52}$$

$$D_{42}(r) = C_{r42}\int_0^\infty e^{R_c(t_r)}e^{R_e(t_r)}L_6^2(R_e(t_r))dt_r \tag{53}$$

$$D_{43}(r) = C_{r43}\int_0^\infty e^{R_c(t_r)}e^{R_e(t_r)}L_6^4(R_e(t_r))dt_r \tag{54}$$

$$D_{44}(r) = C_{r44}\int_0^\infty e^{R_c(t_r)}e^{R_e(t_r)}L_6^6(R_e(t_r))dt_r \tag{55}$$

$$D_{41}(\theta) = C_{\theta 41}\int_0^{\pi/2} T_7^\dagger(\sin[\pi/2\, A_i(t_\theta)])dt_\theta \tag{56}$$

$$D_{42}(\theta) = 1/7 D_{31}(\theta) \tag{57}$$

$$D_{43}(\theta) = 1/7 D_{21}(\theta) \tag{58}$$

$$D_{44}(\theta) = 1/7 D_{11}(\theta) \tag{59}$$

$$C_s = F_c F_{ms} F_{ss} \tag{60}$$

where again the constant F_c is as above, and Equation (18) gives F_{ms} = 2.366,863,9 x 10^{-02}. F_{ss} the shielding or mass defect factor is:

$$F_{ss} = \frac{1}{8}\text{or } \frac{1}{6}\left[\frac{1}{1-1/3!-2^{9/4}/5!-4^{16/9}/7!}\right] \tag{61}$$

and setting $S_4 \approx (1 + 1/6 \times 1/37)$ since the pattern is not well established. Putting all the pieces together the mass of shipa can be calculated as

$$m_s = C_g C_s \sum_{k=1}^{4} S_k D_{4k}(r) D_{4k}(\theta) \tag{62}$$

Tables 1 through 5 show the numerical values for the shipa. As seen the net mass of this hypothetical particle **lays only about 2% of the way from that of the electron mass going towards that of the muon.** The exact value of this mass cannot be predicted since a strong pattern has not been set for several of the particle scale factors.

While this 4^{th} member is not rigorously excluded mathematically, additional physical ramifications of the details of its mathematics need to be considered. While the overall particle has a net positive mass, one of its shells has a negative value. Could this mean that the negative mathematical value of the particle's 3^{rd} radial shell seen in Table 1 would in physical reality effectively act as a drain and bleed the particle's overall energy dry? Would this negative value for the 3^{rd} shell effectively prohibit the entire particle's existence? This is unknown. This mathematical calculational versus physical existence issue has not arisen before.

Upon drawing radial-angular plots, polar coordinates, for the angular equations of the muon, tau, and this 4th member, a thumb appears amongst several fingers. The plots of both the angular equations and those of the angular equations multiplied by the initial condition all show a clearly imbalanced lobe amongst the other lobes of the plots. See Figures 2-5. This imbalance gets accentuated the higher the particle is in the series. This out-of-balance angular appearance is probably directly related to the decreasing half life between the muon and tau.

The gross imbalance of the shipa's angular appearance combined with a more complicated radial equation that stabilizes far less energy than the two previous simpler members, muon and tau, also could explain why this particle has never been observed. Additionally, machines on which older low energy collider data was collected, may not have been able to produce a fine enough scattering matrix to indicate that some of the collision products were the result of not only a low energy intermediate, but also an extremely short lived particle only able to travel a short distance.

Aside from the machinery, there is the human element. The known lepton and quark series set up an appearance which could lead to logical trap for the particle physicists studying these series. First these elementary particle series give the appearance of always increasing in mass as progress is made up through the series. Secondly, higher energy for an elementary particle, quark or lepton, always appears to go hand-in-hand with a shorter half life. This may be true, but this apparent pattern sets up invalid logic. High energy yields short half life; therefore short half life must always come from high energy particles.

Should this particle exist then it would only stabilize about 5 times more energy than the electron compared with the approximate 207 times greater amount stabilized by the muon. Since this potential 4^{th} member has more complicated wave patterns but stabilized trivial amounts of energy compared to the muon and tau, a person can wonder why nature would create such an item? That is, other than as a short lived low energy sink. Asking how would such a particle would ever be created is also a legitimate question. Other particles with higher energy such as the muon and tau would not be likely to decay to a wave form which, while it has a lower energy as a system, has more complicated wave patterns. Simple waveforms decaying to more complicated waveforms does not pass the common sense test. Just because this particle may not be prohibited mathematically does not mean that its physical existence is mandatory either. Nature does not create particles just to agree with human mathematical models.

Concluding this section concerning the potential 4^{th} member of the lepton series, the shipa particle, several open ended questions remain. There are mathematical arguments both for and against its actual physical existence. Finally regardless of whether its existence is ever proven or disproven by particle physics this cannot be used to invalidate the entire remainder of this work. This one item is separable from the rest of the other known particles as a family due to its oddball 3^{rd} negative shell.

Table 1 Values Of Lepton Radial Equation Integrals

Particle, Shell	Initial Constant	Integrated Equation	Product
	C_{rpk}	Integ$D_{pk}(r)$	$D_{pk}(r)$
Electron			
Primary	1.618,533,691 x 10^2	3.428,165,302 x 10^2	5.548,601,040 x 10^4
Muon			
Primary	6.760,706,674 x 10^3	1.943,599,062 x 10^4	1.314,010,315 x 10^8
Secondary	1.618,533,691 x 10^2	2.424,078,932 x 10^2	2.424,078,932 x 10^2
Tau			
Primary	2.387,176,656 x 10^4	1.089,901,363 x 10^5	2.601,787,092 x 10^9
Secondary	4.116,467,332 x 10^3	3.699,379,637 x 10^3	1.522,837,542 x 10^7
Tertiary	1.618,533,691 x 10^2	6.997,713,119 x 10^1	1.132,603,445 x 10^4
Shipa			
Primary	1.179,803,559 x 10^2	6.949,401,001 x 10^4	8.198,928,031 x 10^6
Secondary	5.557,260,386 x 10^3	6.878,138,841 x 10^3	3.822,360,851 x 10^7
Tertiary	-7.069,341,479 x 10^3	3.987,278,720 x 10^2	-2.818,743,484 x 10^6
Quaternary	1.618,533,691 x 10^2	1.277,601,775 x 10^1	2.067,841,518 x 10^3

Table 2 Values Of Lepton Angular Equation Integrals

Particle, Shell	Initial Constant	Integrated Equation	Symmetric	Product
	$C_{\theta pk}$	Integ$D_{pk}(\theta)$	Multiplier	$D_{pk}(\theta)$
Electron				
Primary	0.890,365,284	0.941,966,611	4	3.354,777,477
Muon				
Primary	0.442,427,296	0.152,908,897	4	0.270,604,279
Secondary	0.890,365,284	0.313,988,870	4	1.118,259,159
Tau				
Primary	0.331,851,909	0.348,176,984	4	0.462,172,786
Secondary	0.442,427,296	0.030,581,779	4	0.054,120,856
Tertiary	0.890,365,284	0.188,393,322	4	0.670,955,495
Shipa				
Primary	0.276,612,505	0.138,706,718	4	0.153,472,051
Secondary	0.331,851,909	0.049,739,569	4	0.066,024,684
Tertiary	0.442,427,296	0.021,844,128	4	0.038,657,754
Quaternary	0.890,365,284	0.134,566,659	4	0.479,253,925

Table 3 Lepton Radial x Angular Products

Particle, Shell	Radial x Angular Product	Shell Factor	Final Shell Contribution
	$D_{pk}(r) \times D_{pk}(\theta)$	S_{pk}	D_{pk}
Electron			
Primary	$1.861,432,180 \times 10^5$	1	$1.861,432,180 \times 10^5$
Sum of Shells			**$1.861,432,180 \times 10^5$**
Muon			
Primary	$3.555,768,139 \times 10^7$	1	$3.555,768,139 \times 10^7$
Secondary	$4.387,437,724 \times 10^4$	1.013,513,514	$4.446,727,423 \times 10^4$
Sum of Shells			**$3.560,214,867 \times 10^7$**
Tau			
Primary	$1.202,475,190 \times 10^9$	1	$1.202,475,190 \times 10^9$
Secondary	$8.241,727,105 \times 10^5$	1.013,513,514	$8.353,101,796 \times 10^5$
Tertiary	$7.599,265,053 \times 10^3$	1.006,756,757	$7.650,611,438 \times 10^3$
Sum of Shells			**$1.203,318,151 \times 10^9$**
Shipa			
Primary	$1.258,306,297 \times 10^6$	1	$1.258,306,297 \times 10^6$
Secondary	$2.523,701,665 \times 10^6$	1.013,513,514	$2.557,805,741 \times 10^6$
Tertiary	$-1.089,662,926 \times 10^5$	1.006,756,757	$-1.097,025,514 \times 10^5$
Quaternary	$9.910,211,643 \times 10^2$	1.004,504,505	$9.954,852,236 \times 10^2$
Sum of Shells			**$3.707,404,972 \times 10^6$**

Table 4 Lepton Particle Scale Factors

	Common Constant	Constant Factor	Member Factor	Shielding Factor	Particle Constant	Product
	C_g, m (1 L_{Sgs})	F_c	F_{mp}	F_{sp}	C_p	$C_g \times C_p$, m
Electron	$4.893,752,96 \times 10^{-36}$		1	1	1.000,000,000	$4.893,752,96 \times 10^{-36}$
Muon	$4.893,752,96 \times 10^{-36}$	68.517,994,746	0.026,296,567	0.600,000,000	1.081,072,817	$5.290,503,30 \times 10^{-36}$
Tau	$4.893,752,96 \times 10^{-36}$	68.517,994,746	0.024,930,748	0.314,983,211	0.538,055,850	$2.633,112,41 \times 10^{-36}$
Shipa	$4.893,752,96 \times 10^{-36}$	68.517,994,746	0.023,668,639	0.157,955,887	0.256,161,435	$1.253,590,78 \times 10^{-36}$

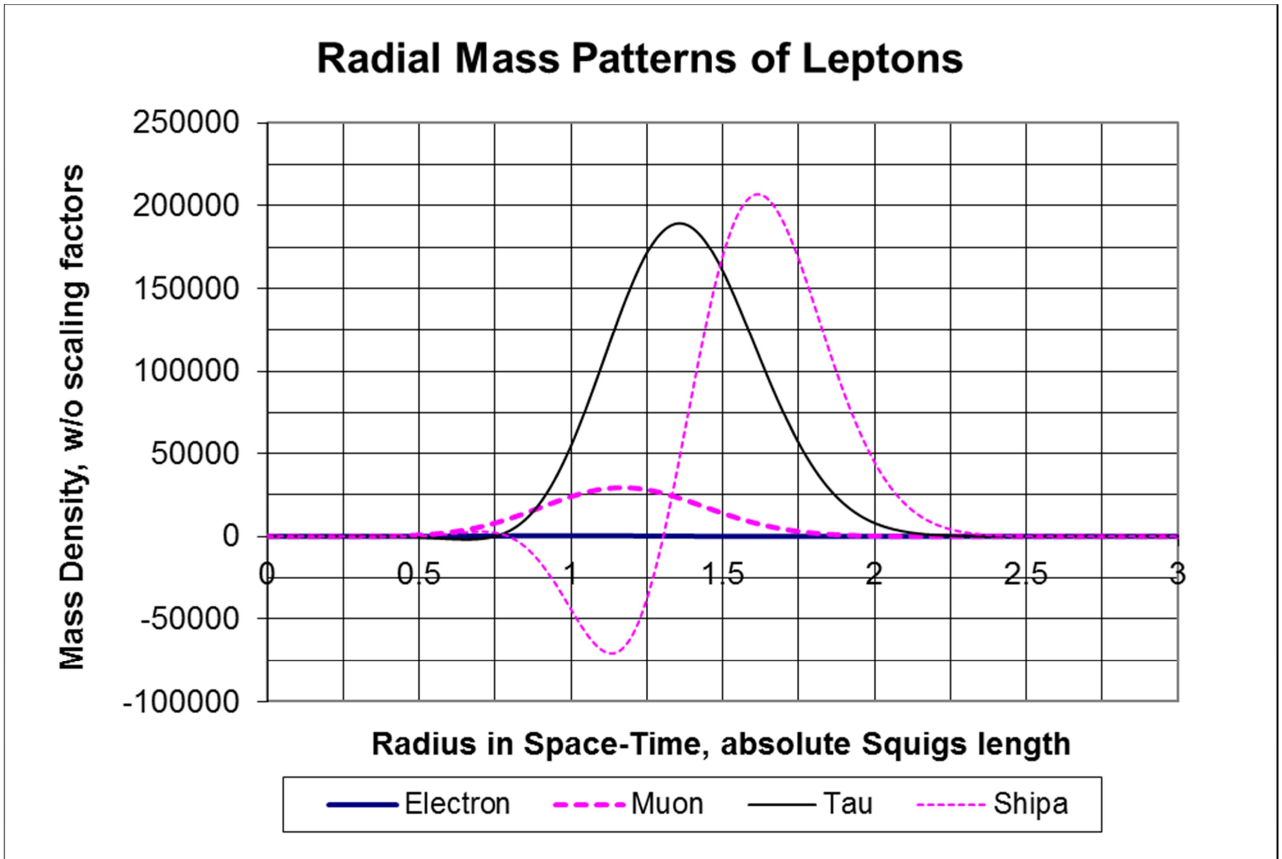
Table 5 Results Of Derivations For Masses Of Leptons

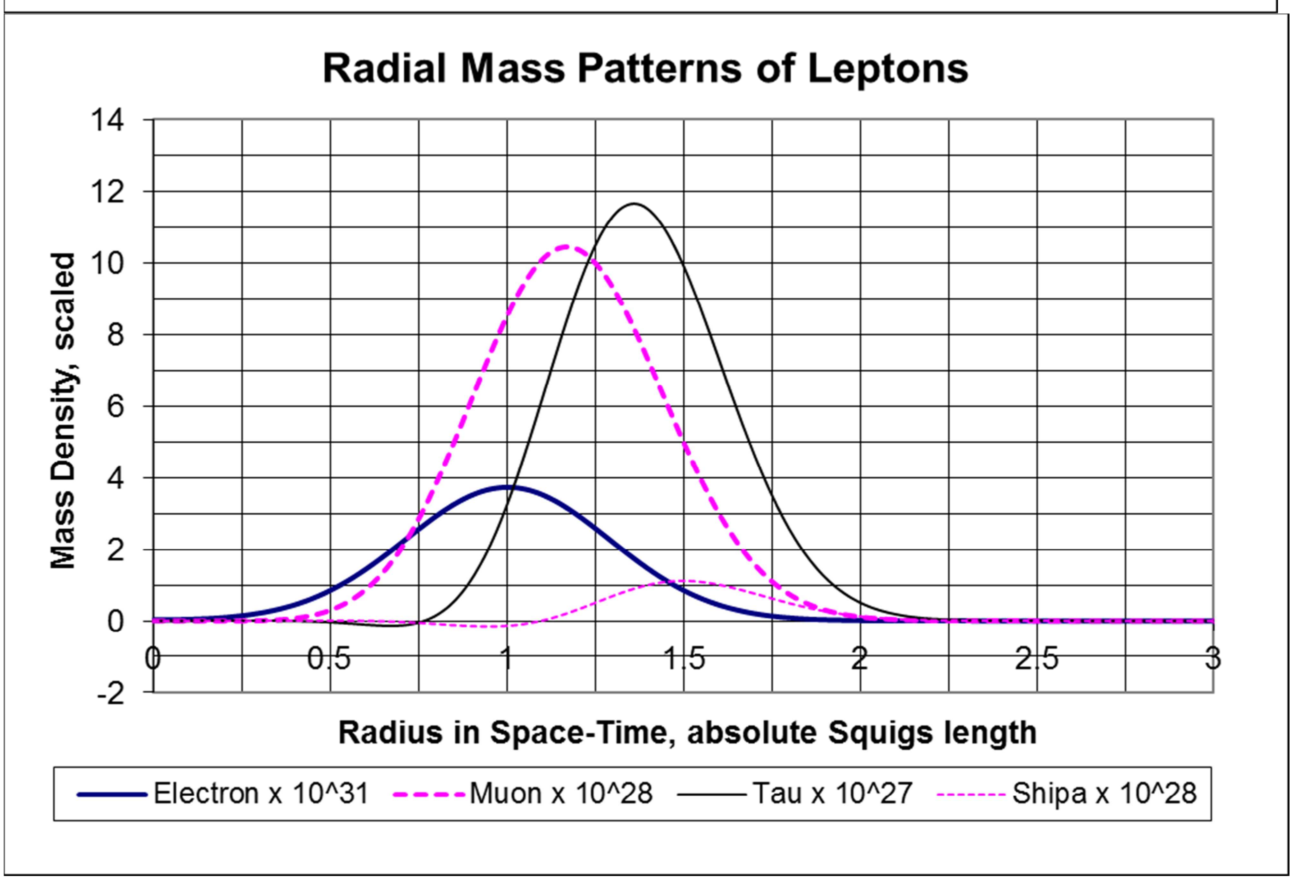
	Radial Angular Product	Scale Multiplier	Calculated Mass
	D_p, kg/m	C_g x C_p, m	m_p, kg
Electron	1.861,432,180 x 10^{5}	4.893,752,96 x 10^{-36}	9.109,389,239 x 10^{-31}
Muon	3.560,214,867 x 10^{7}	5.290,503,30 x 10^{-36}	1.883,532,849 x 10^{-28}
Tau	1.203,318,151 x 10^{9}	2.633,112,41 x 10^{-36}	3.168,471,956 x 10^{-27}
Shipa	3.707,404,972 x 10^{6}	1.253,590,78 x 10^{-36}	4.647,568,702 x 10^{-30}

Table 6 Comparison Of Lepton Mass Derivations To Measurements

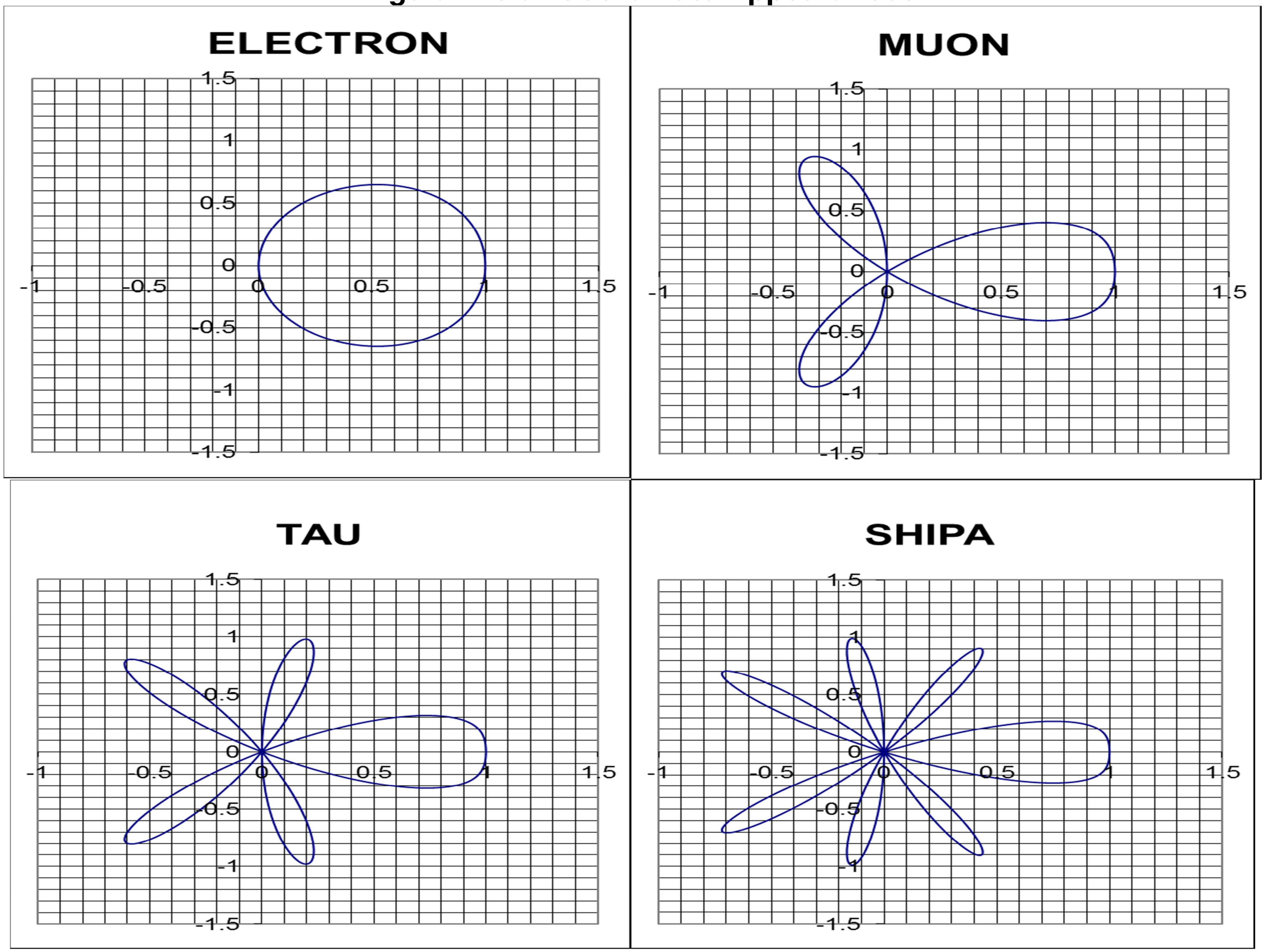
	Measured Mass, kg	Measured / Calculated
Electron		
High	9.109,395,1 x 10^{-31}	1.000,000,6
Mid	9.109,389,7 x 10^{-31}	1.000,000,0
Low	9.109,384,3 x 10^{-31}	0.999,999,45
Muon		
High	1.883,533,8 x 10^{-28}	1.000,000,5
Mid	1.883,532,7 x 10^{-28}	0.999,999,92
Low	1.883,531,6 x 10^{-28}	0.999,999,33
Tau		
High	3.168,39 x 10^{-27}	0.999,976
Mid	3.167,88 x 10^{-27}	0.999,813
Low	3.167,41 x 10^{-27}	0.999,666

FIGURES 1.1 & 1.2

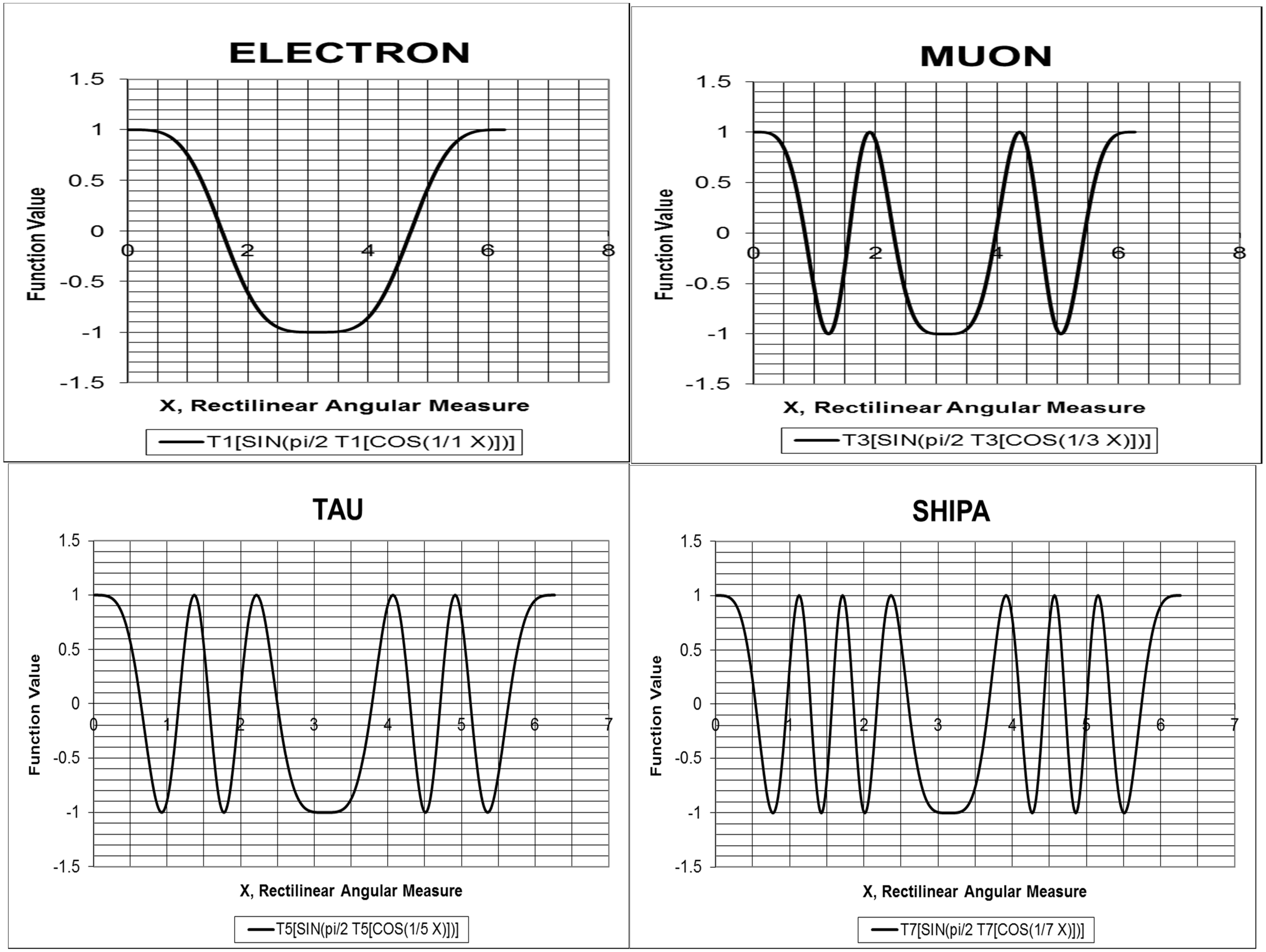




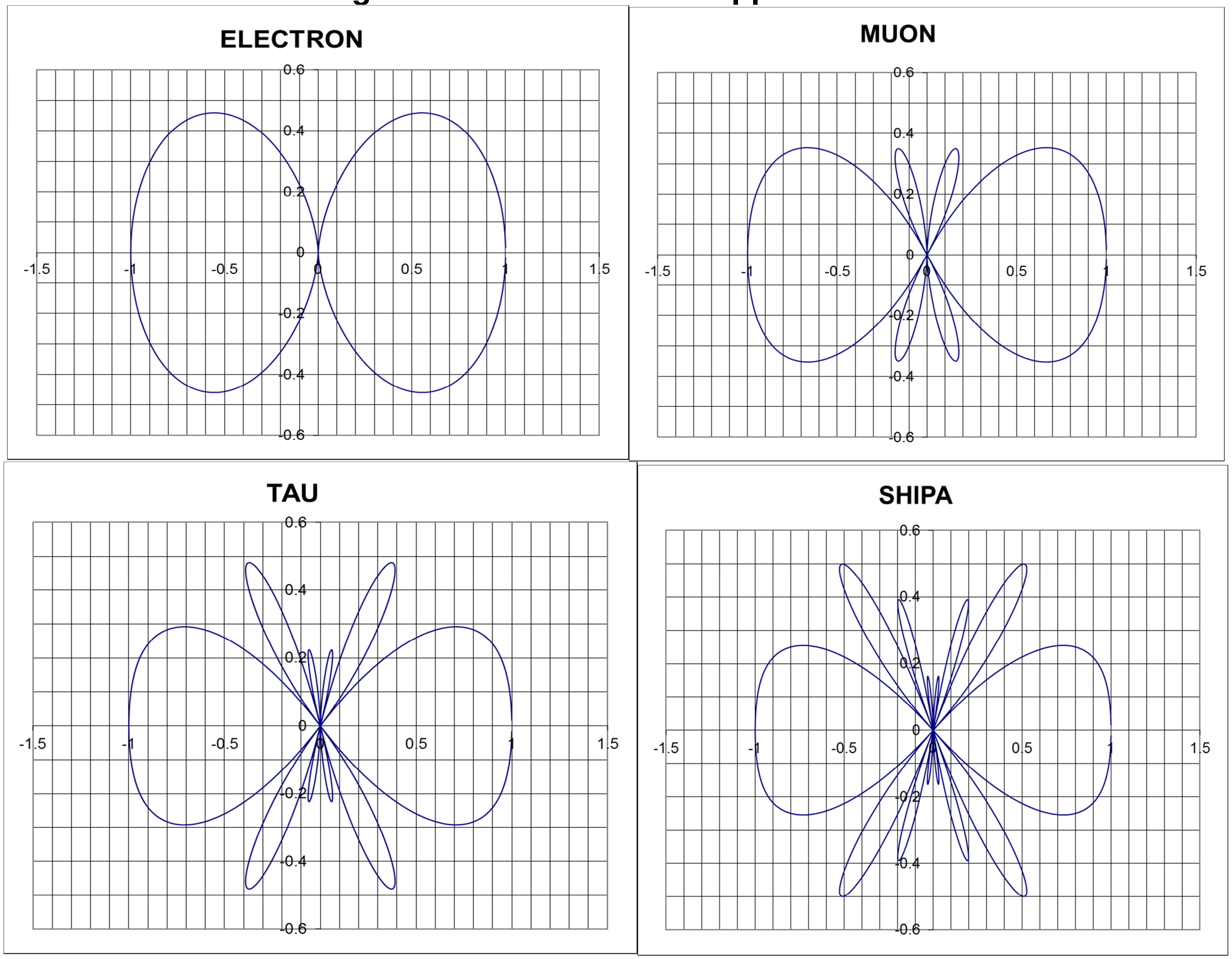
FIGURES 2.1-2.4 Lepton's Mass Density Cross Sections
Angular Polar Coordinate Appearances



FIGURES 3.1-3.4 Lepton's Angular Oscilloscope Appearances in Rectilinear Measure

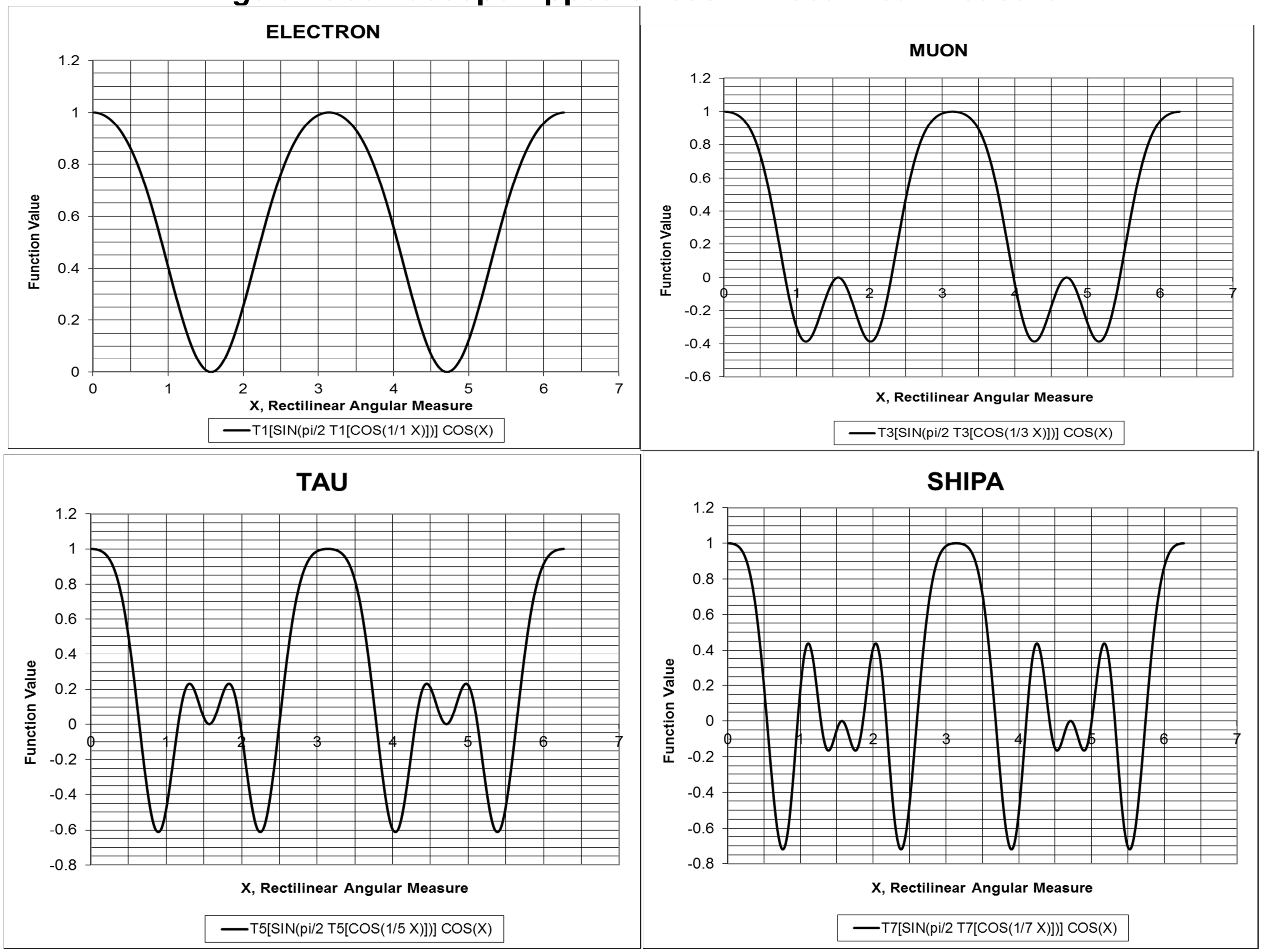


FIGURES 4.1-4.4 Lepton's Final Mass Cross Sections Angular Polar Coordinate Appearances[1]



1 **Angular mass distribution times initial condition**

FIGURES 5.1-5.4 Lepton's Final Mass Cross Sections
Angular Oscilloscope Appearances in Rectilinear Measure[1]



1 Angular mass distribution times initial condition

6 Analysis And Discussions

6.1 The Mass Density Equation Parameters And The Charge Equation Parameter

In the mass density equations for the leptons there are two embedded or implicit parameters. For simplicity these could have been represented as the spatial quantities r and θ, rather than the temporal quantities t_r and t_θ which are used. The integrals then would have read dr and dθ, instead of dt_r and dt_θ which are shown. The use of such spatial variables is particularly appealing since temporal quantities are conceptually more complicated and one step further removed from the ultimate objectives which are clearly spatial quantities D(r) and D(θ). Further throughout the equations there appears to have been no strong reason, driving necessity, or hard proof as to why temporal quantities should be used.

The discussions in this section lay grounds why temporal interpretations for these ultimate implicit variables make more sense. Also as is found repeatedly throughout the discussions in the next report which compares and contrasts the equations for the leptons and those for the photons, there temporal interpretations are almost mandatory. There many of the physical properties of these two most elementary electromagnetic species are shown to depend upon or can best be explained by a temporal interpretation of these implicit variables. Finally in Appendix 2, Time & Space which focuses entirely on the topics of time and space as represented in the human uses of implicit/explicit variables, the temporal choice is again found to be the better one. The conceptual ground work is laid there for the idea that behind or within every exterior spatial phenomena there is an internal or inner reference to a sense of time. There is no surprise or there might even be an expectation that at some scale, for every spatial dimension there would appear a corresponding underlying temporal dimension.

Having just seen the spatial variables r and θ, the lack of a requirement that these two implicit variables t_r and t_θ be the same is easy to see. If this temporal interpretation is to be a permitted interpretation of these variables or is even "the correct view", then this means that time is two dimensional. That is at least specifically within the size realm of these subatomic waveforms, there would be a radial temporal sense of events and an angular temporal sense of events, and further these two senses would necessarily be independent.

Further, if these radial and angular equations are to represent the solutions to some time dependent Schrodinger style wave equation after the separation of variables, then these two expressions of time, t_r and t_θ, must necessarily be independent. Otherwise the radial and angular parameters would be linked and the solutions to the equations would not be separable, as they have been found to be here.

Besides these two possible variables of time, or the interpretation of the two ultimate implicit variables within the mass density equations as both representing time, there is the third temporal parameter. This parameter t was found in the vector expressions of **R**(t) shown in Equation (13) for the electromagnetic formulation or description of the leptons. This then leads to the equation for the charge of the leptons. This unsubscripted form of t implicit within **R**(t) appears to be independent of the two possible variables t_r and t_θ as might be found in the mass density equations. This parameter measures events relating to the circulation of the energy pattern around the center of the donut. While there might be logical expectations that one cycle length or revolution around the donut would coincide with one rotation or spin of the periphery of the radial planar figure about its center, these equations give no mathematical guarantee of this. Likewise, there is no requirement that the t of the vector **R**(t) and t_θ as the implicit variable of $D_{pk}(\theta)$ be integer multiples of one another, or have any relation at all.

Another way of thinking about these temporal variables which helps further clarify this picture is as follows. The two variables t_r and t_θ refer to the inherent 2-dimensional structure of the energy form or pattern of the leptons, while the unsubscripted t of **R**(t) models this 2-dimensional structure's flight path thru the third dimension.

Continuing with the other parameters found within the mass density and charge equations, in Equation (09) concerning the calculation of the charge the geometric parameter A appeared, with units of (Q^2 relative / L radial_absolute) specifically C^2/l_{Sgs}. In that setting what is the meaning of this basic human measuring stick, distance? In this setting of a rectilinear vector expression, distance is a measure of length perpendicular to the electromagnetic surface being discussed.

In the scalar radial-angular mass density equations for the grand total expression in Equation (14) up to the point that the general correlation constant C_g is applied, the expressions result in mass density units of (M relative / L radial_absolute) specifically kg/l_{Sgs}. There distance can be thought of as a measure of length co-linear, parallel, or simultaneous with the stabilized gravitational force or energy pattern being discussed.

Aside from the basic spatial and temporal measuring devices, there are the parameters of C^2 and kg. These two of course are the objectives of the calculational model. These represent different measurements or descriptions of the encapsulated or stabilized energy of the particles. Of course being the result of human measurements these values were naturally placed on relative scales. Even here a very simple pattern can be seen. Coulombs (charge) representing the binary force electromagnetic are described by a 2 dimensional phenomenon the curvature and are squared. Kilograms (mass) representing the unary force gravity is first order. Likewise this measure is described by a simple linear or radial phenomenon, that just happens to tumble around in multiple dimensions with time.

6.2 Discussion Of Radial Equations

Plots of the radial equations of the leptons were seen above. Figure 1 shows both the un-scaled and the final appearances of these equations.

The radial mass density equations of the leptons have the generic form:

$$D(r) = R_{csf} \text{ x } R_{esf} \text{ x } L_n^d(R_e(t_r)) = e^{(R_c(t_r))} \text{x } e^{(R_e(t_r))} \text{ x Laguerre polynomial expression in } R_e(t_r) \qquad (63)$$

where $R_c(t_r)$ and $R_e(t_r)$ are functions of the implicit radial variable as defined earlier in Section 4.2.

There are three factors here:

1 $R_{csf} = e^{(R_c(t_r))} = e^{(-6t_r^2)}$; This is the attenuator or longevity factor. Mathematically, this factor overpowers all other factors, of exponential order, and ultimately terminate the expression or bring it to converge to some value. Physically this factor represents a contractive wave pattern or force.

2 $R_{esf} = e^{(R_e(t_r))}$; Mathematically, this factor is highly expansive. Physically, this is the driver and represents the real force, intensity, or effort that sustains the particle, and represents an expansive wave pattern or force.

3 L_n^d polynomial expression of $R_e(t_r)$; This is the shape factor. It gives form, shape, or direction to the effort of the second factor.

Had the necessity of at least a duality of counter acting forces or wave patterns been realized, then 8 years of brutal trial and error in search of the mathematical-geometric form of the leptons could have been avoided. Viewed from 20:20 hind sight, the necessity for least the two opposing mathematical factors is obvious. Once this concept is grasped, then the necessity of a third factor to step thru the members of the lepton family is immediately obvious.

Why is the necessity of two opposing waves or forces obvious? Going to Chapter 4.1, Methodology there the observation is made that everything in the physical universe is impermanent. That is all things with a known or observed form have a limited duration. They come into being, exist for a period of time, and then go back out of being or cease to exist. This observation was used to conclude that everything, including the leptons, must have a form or else they could not decay or be destroyed. What was forgotten for 8 years is the spatial counter part to this observation. Everything known or observed in the physical universe also has a limited spatial extent. Just as with temporal boundaries, all physical forms also have spatial boundaries, limits, or extents. Had this spatial observation been made, then the question"why is this so" would have followed. There must be some physical mechanisms which inhibit or limit infinite growth, expansion, and size. Likewise there must be some physical mechanisms which create or promote growth or forms would never come into being. Without this expansive impulse on the part of the individual particles, the content of the universe would have consisted of a soup of invisible formless mathematical points. Now the necessity for at least two mathematical expressions to model these two physical mechanisms is obvious.

Examining the mathematical properties of the factors, both the contractive implicit radial factor, $e^{(R_c(t_r))} = e^{(-6t_r^2)}$, and the expansive implicit radial factor $e^{(R_e(t_r))}$ are seen to involve exponentials. They could easily derive from or evolve into differential equations, of either the first or second order. Also just as the working internals of the angular equations are themselves angular in nature, trigonometric functions, the major working mathematical factors for the radial equations here are themselves radial in nature. That is, both exponentials, whether positive or negative, are still exponentials and "radiate" outwards forever.

There is an interesting aspect of the initial radial mass density distribution seen in the Fraunhofer Diffraction Function of Equation (22), repeated here.

$$I(r) = \text{FHDif}[F(r)] = \left[\frac{2J_1[F(r)]}{F(r)}\right]^2 \qquad (22)$$

When this is used as a radial function in energy calculations and when $F(r) = ar^1$, then this function effectively incorporates a modified version of the inverse square law with the factor $F(r)^2$ in the denominator. Obviously the initial radial mass density Fraunhofer diffraction pattern is not the result of diffraction, but rather represents some energy pattern whose mathematics are incidentally identical to that of Fraunhofer diffraction. This initial condition probably represents the two dimensional radial energy pattern resulting from a particle in a flat circular box. A mathematical gratuity is that this initial distribution is self normalized. Further mathematical gratuity is that the same initial condition applies to all the members of the series, as well as to all the shells of the upper members of the series.

Examination of the ultimate variables of D(r) reveals an interesting property. $R(t_r)$ in time and t_r are always to the second power. Although the outside appearance and behavior of the expansive radial factor $e^{(R_e(t_r))}$ seen in Equation (21), also repeated here,

$$R_e(t_r) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} ds\left(\frac{2\pi t_r^2}{k^{1/2}}\right) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}}\left[1 + \left(\frac{4\pi t_r^1}{k^{1/2}}\right)^2\right]^{\frac{1}{2}} dt_r \qquad (21)$$

is that of a pseudo 1st order, internally t_r is squared. Both radial factors can be thought of as being symmetrical in time and/or space, with negative values of t_r extending inward into the past and positive values extending outward into the future. The only conceptual versatility needed is that in visualizing radial-polar plots which extends inward to negative values of t_r. Typical radial-angular plots stop with $t_r = 0$ as a dot at the origin. This origin only needs to be expanded outward into a circle of $t_r = 0$ with inside the circle having negative t_r and outside the circle having positive t_r. Physically this would mean the leptons are in a parabolic energy well in radial time, with the present at the bottom of the well.

When viewing the overall pattern of the radial equations across all the members of the series the even-ness of this series is striking. The members occur only at even values of n for the Laguerre L_n polynomials. Odd n is found to produce negative values for the radial integrals. Additionally the auxiliary shells for the upper members only occur at even numbered derivatives. Looking back at the angular equations, the members of the series are found only to occur at the odd T_n polynomials. Neither the radial equations nor the angular equations occur by a continuous sequential polynomial series.

One major outstanding uncertainty is the exact relationship between the 6 occurring here as the coefficient of t_r^2 in the contractive exponential, $e^{(R_c(t_r))} = e^{(-6t_r^2)}$, and the 6 occurring as the amplitude coefficient of the planar i and j vectors of the toroidal coil which gives meaning to the charge equation.

6.3 Discussion Of Angular Equations

Plots of the angular equations of the leptons, both radial-angular and rectilinear presentations, were seen above. Figures 2-3 show the appearance of the angular equations and Figures 4-5 show the appearance of the angular equations multiplied by the initial condition.

The angular equations of the leptons, both the outer spatial form and the inner or implicit form, are described by the odd membered trigonometrically substituted Chebyshev $T_n^{\dagger}$orthogonal polynomials. The reasoning for this is as follows.

First assuming a second order differential equation description is necessary for a series of stable but otherwise unknown energy patterns, such as the leptons. Next assume the validity of the six major assumptions which permit the separation of the variables representing the various spatial dimensions. Proceeding through the complete separation of variables, the following general equations are produced: See Appendix 5, Separation of Variables for these details. For angle 1 the equation is:

$$\cos^{-1}(\theta_1)\frac{d}{d\theta_1}\left[\cos^0(\theta_1)\frac{dH_2}{d\theta_1}\right] - (qn_2)H_2(\theta_1) = 0 \qquad (64)$$

For angle k, where 1 < k < (n-1)

$$\cos^{-(k-1)}(\theta_k)\frac{d}{d\theta_k}\left[\cos^{k-1}(\theta_k)\frac{dH_{k+1}}{d\theta_k}\right] - (-qn_{k+1}\cos^{-2}(\theta_k) - qn_{k+1})H_{k+1}(\theta_k) = 0 \qquad (65)$$

and for angle n-1, the equation becomes:

$$\cos^{-(n-2)}(\theta_{n-1})\frac{d}{d\theta_{n-1}}\left[\cos^{n-2}(\theta_{n-1})\frac{dH_n}{d\theta_{n-1}}\right] - (-qn_{n-1}\cos^{-2}(\theta_{n-1}) - qn_1)H_n(\theta_{k-1}) = 0 \quad (66)$$

Here H_{k+1} is an arbitrary function of the θ_k argument, and qn_k is an arbitrary constant or quantum number. Here the first dimension is assumed to be the radial parameter, and has H_1 and the other appearance of qn_1 associated with it. Further assume some $F(x_k) = H_{k+1}(\theta_k)$ for each of the n-1 separated angular equations, and assume $x_k = f(\theta_k) = \sin(\theta_k)$. Taking the required derivatives and substituting the results into the first term of the n-1 angular equations, these first term appearances become as in Table 7. Considering the differential equation formulations for the orthogonal polynomials, some natural correspondences arise, as shown.

Table 7 General Angular Equation Correspondences

Angle	Equation 1st Term Appearance	ODE Orthogonal Polynomials
1	$(1-x^2)d^2F/dx^2 - 1x\ dF/dx$	ChebyshevT_n (x)
2	$(1-x^2)d^2F/dx^2 - 1x\ dF/dx$	Jacobi P_n(a,b,x), a=b=0
3	$(1-x^2)d^2F/dx^2 - 3x\ dF/dx$	Ultraspherical C_n(a,x), a=1
k even	$(1-x^2)d^2F/dx^2 - kx\ dF/dx$	Jacobi P_n(a,b,x), a = b= (k-2)/2
k odd	$(1-x^2)d^2F/dx^2 - kx\ dF/dx$	Ultraspherical C_n(a,x), a = (k-1)/2

A definitive discussion of all these orthogonal polynomials can be found in Chapter 22 of Abramowitz and Stegun [6].

There is a systematic orderly progression of correspondences between the differential equations for the orthogonal polynomials and the first term of the trigonometrically substituted angular equations which could result from some stable n-dimensional energy pattern.

The quantum mechanic description of the hydrogenic electron orbital shells follows this progression for the third spatial dimension of discussion. The Legendre P_n polynomials, technically the Jacobi $P_n[0,0,\cos(\theta_2)]$ polynomials and their derivatives, are used successfully to describe the geometric appearance of the spherical or second angular dimension. For the planar mathematics or first angular dimension, though, the mass density description assumed for the hydrogenic orbitals deviates and is described as:

$$D(\theta_1), \text{ hydrogenic electron orbitals} = qn_2 e^{(-i\theta_1\sqrt{qn_2})} \tag{67}$$

where $i = \sqrt{-1}$

Here with the leptons for the first and only angular parameters, for both the external and the internal angular expressions, the logical pattern of using the trigonometrically substituted Chebyshev $T_n^\dagger$ polynomials is simply followed. As seen in Tables 2 through 6, this assumed description gives the desired results.

Referring to the generalized cylindrical figure of Equation (2) to produce the constant value of the electrical charge of the leptons G'(t) was required to equal F'(t). Applying this to the angular mass density equations for the leptons results in the curious appearance of:

$$D(\theta) \text{ for the leptons} = T_n^\dagger(\sin[\pi/2A_i(t_\theta)]) \tag{68}$$

where the embedded $A_i(t_\theta)$ is itself $T_n^\dagger(\sin[n^{-1}t_\theta])$.

The $\pi/2$ is necessary so that the exterior $T_n^\dagger$ polynomial covers its full range of 0 to +1. The n^{-1} assures that the outside spatial sin() function covers a range of π over the integration. Again there is the reminder that the integrals for the angular expressions of the leptons, both those for the initial constant and for the angular equation itself, are integrals of substituted rectilinear orthogonal polynomials. These represent the solutions to some unspecified second order differential equations. The various angular equations are integrated from,-$\pi/2$ to $\pi/2$, or 2 x (0 to $\pi/2$), values which correspond to the valid range of the substituted original orthogonal polynomials. These are not polar or spherical mathematical forms. The simple rectilinear integrals dt_θ are used, and not the form $rdrdt_\theta$ used to find polar areas. Finally while these forms ultimately are for rectilinear mathematics, the working internals of them, the sin() and cos() functions, are indeed angular in nature in that they are cyclic trigonometrics. This applies to the internal or implicit function as well as to the external spatial function.

The two appearances $\sin[a \cos(bt_\theta)]$ and $\sin[a \sin(bt_\theta)]$ are found to work equally well, are orthogonal to each other, and are just phase shifted or represent two possible orientations of the angular function t_θ about the arbitrary starting point of the t_θ polar line. The implicit trigonometrics of $\sin(t_\theta)$ and $\cos(t_\theta)$ result in stable cyclic and bounded figures, unlike the open ended λ which results in the unbounded photon.

The ultimate embedded variable representing time, t_θ, produces an interesting concept. Thinking of the t_θ polar line as the present, proceeding angularly clockwise away from the t_θ polar line can be viewed as going into the past, and anticlockwise as going into the future.

The secondary and tertiary shells of the leptons are not described by the derivatives of the $T_n^\dagger$ polynomials. Reviewing the higher hydrogenic electron shells, the derivatives of the $P_n(\theta_2)$

polynomials give correct mathematical descriptions. This is because a mathematical "trick" can be employed to maintain the orthogonality of the derivatives of the P_n polynomials. The derivatives of the P_n polynomials can be multiplied by $(1\text{-}x^2)^{\text{derivative order / 2}}$ to force them to comply with the defining differential equations for the original functions and to simultaneously still maintain their orthogonality. In other words the definition of the weight factor for the original Legendre polynomials, numerically 1, has been redefined to this expression containing the variable x for the Legendre derivatives. There appears to be no similar trick which can be used to modify either the form or the weight factors of the derivatives of the $T_n^{\dagger}$polynomials.

The initial condition, again angular in nature, of $\cos(t_\theta)$ probably represents the linear or one dimensional energy pattern of a particle-wave running unimpeded around a smooth circular ring. The angular equations of the upper lepton members then mature into flower petal-like appearances as mature functions in time. Again there is a physical-mathematical gratuity that this simple initial angular condition applies to all the members of the series, and as with the radial initial condition is a self normalized distribution.

6.4 Discussion Of Scale Factors

Aside from the three factors (driving, shaping, and attenuating) of the radial equation, a fourth factor is necessary for a real particle to come into being. A factor is needed which gives concreteness or materialization to the mathematical expressions. This is the scale factor which mathematically translates from the arbitrary scale of math-geometry to the scale of the consensus world of humans. Here is where the real world intrudes upon what to this point has been purely sterile mathematic-geometric equations. This factor, typically a premultiplier external to any exponentials, trigonometrics, et cetera, is composed of physics constants. A typical example of this might be the $8m(\pi/h)$, with SI units of (s/m^2), found in the Schrodinger wave equation for the electron shells of the hydrogen atom. Another example is the conversion factor G found in $F = Gm_1m_2/r^2$, where the relative SI measurement units of G are $(m/kg)(m/s)^2$. It is this correlation constant which turns what otherwise would remain a correlation into an actual equation. At least one general scale factor must be involved in these equations. The math-geometric portion of the mass density equations results in units of (M relative/L radial_absolute) specifically kg/l_{Sgs} and needs to be multiplied by a quantity, distance absolute. As discussed with the charge of the leptons and in Equation (16) the value of $C_g = e\mu_o(G\varepsilon_o)^{1/2} = 1.0$ L absolute = 1.0 l_{Sgs} = 4.893,752,96 x 10^{-36}m needed the accuracy of G to be improved by back calculation from the equation for the charge.

One finds as they step through the Periodic Table of the Elements of Chemistry (PTEC), the best known mathematical-physical series, that each member has some uniqueness, some specific quirks of their own. The details of each member of the whole periodic table cannot be predicted by just examining the first element, hydrogen. Likewise, here the mass density equations for the first lepton member, the electron, are so simple that logically some added complications can be expected to arise when moving to the higher members of the series. The first factor contributing to the individual particle's uniqueness was called the series member factor in Section 4.2.1. This factor has the generic form:

$$F_k = \left[\frac{1}{2\alpha}\right]\left[\frac{a}{a^2 + b^2}\right]^2 \qquad (69)$$

where a = 6 and b = $(k\text{-}1)^{1/2}$.

This again has the appearance of ρ^2 found in the equation describing the charge of the leptons, and ultimately has the form of the curvature or torsion of a generalized cylindrical spiral, or a toroidal coil in this case.

The second factor contributing to the particle's uniqueness has been called the shielding factor. This modification or mathematical factor appears to describe some sort of "shielding", "binding energy", or "mass defect" in going from the electron to the muon to the tau. This factor appears to have the form:

$$F_\mu = 1/2\left[1 - \frac{1^{4/1}}{3!}\right]^{-1} \tag{70}$$

$$F_\tau = 1/4[1 - \frac{1^{4/1}}{3!} - \frac{2^{9/4}}{5!}]^{-1} \tag{71}$$

where the general form appears as follows:

$$F_p = 1/2^{n-1}[1 - 1^{4/1} /3! - 2^{9/4} /5! - 4^{16/9} /7! - \ldots]^{-1} \tag{72}$$

Unfortunately since the muon and tau are only two members of a series, and since the mass of the tau has not been measured to the accuracy of that of the electron and muon, the pattern is not well established.

One more individualizing factor was discernible, that which gave uniqueness to the individual shells of the higher members of the series. For the muon with a 7 decimal mass measurement and the contribution of its secondary shell only 3 orders of magnitude smaller than that of its primary shell, this factor is absolutely necessary and is mathematically precise. The mathematical accuracy and simplicity of this factor tend to preclude it from being a coincidence. The form found for this multiplication factor for the secondary shell of the muon is:

$$S_{\mu 2} = \left[1 + \left(\frac{1}{2}\right)\left(\frac{1}{37}\right)\right] \tag{73}$$

Little effort is needed to recognize this 1/37 as$1/(6^2 + 1^2)$. For the tau with only 5 decimals of measurement accuracy and with its secondary shell contributing 4 orders of magnitude less energy than its primary, the effects of this factor are not discernible. Similarly, for the tertiary shell of the tau this individualizing factor cannot be specified.

6.5 Accuracy, Convergence, And Sources Of Error

Unlike the derivation of the charge of the leptons, e, the derivations of the masses of the leptons each involve four or more integrals for which there are no analytical expressions. Aside from the issue of the accuracy of the physics constants used in the final scaling factors, the accuracy of the evaluation of these iterated numerical integrals becomes an issue. The accuracy of the evaluation of these iterated integrals is directly related to the step size used in their evaluation. In general for each order of magnitude of decrease in step size there is usually a corresponding increase of one decimal of accuracy in the result, and unfortunately also a corresponding order of magnitude increase in time required for the same computer to calculate the integral. The first source of calculational error is a matter of time and computer speed.

For example on the computers used, to evaluate the initial conditions of the radial functions over the mere range of t_r = (0 to 4) required six days at a step size of 10^{-7}, for each power of $R_e(t_r)$ in the L_n polynomials. This is due to the initial condition being the Fraunhofer Diffraction Function, FHDif(kr^1). While calculation of this integral of the initial condition x radial function is important in producing the initial constant for each radial equation, it converges fairly rapidly due to the factor of $e^{-6t_r^2}$ in the radial function.

Table 8 Stepwise Values Of Radial Equation Integrals

Values of $\int D_{pk}(r)$, (radial equation $D_{pk}(r)$ w/o C_{rpk}), varying $R_e(t_r^n)$			
n	Step Size = 10^{-5}	Step Size = 10^{-6}	Step Size = 10^{-7}
0	3.428,165,605 x 10^2	3.428,165,330 x 10^2	3.428,165,302 x 10^2
1	4.167,523,186 x 10^3	4.167,52,3151 x 10^3	4.167,523,147 x 10^3
2	5.485,644,082 x 10^4	5.485,644,078 x 10^4	5.485,644,077 x 10^4
3	7.688,701,888 x 10^5	7.688,701,887 x 10^5	7.688,701,886 x 10^5
4	1.135,987,718 x 10^7	1.135,987,718 x 10^7	1.135,987,718 x 10^7
5			1.756,993,752 x 10^8
6			2.830,089,007 x 10^9

Values of Initial Constant C_{rpk}, ($\int$initial cond x radial eq), varying $R_e(t_r^n)$				
n	Step Size = 10^{-4}	Step Size = 10^{-5}	Step Size = 10^{-6}	Step Size = 10^{-7}
0	1.618,536,745 x 10^2	1.618,533,994 x 10^2	1.618,533,719 x 10^2	1.618,533,691 x 10^2
1	1.726,753,477 x 10^3	1.726,753,132 x 10^3	1.726,753,098 x 10^3	1.726,753,094 x 10^3
2	2.010,471,947 x 10^4	2.010,471,904 x 10^4	2.010,471,899 x 10^4	2.010,471,899 x 10^4
3	2.502,709,184 x 10^5	2.502,709,179 x 10^5	2.502,709,178 x 10^5	2.502,709,178 x 10^5
4	3.291,601,133 x 10^6	3.291,601,132 x 10^6	3.291,601,132 x 10^6	
5		4.537,587,651 x 10^7	4.537,587,651 x 10^7	
6		6.518,237,500 x 10^8	6.518,237,500 x 10^8	

Notes:
These values for $R_e(t_r^n)$ must be combined appropriately and normalized to produce the $L_n[R_e(t_r)]$ polynomials.
The definitions of $R_c(t_r)$ & $R_e(t_r)$ are defined in Equations (21) & (22), in Section 4.2
The initial condition I(r) is defined in Equation (23), in Section 4.2
The value used here for the Fraunhofer Diffraction Integral of (1.0 r), as if it were exact, is k = 1.697,525,53
Limits of the integral of the radial expression were; Lower = 0, Upper = 8
Limits of the initial condition times the radial expression were; Lower = 0, Upper = 4
A step size of 10^{-n} means 1.745,329,925 x 10^{-n} = (π/180) x $10^{(2-n)}$

Table 9 Stepwise Values Of Angular Equation Integrals

Value of $\int D_{pk}(\theta)$, varying $T_n^{\dagger}(\sin[\pi/2\ A_i(t_\theta)])$			
n	Step Size = 10^{-5}	Step Size = 10^{-6}	Step Size = 10^{-7}
1	1.180,588,710	1.180,580,856	1.180,580,071
3	0.191,651,522	0.191,643,668	0.191,642,882
5	0.436,383,775	0.436,375,921	0.436,375,136
7	0.173,851,730	0.173,843,876	0.173,843,091

Values Initial Constant $C_{\theta pk}$, varying $T_n^{\dagger}(\sin[\pi/2\ A_i(t_\theta)])$			
n	Step Size = 10^{-5}	Step Size = 10^{-6}	Step Size = 10^{-7}
1	0.890,373,923	0.890,366,069	0.890,365,284
3	0.442,435,935	0.442,428,081	0.442,427,296
5	0.331,860,548	0.331,852,694	0.331,851,909
7	0.276,621,144	0.276,613,290	0.276,612,505

Notes:
$A_i(t_\theta) = T_n^{\dagger}(\cos[n^{-1}t_\theta])$, Initial condition = $\cos(t_\theta)$
Normalizing factors of $(\pi/2)^{-1/2}$ for the exterior $T_n^{\dagger}$orthogonal polynomials are not included in the presentation here.
Lower limits = 0, Upper limits = $\pi/2$
A step size of 10^{-n} means 1.745,329,925 x 10^{-n} = (π/180) x $10^{(2-n)}$

"Final" values for the integrals of the radial and angular equations were presented in Tables 1 and 2. What would be nice is for the reader to see what is behind these formal looking results, just so they would know that these answers do have a rigorous basis. Tables 8 and 9 following show just how these "final" values were obtained. As seen there is a fairly rapid convergence of these integrals towards the number of decimal places required in this work.

Even more important to this work, than the initial conditions for the radial integrals, is the exact evaluation of the overall or free standing Fraunhofer Diffraction Integral.

$$\mathrm{FHDif}[F(r)] = \int_0^\infty \mathrm{FHDif}(1.0r^1)\,dr = k = 1.697{,}525{,}53\ldots \qquad (74)$$

This is used in the initial condition $\mathrm{FHDif}(kr^1)$ and throughout the radial equations within $R_e(t_r)$. Only five of the decimals shown for k could be obtained with absolute certainty. To have obtained the seven decimal accuracy necessary for this free standing integral would have required a step size of 10^{-8} or 10^{-9} over the range r = (0 to ~10π), when each π multiple already required four days at a step size of 10^{-7}. The cure for this source of error is obvious, access to higher speed computers. After having spent 12 1/2 years to produce the initial results of this research and receiving an utterly hostile reaction from the academic hypothetical physics community, why pursue any more decimals? Would it be possible to please an in-group who hasn't produced any decimals of their own matching real world data? This pursuit of further accuracy is somewhat moot though, since the measured decimal accuracy of the lepton masses would be surpassed and the consensus world would provide no feedback to what then would be purely an intellectual effort.

Examining the application in this work of the Fraunhofer Diffraction Function,

$$I(r) = \mathrm{FHDif}[F(r)] = \left[\frac{2J_1[F(r)]}{F(r)}\right]^2$$

one finds the source of multiple difficulties, the Bessel functions, $J_1[F(r)]$. Due to these Bessel functions, evaluating either

$$\int_0^\infty \mathrm{FHDif}(1.0r^1)\,dr = k \qquad (75)$$

to determine the exact value of k, or

$$\int_0^\infty \mathrm{FHDif}(kr^1)\,dr = 1.0 \qquad (76)$$

to guarantee that this integral converges to 1.0 and is self normalized, both entail the same difficulty. These integrals converge very slowly. Values of the upper limit reach past 1000π before 7 decimals of convergence were obtained.

A second difficulty is the question of the range of applicability of the general series solution for the Bessel functions. The range approaching zero, $< 10^{-4}$, contributes significantly to both of these integrals, as well as to those determining the initial constants of the radial equations. For these two full range integrals (0 to ∞) at large values, > 10π, or for very large values, > 100π what algorithms should be used? There are several asymptotic and approximation formulas. Against what would the 7 decimal accuracy of these algorithms, of lack thereof, be tested?

A third difficulty occurs in the evaluation of Bessel functions J_n at large values. The series solution formula for the Bessel functions J_n is as follows

$$J_n(x) = \sum_{k=0}^{\infty}(-1)^k \left(\frac{x}{2}\right)^{(2k+n)} /(k!\,(n+k)!) \tag{77}$$

As the argument x for J_n gets large the series solution must be driven to a higher and higher number of summation steps, k, to bring it to a convergence. At arguments $x > 10\pi$, the number summation steps, k, may reach past 100 to obtain 7 decimals of convergence. Examining this general series solution to the Bessel equation there is seen one numerator and two denominators. As the number of k steps gets large this numerator and both denominators each individually tend to go to infinity. Only through a very delicate balancing act between this numerator and these denominators does each individual term of the series converge to a value. This value of each term becomes decreasingly small as k grows and the whole summation converges. This is what is supposed to happen in theory. What actually happens is that as the number of terms grows large, then the numerical difference between the largest and the smallest grows to > 10^{15}. When the difference in size of the terms of the series becomes greater than 15 orders of magnitude most computers develop overflow-underflow errors. This is due to the math packages used by the operating systems of most personal computers only carrying 15 decimals of accuracy. Ultimately as arguments for Jn become large these inherent computer overflow-underflow problems drive the series solution to diverge long before it can converge. The asymptotic and other large value approximations are of no benefit because they too involve infinite series with terms also having delicately balanced numerators and denominators. This problem can be solved by a knowledgeable computer programmer or by access to computers with entirely differently based math packages.

With the use of computers at large and very large values of the argument of the Bessel functions there is also the choice of just using the defining differential equations for the Bessel functions to determine the contribution of these regions to the overall constant Fraunhofer Diffraction Constant, k. Performing the necessary calculations to evaluate

$$x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + (x^2 - n^2)y \tag{78}$$

are relatively easy but as to be expected yield every so slightly different values from the series solution formula.

A final potential source of error is not calculational but is hypothetical or a matter of assumptions. The question needs to be asked whether or not the Fraunhofer Diffraction Function is applicable or is the correct representation of the initial radial energy distribution over all regions of r, with the very small and very large being of most concern.

6.6 Final Miscellaneous Observations

While developing these equations or reading thru them carefully several other items of interest can be noticed.

Examining the secondary shell of the muon in Tables 1 and 2, some insightful information is found. This integrated radial equation is identical with that of the electron, except that it represents a 2nd derivative and has its own normalizing factor of $2^{-1/2}$, instead of simply 1 as for the electron. The integrated angular equation is again identical with that of the electron, except just divided by 3. Before any individual shell multiplier is applied, the radial x angular product for this muon secondary shell is the same as that of the electron, simply divided by $3\sqrt{2}$. In short, this secondary shell could be thought of as a low energy electron wave pattern embedded within the muon. Likewise, the tertiary shell of the tau is in fact another electron mass simply divided by $5\sqrt{24}$. For the secondary shell of the tau such a simplistic analogy to the muon could not be obtained.

7 Conclusions

The specific objective of this work was: starting with the three universal force constants G, μ_o, ε_o as logically a-priori to develop mathematical equations which explain some of the fundamental measured physical properties of the leptons. Equations were found which predict or match the measured charge of the leptons, and the measured masses of the three leptons to the required accuracy.

The nature of these equations is as follows. In general form, the mass density equations are similar to those describing the electron shells of the hydrogen atom. They contain a radial planar equation in space and one angular equation in space. These spatial equations in turn both contain embedded or implicit temporal equations, and each have initial conditions in time that result in multiplying factors. The radial equations have two exponentials multiplying an appropriate member of the Laguerre orthogonal polynomial series. The angular equations, both the external spatial equations and the internal temporal equations, are trigonometrically substituted Chebyshev $T_n^{\dagger}$ orthogonal polynomials. There are also a series of well defined factors which serve to scale from an arbitrarily sized realm of math-geometry to the consensus world of physics.

The general geometric appearance found for these particles was that of a toroidal coil. The equation for the charge of the leptons follows simply and directly from the vector formulation of the curvature or torsion of this toroidal description.

This mathematical-geometric model leads to several additional conclusions.

1 The leptons, and logically the neutrinos and quarks, have definable structures and are not mathematical points. Albeit, the diameters of these structures or wave patterns probably are many orders of magnitude too small for physicists to ever measure.

2 Time may be two dimensional, at least. In the specific case of the mass density structures of the leptons, there is one temporal parameter for each of the two spatial dimensions. There appears to be no mathematical requirement that these two temporal parameters or dimensions be linked or that one be dependent on the other. Even further, the independence of these two temporal parameters appears to be mandatory.

3 Mathematically there can be a fourth lepton, the shipa, with a net positive mass, and logically a fourth neutrino and family of quarks. The equations for the shipa were briefly investigated. Although mathematically possible, such a particle appears somewhat improbable. Its mass structure appears inefficient in comparison to the muon and tau. It has one energy shell with a net negative value. Also its angular geometric appearance indicates that it probably would rapidly self-destruct similar to a badly imbalanced airplane propeller.

4 One of the factors involved in the scaling of the mass density equations from math-geometry to the consensus world of physics suggests shielding, binding energy, or the lowering-minimizing of the energy state of a composite structure. That is, in going from the electron to the muon to the tau an effect was found similar to the mass defect found in going from H to He to Li. This implies that the muon and the tau probably have composite internal structures analogous to He and Li.

5 By rearranging the vector formulation of the structural appearance which gives rise to the leptons' charge, the value of G can be back calculated to three orders of magnitude more accuracy than that to which it can be measured. Although this rearrangement, interchanging of parametric dependencies, runs against the grain of the logic in this work, such a reordering of variables from $e = F_1(G, \varepsilon_o, \mu_o,$ and geometry) to become $G = F_2(e, \varepsilon_o, \mu_o,$ and geometry) is simple and legitimate mathematically and serves a beneficial purpose.

6 The Laguerre orthogonal polynomials, which are solutions to second order differential equations, have been used to step thru the lepton mass series of electron, muon, and tau. Therefore a person could say that the mathematical solutions which were found for the lepton masses are essentially quantum mechanical in nature.
7 Additionally the geometric forms found for the leptons consist of spinning radial-angular plane waves, which propagate into space with time. Therefore a person could say that these elementary energy forms could be ascribed as being membranes of the string-membrane school of hypothetical particle physics.

8 References

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CHAPTER 1.2 A MODEL FOR DETERMINING PHYSICAL PROPERTIES II: PROPERTIES OF PHOTONS

1 Introduction

1.1 Abstract

The mathematical forms which describe the structures of all the basic electromagnetic waveforms, "particles", leptons and photons alike, are discussed in this report. Specifically, an equation is presented which explains the value of the Planck constant *h*, 6.626,075,5 x 10^{-34} (kgm)(m/s). This equation has a general form very similar to those which were discovered that predict the masses of the leptons. From a mathematical view, only a few modifications are required to go from the leptons to the photons and vice versa. The form of this photon equation contrasted with the general form of the lepton equations points toward explanations of the notable physical property differences between the photons and the leptons, e.g. the photons have no mass, display no charge, and have a spin different from the leptons, et cetera. The nature of this photon equation and the geometry that it represents has several profound implications for cosmology and particle physics.

1.2 Objective & Scope

The general objective of this report is to show observed mathematical organizing principles and patterns which describe the structures of the elementary electromagnetic waveforms, "particles", leptons and photons alike. The specific objective of this work is to show a derivation for the Planck constant *h*. This derivation is based on the known geometric structural form of photons, the cylindrical helix. Also a desirable objective is answering why the Planck constant *h* is constant while the photons' wave lengths can vary over a known range of many orders of magnitude.

Additional objectives are to compare the equations found for the photons with those which were found for the mass densities of the leptons discussed in the lepton report. This mathematical analysis point toward explanations of the notable physical property differences between these two waveform, "particle", species.

The formula discovered for the photon (ML)(L/T) is simply presented here. Its parts (factors) and the analogous mathematical forms from the lepton equations are described and analyzed to ensure understanding of their correct calculation and use. This work is best described as a mathematical analysis of some very limited and exact data which has resulted in an applied mathematical model. This presentation should be thought of as only a demonstration. The descriptive equations found are not proven nor derived in the formal or rigorous mathematical sense of those words.

The overall model discovered for the electromagnetic waveforms, "particles", was not derived from a hypothesis but was the result of a correlative approach. The model presented here does not start with a hypothetical platform, does not specifically support any particular hypothesis, has not been embedded into any preexisting hypothesis, nor has a hypothesis been constructed around it. While there may be implications concerning particle hypotheses indirectly supported by this work, only those conclusions directly supported by the mathematics of the equations that were found are reported. Implications of this work for the multitude of current grand particle hypotheses are beyond the scope of this work.

1.3 Historical And Current Particle Research Efforts

Examining major physics journals such as Physical Review D, Nuclear Physics B, Physics Letters B, Progress in Particle and Nuclear Physics over the last 10, 20, and even 30 years one can find thousands and even tens of thousands of articles focusing on the fermions and new exotic bosons. Specifically the leptons and quarks are analyzed from about every possible hypothetical, mathematical, and experimental angle possible. In the near past much effort centered around the weak "force" species of bosons.

Currently much speculation is devoted to the neutrinos. There are a plethora of such efforts. Some of the typical ones are referenced in Chapter 4.2, How Not To Approach This Work.

What are conspicuous by their absence are reports of research work devoted to examining and explaining the photons. With the conceptualization and quantification of the Planck constant, all interest in the photons appears to have dropped. Additionally, the photons have long since been shown to have some form of a structure which rotates as it progresses forward, making the outline of a cylindrical helix as it passes an observer. This structure has an electrical and a magnetic vector at right angles to each other, which are also oriented in a radial manner sideways to the photon's flight path. With this basic knowledge amassed, now the photons are apparently only studied for their use in relation to practical applications, such as polarized sunglasses or lasers. Photons have also been involved in searches for other basic physics knowledge such as with beam splitters examining the question of super-luminal information transfer. But the study of the photon itself for its own sake appears to be a thing of the past.

This is an unfortunate state of affairs because there are still several major unexplained aspects or assumptions made about the photon, one of the most basic of all physics waveforms, "particles". First and foremost, the Planck constant has never been explained but is treated as a basic assumption. Much effort is made to explain the unexplained masses of the other elementary electromagnetic particles, the leptons, but none to explain this energy equivalent for the photons. Secondly, there is the obvious question, how can the Planck constant be constant when the wavelength of the photons' varies over a known range of many orders of magnitude?

2 Outline Of Work

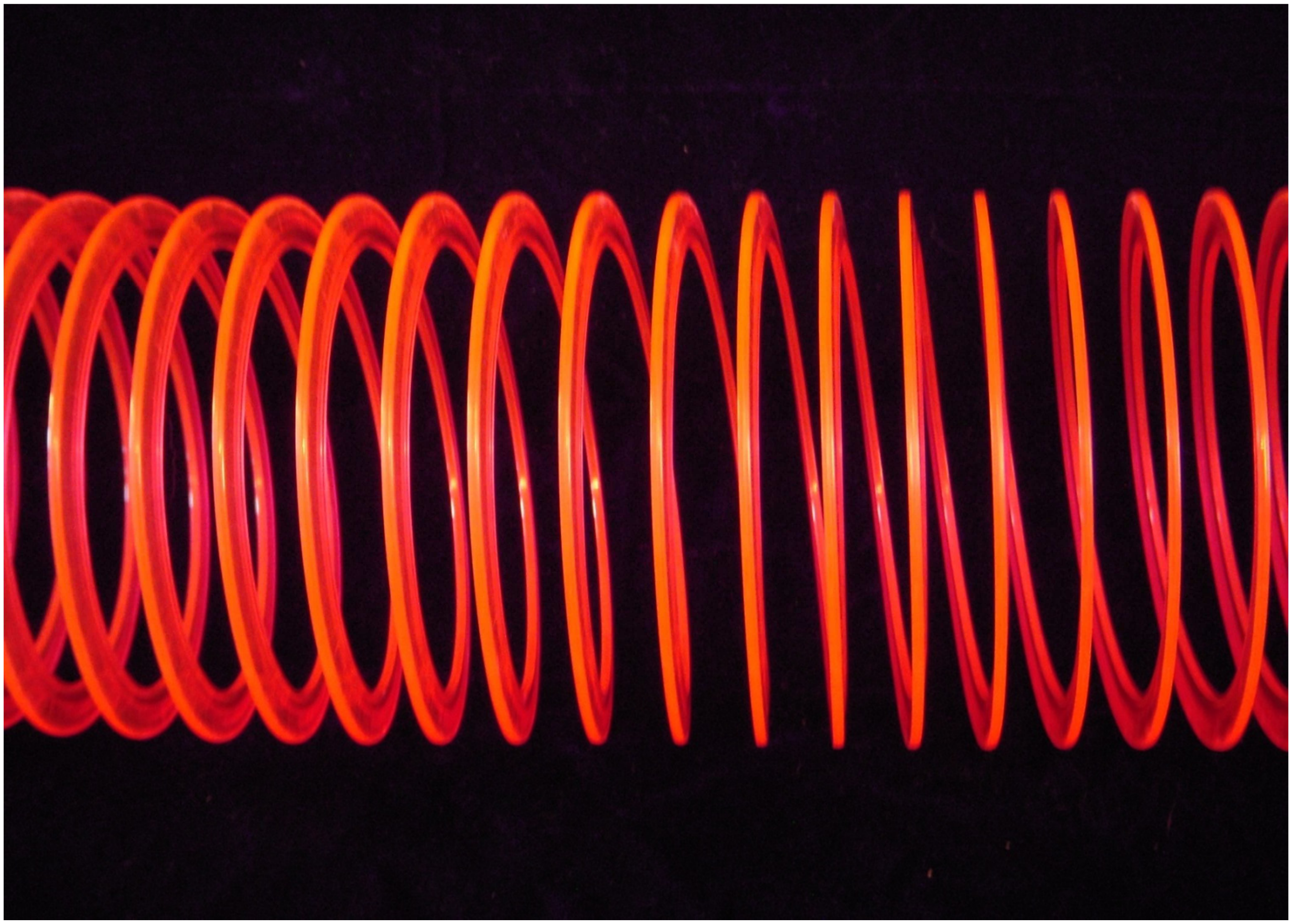
This report examines the common mathematical structural phenomena which can explain the contained energy of both of the elementary electromagnetic waveform species. The word energy is used here to mean a measure of the entrapped or enclosed gravitational energy stabilized by the waveform. A tit-for-tat mathematical picture is developed for the gravitational structure of the photons comparable with that discussed in the lepton report which lead to the precise calculation of masses of the leptons.

What is amazing here is that not only can structural equations be developed which lead to the exact calculation of the masses of the leptons in kg and the (ML)(L/T) for the photons in (kgm)(m/s), but also that all the component factors within these equations have real world meaning. All the component factors within the equations discovered have common simplistic geometric mappings to the physical world as humans understand it. Further, analysis of the various factors and implicit variables within these equations directly lead to explanations for many of the other observed physical properties of the waveforms. The equations discovered lead to much more than just their precise target or objective numerical values. These references to common sense features of the world as experienced by humans added with the cross referencing of other physical property information tend to virtually eliminate the possibility of these equations being happen-stance or coincidence.

The common core of structural descriptions found for both waveform species is a radial planar energy density pattern. This radial plane is set at right angles to the flight path of the waveforms, a straight line for the photons and a circular loop for the leptons. This two dimensional feature then immediately answers the mystery of the independence of the photon's quantum of energy from its wavelength. This is in the same sense that the leptons' rest masses are independent of their activated or excited states. The mathematical nature of this radial planar structure is outlined in the next immediate Section 2.1 and discussed in full detail for both waveform species in Section 4.1.

Structures are detailed in depth in this report that describe the gravitational picture of both of the elementary electromagnetic waveforms, "particles", photons and leptons alike. Additionally the electromagnetic structure of both species is discussed in the last section of this report. An analogy is made for the photons to the mathematical description of the electromagnetic structure discussed in the lepton report for the leptons. This mathematical analogy explains why the photons, while they may be "carriers" of charge, display no measurable charge themselves.

PICTURE 1 SNAPSHOT OF A HIGHLY MAGNIFIED FROZEN PHOTON



2.1 Mathematical Model For Structures Of Electromagnetic Waveforms, "Particles"

In this section, the common mathematical-geometric features found for all the elementary electromagnetic waveforms, both leptons and photons alike, are described. These are as follows.

1 A radial energy density equation (2 dimensional, planar), $D_L(r)$ for the leptons and $D_P(r)$ for the photons. Note when used within the lepton report describing the leptons' mathematics, $D_L(r)$ appears as $D_{pk}(r)$ where p designated the electron, muon, or tau and k designated the different energy shells.
2 Within the radial equations $D_L(r)$ and $D_p(r)$ there is a Contractive Radial Spatial Factor, R_{csf}, which is identical for the two species leptons and photons. Within these contractive factors of each species there are identical embedded implicit functions $R_c(t_r)$ of the radial implicit variable t_r. Using other choices of words, R_{csf} could be called a contractive function and $R_c(t_r)$ its argument.
3 Within the radial equations $D_L(r)$ and $D_p(r)$ there is an Expansive Radial Spatial Factor, R_{esfL} for the leptons and R_{esfP} for the photons. These two factors have the identical generic appearance or form R_{esf} seen later.

4 Within the Expansive Radial Spatial Factors of each species there are embedded or implicit functions $R_e(t_r)$, again of the ultimate radial implicit variable t_r. These implicit functions, or arguments, are distinct for the two species and becomes the subject of much discussion. Here these functions are labeled $R_{eL}(t_r)$ and $R_{eP}(t_r)$ when being applied to the specific species.
5 An angular energy density equations (a single angle), $D_L(\theta)$ for the leptons and $D_P(\theta)$ for the photons. Note when used within the lepton report describing the leptons' mathematics, $D_L(\theta)$ appears as $D_{pk}(\theta)$, where p designated the electron, muon, or tau and k designated the different energy shells.
6 The angular equation $D_L(\theta)$ has an Outer or exterior Angular Spatial Functional appearance A_{osfL} for the leptons and A_{osfP} for the photons. These two have the identical generic appearance or form A_{osf}. This function is based on the Chebyshev $T^{\dagger}$orthogonal polynomials.
7 The angular equation $D_L(\theta)$ has an Inner or implicit angular functional appearance $A_{iL}(t_\theta)$ and $A_{iP}(t_\theta)$ as the argument of the Outer Spatial Function. These are again identical in generic appearance or form $A_i(t_\theta)$ of the angular implicit variable t_θ. Although these angular inner or implicit functions have identical generic functional forms, within them the use of the ultimate implicit variables t_θ, their arguments, are distinct. This distinction between the two species is critically important and becomes the subject of much discussion.
8 Initial temporal, boundary, or normalizing conditions for both the radial and angular equations, which lead to initial multiplying factors or constants. For the lepton report these are $I(r)$ which leads to the factor C_{rpk} and $I(\theta)$ which leads to the factor $C_{\theta pk}$. Here the designations $I_{rL}(t_r)$, $I_{rP}(t_r)$ are used which lead to C_{rL}, C_{rP}, respectively, and $I_{\theta L}(t_\theta)$, $I_{\theta P}(t_\theta)$ leading to $C_{\theta L}$ and $C_{\theta P}$.
9 A general scale factor or correlation constant for all the waveforms, composed of basic a-priori measured physical constants. This appears as C_g in the lepton report, and are designated C_L and C_P here.
10 An overall equation combining the radial equations, angular equations, and the final scale factors as multipliers. This appears as m_p in the lepton report, and describes the mass of the particle in kg. Here this quantity is designated as e_p to describe the "energy" of an electromagnetic waveform.

Here the word energy is used in a very loose sense to describe the result of these equations. The ultimate result of the lepton equation is mass, where the ultimate result of the photon equation is (ML)(L/T), i.e. units of the Planck constant.

Finally a rationale is needed for the dimensional nature that is used in this report for the ultimate implicit variables found within both the radial and angular energy density equations of the elementary electromagnetic waveforms, "particles". In the report concerning the leptons, a meaning or interpretation for these parameters could have been assigned as being either spatial or temporal in nature. For the leptons there was no strong reason, driving necessity, or hard proof as to why temporal quantities should be used. Temporal variables were chosen mainly just to be in keeping with this report.

Here in this report in the discussions later comparing/contrasting the physical properties of the two elementary species, leptons and photons, then the major points of these discussions are much simpler to follow when the ultimate implicit variables are stated in terms of temporal phenomena. Particularly in Sections 5.1 General Observations, Section 5.4 Parity, Section 6.1 Mass-Masslessness, and Section 6.3 Spin, the mathematical conceptual approaches are easier to understand and generally make much more sense when temporal interpretations are assigned to the ultimate implicit variables. In this report the following are found; t_r, t_θ, $R_c(t_r)$, $R_e(t_r)$, $A_i(t_\theta)$, dt_r, dt_θ, and so on.

3 Mathematical Preliminaries

For this work three mathematical tools or features are of importance.

1 The Fraunhofer Diffraction Function, FHDif().
2 The distance function, ds().

3 The exponential form, $e^{-at_r^2}$. This last form is discussed in the report concerning possible mathematical descriptions for the quarks, Chapter 2.3.

First, the less commonly found Fraunhofer Diffraction Function FHDif[F(r)] is first used and discussed in the lepton report. Reviewing, the key features of this function are as follows:

$$\mathrm{FHDif}[\mathrm{F(r)}] = \left[\frac{2J_1[\mathrm{F(r)}]}{\mathrm{F(r)}}\right]^2 \qquad (01)$$

Where J_1 being the Bessel Function of the First Kind, Order 1. Specifically in this work F(r) is found to be = $k_L r^1$ for the leptons and = $k_P r^{1/2}$ for the photons. Where

$$k_L = 1.697{,}525{,}53... = \int_0^\infty \mathrm{FHDif}(1.000{,}000,\ldots r^1)dr \qquad (02)$$

Note that $\int_0^\infty \mathrm{FHDif}(k_L r^1)dr = 1.000{,}000,\ldots$ and could be used as a self normalizing initial distribution.

$$k_P = 1.980{,}416{,}377\ldots = [\int_0^\infty \mathrm{FHDif}(1.000{,}000,\ldots r^{1/2})dr]^{1/2} \qquad (03)$$

Note that $\int_0^\infty \mathrm{FHDif}\left(k_P r^{1/2}\right)dr = 1.000{,}000,\ldots$ and could be used as a self normalizing initial distribution.

More generalized where F(r) is the monomial $k_n r^n$, if k_n is calculated as:

$$k_n = [\int_0^\infty \mathrm{FHDif}(1.000{,}000,\ldots r^n)dr]^n \qquad (04)$$

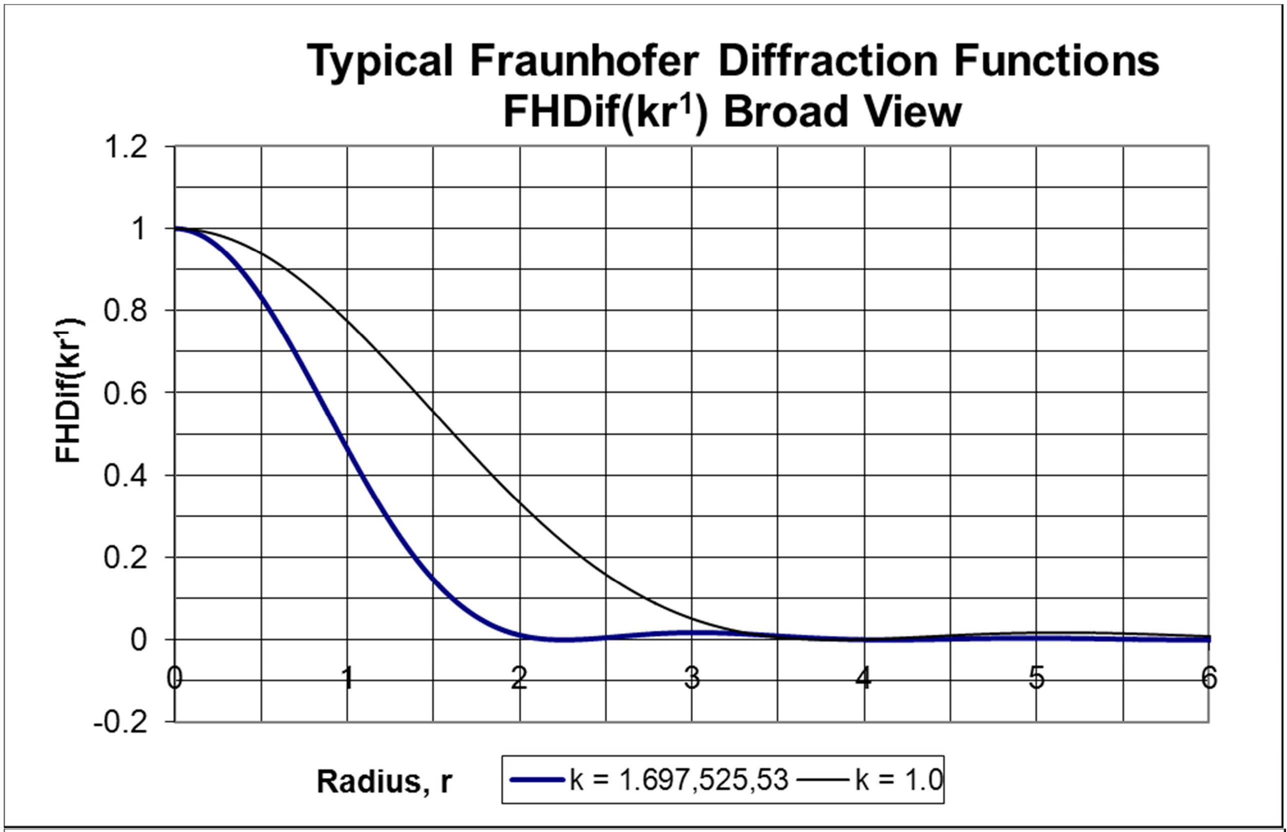
then $\int_0^\infty \mathrm{FHDif}(k_n r^n)dr = 1.000{,}000,\ldots$. A candidate can always be created for a self-normalized initial distribution using the Fraunhofer Diffraction Functions. Of course the variable in this function presented here as the spatial r can be anything, such as the temporal variable t_r.

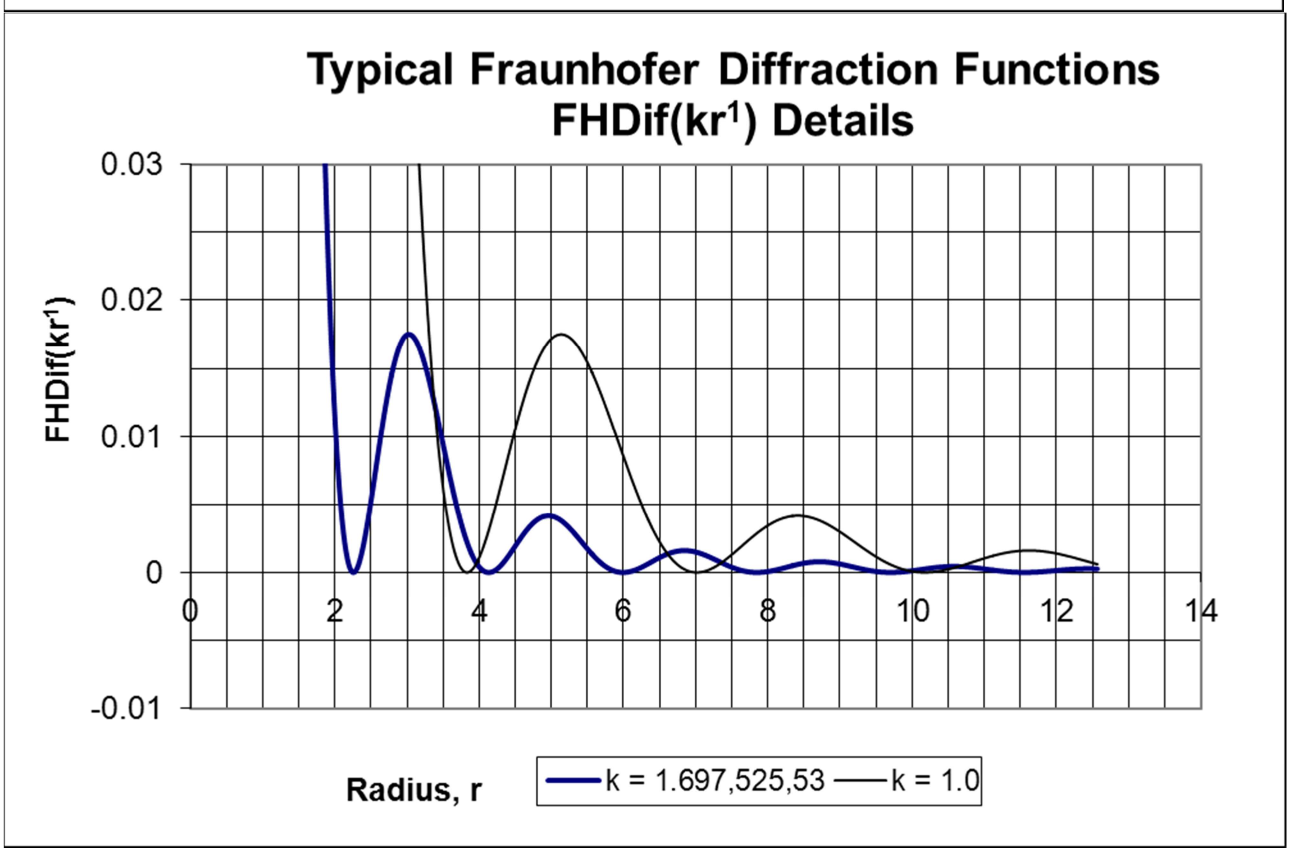
Graphical presentations of both of the functions FHDif[$k_L r^1$] and FHDif[$k_P r^{1/2}$] are seen in Figures 1 and 2. This Fraunhofer Diffraction Function has many interesting properties. For example, with this function the monomial = ar^1 has a reciprocal scaling property,

$$\int_0^\infty \mathrm{FHDif}[ar^1]dr = 1.679, 525\ .../a.$$

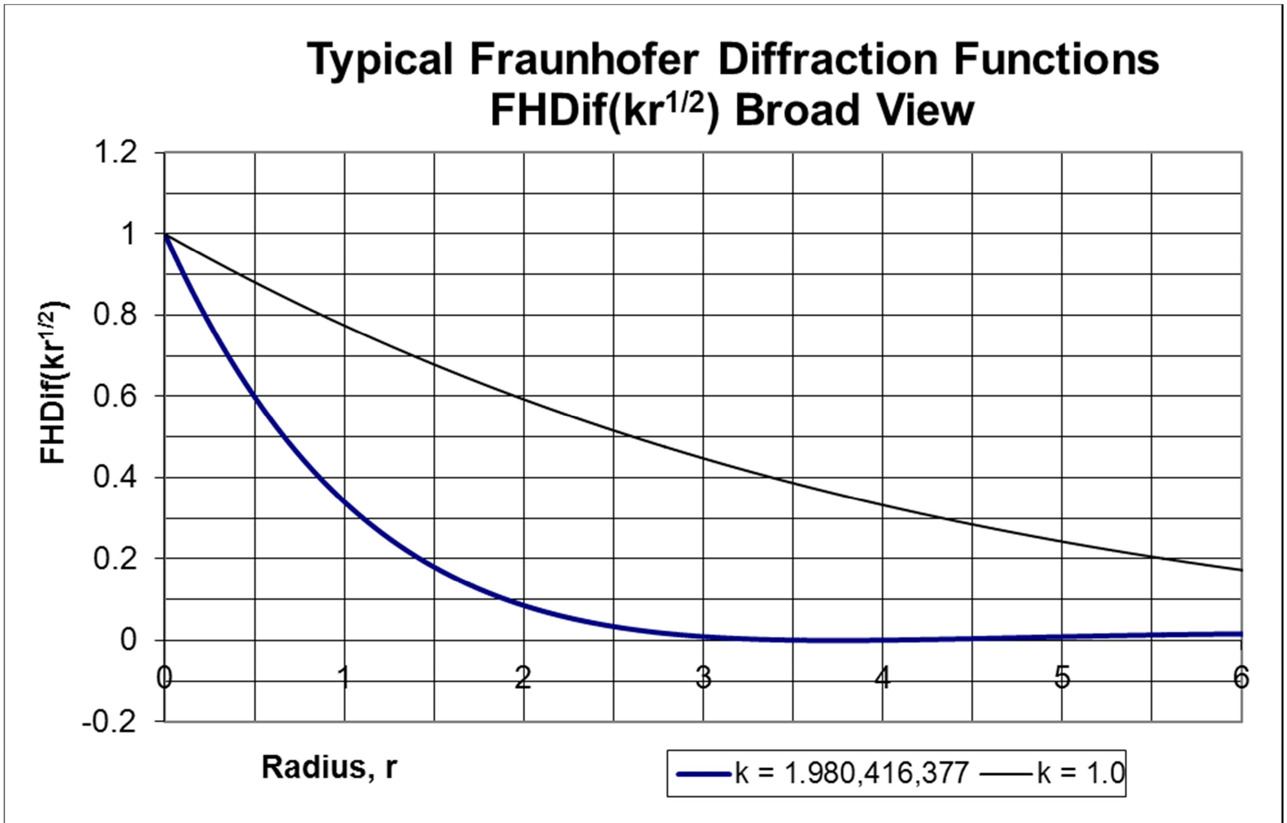
For calculational purposes, the most important property is the slowing of convergence with decreasing value of n, toward 0, when using the function F(r) = $k_n r^n$.

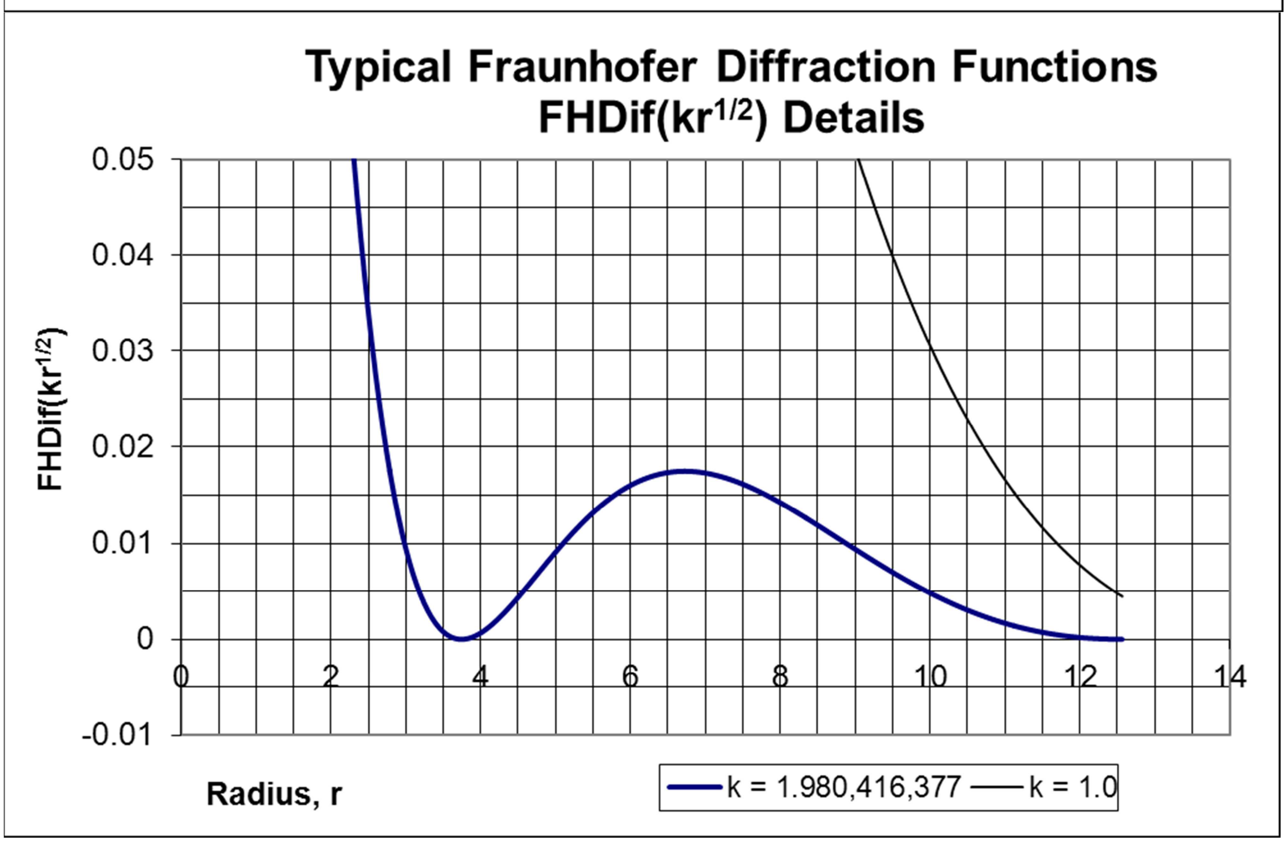
FIGURES 1.1 &1.2





FIGURES 2.1 & 2.2





The second tool, the distance function, is a common heavily used mathematical tool taught in first semester integral calculus. To clarify for the reader the usage of this function here is in a two dimensional rectilinear setting, as found below.

$$\frac{ds}{dt} = \left[1 + \left(\frac{dy}{dt}\right)^2\right]^{1/2} \qquad (05)$$

Specifically, for the form $Y_{parabolic} = R_e(t_r) = (ak_2^{-1/2}\ \pi t_r^{2})$,
which describes a parabolic curve that happens to be the area of an expanding circle weighted by $ak_2^{-1/2}$, the instantaneous distance along this parabolic curve in rectilinear coordinates is:

$$ds(Y) = ds\left(\frac{a\pi t_r^2}{k_2^{1/2}}\right) = \left[1 + \left(\frac{2a\pi t_r^1}{k_2^{1/2}}\right)^2\right]^{1/2} dt \qquad (06)$$

More generally, for the form$Y_{area} = R_e(t_r) = (ak_{2n}^{-1/2}\ \pi t_r^{2n})$
this is the weighted "circular area" producible from an initial curve $Y_{circ} = (a^{1/2}k_{2n}^{-1/4}t_r^{n})$.
The instantaneous distance along this "area" figure is:

$$ds(Y) = ds\left(\frac{a\pi t_r^{2n}}{k_{2n}^{1/2}}\right) = \left[1 + \left(\frac{2na\pi t_r^{2n-1}}{k_{2n}^{1/2}}\right)^2\right]^{1/2} dt \qquad (07)$$

4 Applications To Physical Property Determinations

4.1 General Features Of Equations

The general or generic forms of the mass density equations discovered for the leptons is discussed in the lepton report. The specific application of these mathematical forms to the individual leptons to give exact physical property information is also detailed there. Here structural equation forms are compared/contrasted between the two electromagnetic species the leptons and the photons.
The general or generic form of the equations for the mass density of the leptons and the (ML)(L/T) of the photons are given below. The specific application of these equations to the photons are given in Subsection 4.4.
The numerical results of using these equations are shown in Tables 1 and 2. Since this work is a mathematical endeavor, these results are presented so that the reader can reproduce, verify, and validate the "experimental" findings, if they so choose, before beginning any discussions as to their meaning. Likewise in the tables nine decimals are intentionally carried so that questions of computer calculation abilities and programming techniques can be settled.
Again the reader is reminded that the equations here and throughout Section 4 and the remainder of this report are the result of 14 years of mathematical correlative developmental effort and do not derive from a hypothetical approach. No hypothesis is expounded upon here before the equations are presented.
All the equations in Sections 4 & 5 are in terms of consensus real physical world units, and are not in terms of probability space, momentum space, or other such conceptual or mathematical spaces.

Beginning with the overall or final equation for calculating the energy of an electromagnetic waveform, e_p:

$$e_p = C_g C_p D_p \qquad (08)$$

where C_g is a general correlation constant or universal scaling constant for each species. C_p is the specific correlation constant for each lepton member. For the electron this is equal to 1.0. For the photons C_p also is equal 1.0 or may not apply at all. Here these two correlation constants are simplified to a single constant C_L and C_P for the leptons and photons respectively.
D_p is immediately composed of a radial equation D(r) and an angular equation D(θ). Additionally it is summed over the various applicable shells for the leptons. To maintain clarity of focus here only the mathematical appearances of the electron, which only has one shell, is referenced. Specifically the final overall equations for the two species are:

$$e_{Lepton} = e_L = C_L D_L(r) D_L(\theta) \quad \text{, units of M relative, kg} \tag{09}$$

$$e_{Photon} = e_P = C_P D_P(r) D_P(\theta)\text{, units of (ML)(L/T) relative, (kgm)(m/s)} \tag{10}$$

Beginning at the beginning:

$$C_L = e\mu_o(G\varepsilon_o)^{1/2} = 4.893{,}752{,}96 \times 10^{-36} \text{m}/l_{Sgs} \tag{11}$$

and

$$C_p = e^2(\mu_o/\varepsilon_o)^{1/2} = 9.670{,}562{,}404 \times 10^{-36} \text{ (kgm)(m/s) / } (m_{Sgs} l_{Sgs})(l_{Sgs}/t_{Sgs}) \tag{12}$$

These constants are the ultimate scaling factors which turn what otherwise would remain as just correlations into actual equations. They scale from the arbitrarily sized conceptual realm of math-geometry to the physical size of the consensus world.
The units of both these constants need some clarification. Both are actually conversion constants to the common or relative SI set of units from the units of the absolute physics Squigs system. They come from equations of the form, (1.0 absolute unit = #.## x $10^{-\#\#}$ relative units). These Squigs scales are based upon the measurement units put forth by George Johnstone Stoney in 1874. Except the Squigs scales have had his assumed 2 or 3 dimensional π constants removed. The universality of these units is discussed at great length and "proven" in Part 3, Analyses of Measurement Systems I, II, & III.

4.2 Radial Features Of Equations

For both species, D(r) is composed of an initial constant C_r, a contractive spatial factor R_{csf}, and an expansive spatial factor R_{esf}. The radial equations for the two species are:

$$D_L(r) = C_{rL} \int_0^{\infty} R_{csf} R_{esfL} dt_r = C_{rL} \int_0^{\infty} e^{(R_c(t_r))} e^{(R_{eL}(t_r))} dt_r \tag{13}$$

$$D_P(r) = C_{rP} \int_0^{\infty} R_{csf} R_{esfP} dt_r = C_{rP} \int_0^{\infty} e^{(R_c(t_r))} e^{(R_{eP}(t_r))} dt_r \tag{14}$$

The radial initial, boundary, or normalizing constants are derived from initial conditions as follows.

$$I(r) = \text{FHDif}[F(r)] = \left[\frac{2J_1[F(r)]}{F(r)}\right]^2 \tag{15}$$

More correctly this needs to be framed in terms of an ultimate implicit variable of t_r. Specifically:

$$I_{rL}(t_r) = \text{FHDif}[k_L t_r^{\,1}] = \text{FHDif}[1.697{,}525{,}53\ldots t_r^{\,1}] \tag{16}$$

This then leads to the initial or normalizing constant for the leptons' radial equation:

$$C_{rL} = \int_0^\infty I_{rL}(t_r)\, e^{(R_c(t_r))} e^{(R_{eL}(t_r))} dt_r \tag{17}$$

Likewise:

$$I_{rP}(t_r) = \text{FHDif}[k_P t_r^{1/2}] = \text{FHDif}[1.980{,}416{,}377\ldots t_r^{1/2}] \tag{18}$$

Which leads to the initial or normalizing constant for the photons' radial equation:

$$C_{rP} = \int_0^\infty I_{rP}(t_r)\, e^{(R_c(t_r))} e^{(R_{eP}(t_r))} dt_r \tag{19}$$

Where the derivation of the constants k_L and k_P were given in Equations (02) and (03) above.
The contractive spatial factor R_{csf} is identical for the two electromagnetic species. It has the form:

$$R_{csf} = F(R_c(t_r)) = e^{(R_c(t_r))} = e^{(-6t_r^2)} \tag{20}$$

The generic form of this function, $e^{(-ar^2)}$, has many special properties to be discussed in the Chapter 2.3.
The expansive spatial factor R_{esf} contains the embedded or implicit function $R_e(t_r)$ as its argument which is distinct or specific for the two electromagnetic waveforms. These are derived as follows. For the leptons:

$$R_{esfL} = e^{(R_{eL}(t_r))} L_n^d(R_{eL}(t_r)) \tag{21}$$

Where the L_n^d is a d^{th} derivative of the n^{th} Laguerre orthogonal polynomial. Where both n and d are even for the leptons, and specifically n = d = 0 for the electron. Since $L_0^0 = 1.0$ and since the normalizing factor for L_0^0 also = 1.0, then this polynomial factor is implied in the discussions here. This factor is absolutely necessary in correctly calculating and making a distinction between the masses for the higher members of the lepton series. The implicit function or argument for expansive radial factor of the leptons is:

$$R_{eL}(t_r) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} ds\left(\frac{2\pi t_r^2}{k_L^{1/2}}\right) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \left[1 + \left(\frac{4\pi t_r^1}{k_L^{1/2}}\right)^2\right]^{\frac{1}{2}} dt_r \tag{22}$$

Likewise for the photons:

$$R_{esfP} = e^{(R_{eP}(t_r))} L_n^d(R_{eP}(t_r)) \tag{23}$$

The Laguerre orthogonal polynomial factor L_n^d may not apply at all for the photons. Even if this factor does apply, just as with the electron this factor is likewise invisible here. The implicit function or argument for expansive radial factor of the photons is:

$$R_{eP}(t_r) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} ds\left(\frac{\pi t_r^1}{k_P^{1/2}}\right) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} \left[1 + \left(\frac{\pi t_r^0}{k_P^{1/2}}\right)^2\right]^{\frac{1}{2}} dt_r \tag{24}$$

Although ultimately in $R_{eP}(t_r)$ the implicit variable t_r is raised to the 0 power, its appearance is intentionally kept to show a pattern for the two species. The difference between t_r ultimately raised to the 2^{nd} power for the leptons and t_r raised to the 0 power for the photons is critical in understanding the physical property differences between the two waveforms. These differences are discussed in detail later.

4.3 Angular Features Of Equations

For both species $D(\theta)$ is composed of an initial constant, a symmetry multiplier, and an Outer or exterior angular Spatial Function, A_{osf}. This outer spatial factor is identical for the two electromagnetic species, and in tern contains an embedded Inner or implicit angular function as its argument, $A_i(t_\theta)$. This implicit function or argument is identical in generic form for the two species. Although the generic form is identical, these inner functional appearances, $A_{iL}(t_\theta)$ and $A_{iP}(t_\theta)$, make very different use of their ultimate implicit variables t_θ.

The outside or primary spatial function within the angular equation $D(\theta)$ has the form:

$$A_{osf} = T_n^{\dagger}(\sin[\pi/2\ A_i(t_\theta)]) \qquad (25)$$

These are trigonometrically substituted Chebyshev orthogonal polynomials and require normalizing factors of $(\pi/2)^{-1/2}$. For both the electron and the photons n = 1. There is a single sin() term present, which keeps the appearances simple. The angular equations for the two species are:

$$D_L(\theta) = C_{\theta L}\ 4\left(\frac{\pi}{2}\right)^{-\frac{1}{2}} \int_0^{\pi/2} T_1^{\dagger}\ (\sin[\pi/2\ \ A_{iL}(t_\theta)])dt_\theta \qquad (26)$$

$$D_P(\theta) = C_{\theta P}\ 2\left(\frac{\pi}{2}\right)^{-\frac{1}{2}} \int_0^{4/\pi} T_1^{\dagger}\ (\sin[\pi/2\ \ A_{iP}(t_\theta)])dt_\theta \qquad (27)$$

The angular initial, boundary, or normalizing constants are derived from initial conditions as follows.

$$I_{\theta L}(t_\theta) = \cos(t_\theta) \qquad (28)$$

Which leads to the initial or normalizing constant for the leptons' angular equation:

$$C_{\theta L} = \int_\theta^{\pi/2} I_{\theta L}(t_\theta)T_1^{\dagger}(\sin[\pi/2\ A_{iL}(t_\theta)])dt_\theta \qquad (29)$$

Likewise for the photons:

$$I_{\theta P}(t_\theta) = \cos(\pi^2/8\ t_\theta) \qquad (30)$$

Which leads to the initial or normalizing constant for the photons' radial equation:

$$C_{\theta P} = \int_0^{4/\pi} I_{\theta P}(t_\theta)T_1^{\dagger}\ (\sin[\pi/2\ A_{iP}\ (t_\theta)])dt_\theta \qquad (31)$$

As seen above in $D_L(\theta)$ the net symmetric factor for the leptons is 4. This is discussed in the lepton report as a result of symmetry of the integral about zero, and as a result of 2 orthogonal forms being applicable. As seen above in $D_P(\theta)$ the symmetry factor for the photons is 2, which is again the result of symmetry of the integral about zero.

The implicit angular function $A_i(t_\theta)$ is specific for the two electromagnetic species and is derived as follows. For the leptons:

$$A_{iL}(t_\theta) = T_n^{\dagger}(\cos[n^{-1}t_\theta]) \tag{32}$$

where again n = 1 for the electron.
Likewise for the photons:

$$A_{iP}(t_\theta) = T_n^{\dagger}([1 - (\pi/4\ t_\theta)^2]^{1/2}) \tag{33}$$

where again n = 1 for the photons.

Since in $A_{iL}(t_\theta)$ for the leptons the cos() function can be restated as:

$$\cos(n^{-1}t_\theta) = [1 - \sin^2(n^{-1}t_\theta)]^{1/2} \tag{34}$$

then this implicit angular function $A_i(t_\theta)$ can be seen to have the same generic or meta-form for both species:

$$A_i(t_\theta) = [1 - f^2(t_\theta)]^{1/2} \tag{35}$$

This concludes the detailing of the mathematical forms which describe the bases of the structures of the leptons and the photons. The specific application of these mathematical equations to calculate the masses of the leptons, to the required accuracy, are detailed in the lepton report. In the next section, the application is given for using these equations to calculate the (ML)(L/T), the Planck constant, for the photon.

These first three sections are ended seeing that the mathematics of the structural forms for the two basic or elementary electromagnetic waveforms, the leptons and the photons, have analogous and often identical features. The minor differences between their mathematical forms lead to profound differences in their measured and observed physical properties. These differences and their results are discussed in later sections. The similarities in their mathematics also lead to profound implications not only for these two waveforms but for all the other elementary "particles" and even for cosmology. Discussions of these implications are left for further reports.

4.4 Application Of Structural Forms To The Photons

As seen the equations found in this work show the (ML)(L/T), the Planck constant, for the photons to be the mathematically analogous counterpart or resulting structural equivalence to the mass of the leptons. Table 1 which follows, details the exact use of these structural equations to calculate the Planck constant for the photons. For comparison Table 2 shows the analogous calculations for the mass of the electron.

As seen in the Section 4.2 the overall radial equation for the photons, $D_P(r)$, was composed of two factors inside the integral: a contractive spatial factor, R_{csf}, and an expansive spatial factor, R_{esfP}. Further, although initially R_{esfP} was composed of a monomial of the variable t_r as follows:

$$\left(\frac{\pi t_r^1}{k_P^{1/2}}\right) \tag{36}$$

when the distance function ds was applied to this monomial, then the variable t_r effectively vanished. This second factor of time reduces to a constant and the remaining contractive radial spatial factor then can be integrated analytically. This same simplification does not occur with the radial equation of the leptons. Putting all this information together, the models for the ultimate physical property determinations are presented in Tables 1 and 2.

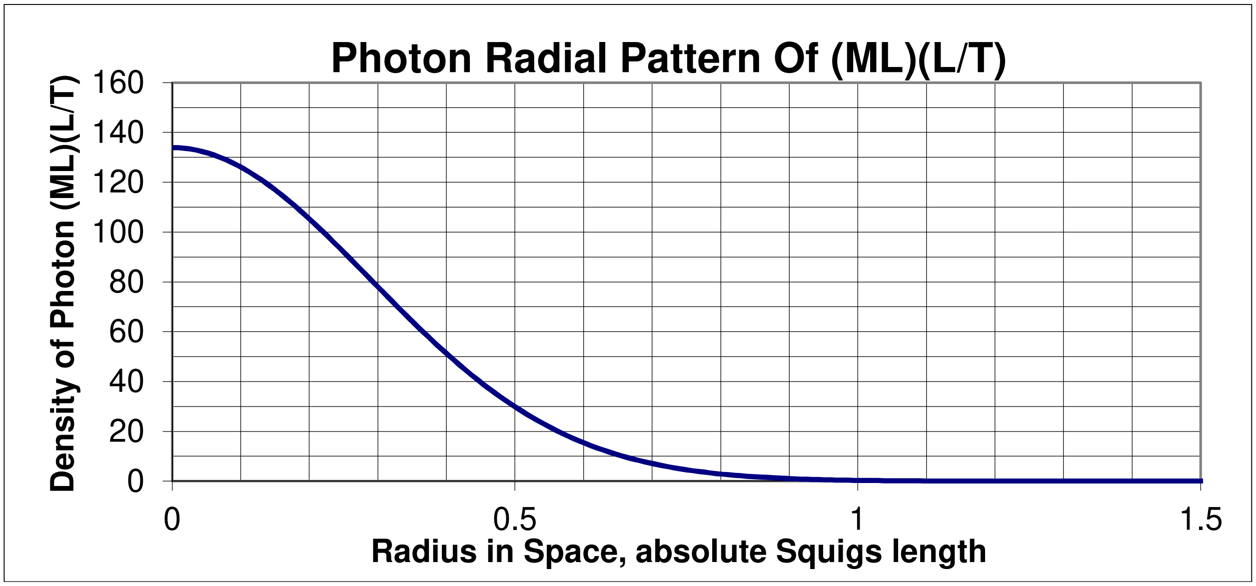
Table 1 Calculations Of The Planck Constant

FUNCTION	SYMBOLS	EXPRESSION	INTEGRATED VALUE
Radial Parameters			
Initial Constant	C_{rP}	$\int_0^\infty \mathrm{FHDif}[k_P t_r^{1/2}] R_{csf} R_{esfP} dt_r$	6.242, 125, 254 from iteration
Contractive Radial Spatial Factor	R_{csf}	$\int_0^\infty e^{(-6t_r^2)} dt_r = \sqrt{\pi}/(2\sqrt{6})$	0.361,800,627 from definite integral
Expansive Radial Spatial Factor	R_{esfP}	$e^{\left[(\frac{\pi}{2})^{1/2} ds\left(\frac{\pi t_r^1}{k_P^{1/2}}\right)\right]}$	21.451,225,729 from algebra
Net Integration	$\mathrm{Integ}D_P(r)$	$\int_0^\infty R_{csf} R_{esfP} dt_r$	7.761,066,925
Final Value	$D_P(r)$	$C_{rP} \int_0^\infty R_{csf} R_{esfP} dt_r$	**48.445,551,845**
Angular Parameters			
Initial Constant	$C_{\theta P}$	$\int_0^{4/\pi} \cos(\pi^2/8\ t_\theta) A_{osfP} dt_\theta$	0.781,812,090 from iteration
Symmetric Factor	sym	———	2.0
$T_1^\dagger$ Normalizing Factor	norm	$(\pi/2)^{-1/2}$	0.797,884,561
Inner Angular Equation	A_{iP}	$T_1^\dagger([1 - (\pi/4\ t_\theta)^2]^{1/2})$	not integrated independently
Outer Angular Equation	A_{osfP}	$\int_0^{4/\pi} T_1^\dagger(\sin[\pi/2\ A_{iP}(t_\theta)]) dt_\theta$	1.133,648,187 from iteration
Final Value	$D_P(\theta)$	$C_{\theta P}$ x sym x norm $\int_0^{4/\pi} A_{osfP} dt_\theta$	**1.414,329,947**
Combined Results			
Radial x Angular Product	$D_P(r)D_P(\theta)$	———	68,517,994,75 (ML)(L/T) Sgs units
Scaling Factor	C_P	$e^2(\mu_o/\varepsilon_o)^{1/2}$	9.670,562,404 x 10^{-36}SI/ Sgs units
Final Calculated Value	e_{Photon}	$C_P D_P(r) D_P(\theta)$	**6.626,075,440 x 10^{-34} (kgm)(m/s)**

Table 2 Calculation Of The Electron Mass

FUNCTION	SYMBOLS	EXPRESSION	INTEGRATED VALUE
Radial Parameters			
Initial Constant	C_{rL}	$\int_0^\infty \mathrm{FHDif}[k_L t_r^1] R_{csf} R_{esfL} dt_r$	1.618,533,691 x 10^2 from iteration
Contractive Radial Spatial Factor	R_{csf}	$e^{(-6t_r^2)}$	not integrated independently
Expansive Radial Spatial Factor	R_{esfL}	$e^{\left[(\frac{\pi}{2})^{1/2} ds\left(\frac{2\pi t_r^2}{k_L^{1/2}}\right)\right]}$	not integrated independently
Net Integration	$\mathrm{Integ}D_L(r)$	$\int_0^\infty R_{csf} R_{esfL} dt_r$	3.428,165,302 x 10^2 from iteration
Final Value	$D_L(r)$	$C_{rL} \int_0^\infty R_{csf} R_{esfL} dt_r$	**5.548,601,040 x 10^4**
Angular Parameters			
Initial Constant	$C_{\theta L}$	$\int_0^{\pi/2} \cos(t_\theta) A_{osfL} dt_\theta$	0.890,365,284 from iteration
Symmetric Factor	sym	— — —	4.0
$T_1^\dagger$ Normalizing Factor	norm	$(\pi/2)^{-1/2}$	0.797,884,561
Inner Angular Equation	A_{iL}	$T_1^\dagger(\cos[n^{-1}t_\theta])$	not integrated independently
Outer Angular Equation	A_{osfL}	$\int_0^{\pi/2} T_1^\dagger(\sin[\pi/2\ A_{iL}(t_\theta)])dt_\theta$	1.180,580,070 from iteration
Final Value	$D_L(\theta)$	$C_{\theta L}$ x sym x norm $\int_0^{\pi/2} A_{osfL} dt_\theta$	**3.354,777,477**
Combined Results			
Radial x Angular Product	$D_L(r)D_L(\theta)$	— — —	1.861,432,180 x 10^5(kg/l_{Sgs} abs)
Scaling Factor	C_L	$e\mu_o(G\varepsilon_o)^{1/2}$	1 l_{Sgs} = 4.893,752,96 x 10^{-36}meter
Final Calculated Value	e_{Lepton}	$C_L D_L(r) D_L(\theta)$	**9.109,389,239 x 10^{-31}kg**

FIGURE 3



5 ANALYSIS AND DISCUSSIONS

5.1 Initial General Observations

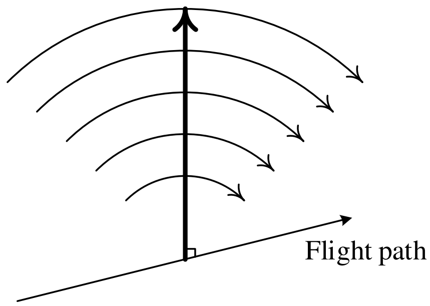
First, the question needs to be asked: What do these two Tables 1 and 2 represent in terms of practical physics? The equations in these tables offer straightforward, relatively simple mathematical means to calculate measures of the entrapped or stabilized gravitational energy of the two waveform species. For the leptons these equations directly result in the units most closely associated with gravity, those of mass, kg. For the photons, for reasons to be discussed more fully later, the equations result in the more indirect units of (ML)(L/T) in universal or meta units, and (kgm)(m/s) in relative human measurement units.

This algebraic collection of measurement units for the photon can be interpreted in two different ways. This composite can be viewed as a discussion of the mass which arises along a length multiplied by a velocity, (ML)(L/T), a spinning string. Or the meaning could be ascribed that this is a discussion of the mass which appears over an area per a unit of time, $(ML^2)(1/T)$, a vibrating membrane. The mathematics given here for the photon focus on a radial-angular plane, therefore the second interpretation of its measurement units seems beneficial here. Likewise, the photon could be thought of as having a radial mass density, just like the leptons, which rotates with a given velocity. Therefore the first interpretation also appears equally valid. This is in the same manner that the use of the word energy in a mechanical context can be interpreted to mean either potential energy $(ML)(L/T^2)$ or kinetic energy $(M)(L/T)^2$.

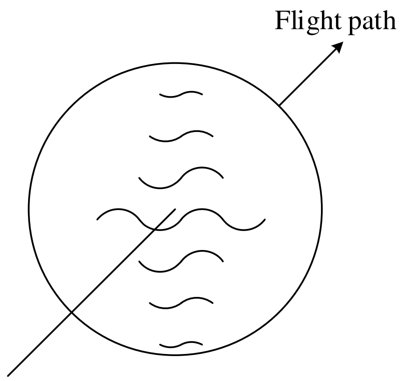
The classical narrative in which the photon constant is construed to mean energy x time is of course still valid. This descriptive use of the composite or collective measurement unit, energy, was obviously the result of measurements of how much energy originating as electromagnetic photons was deposited in a given target over a given period of time. So the assignment of this expression of energy x time to the photon constant was and still is a descriptive mechanism of what happens in an experiment.

This "explanation" though is a typical classical misunderstanding of the nature of forms that are outside of the range of human's ability to experience and understand. A description of what happens when two "objects" or forms interact, does not necessarily describe the internal structural nature of the

FIGURE 4
NEW PHOTON INTERPRETATIONS DISCOVERED FOR PLANCK'S CONSTANT



Spinning String
(a mass over a length) at
(a rotational velocity)
(M x L) X (L / T)



Vibrating Membrane
(mass generated over an area)
per (time)
$(M \times L^2) / (T)$

These new interpretations based on structural considerations explain HOW-WHY Planck's Constant, h, is constant, even though the photons' wave lengths vary wildly.

Note; neither of these are the classical interpretation of Planck's Constant, *h* as (energy) times (time), $(M \times L^2 / T^2) \times (T)$.
This $(M \times L^2 / T^2) \times (T)$ "explanation" was a result of describing WHAT happened during classical experiments. That is, a certain amount of energy from light or photons was deposited in a target in T amount of time. This was not really an explanation. It was just an experimental observation or description. This mathematical grouping of measurement units was a human imposition upon the particles. No-one asked the photons.

interacting forms. This description was and still is an externally based human lack of knowledge posing as an "explanation". This grouping of measurement units as energy x time is not internally based on the particles themselves, alone, the photons in this case, and is not an explanation of how or why the observed phenomena occurs. This is an example of the multitude of human intellectual impositions upon the particles, the photons in this case. No-one asked the photons.

Now the two new interpretations for the photon constant can be said to explain how or why this phenomena occurs. These new interpretations are based upon the structural considerations of the photons themselves as having rotating "mass" densities perpendicular to their flight paths. These explanations are independent of the wave length of the photons and anything to due with the presence or absence of an external experimental target.

The classical narrative in which the photon constant is construed to mean energy x time is not useful at all in this context, and is not used in these reports by J Fisher. By returning to the four basic universal or meta measurement units of distance-length L, duration-time T, mass M, and charge Q for the subatomic scale of endeavors maybe some of the typical past self deceptions can be avoided, such as that scientists and engineers knew what they were talking about. Such deceptions frequently have been set up by assigning composite names to collections of units, such as energy x time. The two interpretations of (ML)(L/T) given above stick strictly with the exact measurement units which are there. In the classical narrative of energy x time humans have added that which is not there. These are two extra units of time, one in the denominator of the composite called energy and one free or independent multiplier of time. Of course these two tack-ons are mathematically situated so as to cancel themselves out.

Continuing, these equations are in radial planar coordinates, and are represented by regular or scalar mathematics. Both the radial and angular factors probably represent the solutions to independent second order differential equations. Much valuable information can be learned here, which may give pointers on how to extend the current analysis to other bosons or fermions. In the lepton report a different mathematical description, rectilinear vectors, was presented which gave the electromagnetic structural picture for the leptons. The probable analogous electromagnetic structural picture of the photons is discussed later.

Another important note is that the final equations for both the mass of the leptons and the (ML)(L/T) of the photons are integrals in form, over appropriate regions of 2 dimensional space. This is vitally important in the interpretation of these mathematical descriptions. This means that these bodies are distributed in nature and are not point sources. Temporarily ignoring the angular equations for simplicity, what is found is that every section of the entire radial distribution carries some portion of the total content of these wave patterns' mass. Specifically, the initial portion of the wave near the origin and their final outwards extents all contribute to the final integrated value. That is, these inner and outer extents exist and they contribute, even though their contribution might be exceedingly tiny compared to the whole. This is in the same sense that a human dinner table is a distributed object. The top of a table exists. The legs exist even though they might only contain 5% of the mass of the table.

Since every portion of the wave patterns are needed to contribute to the final integrations, a conclusion can be made that these equations represent real physical quantities and are not probabilities. This is very different from the narrative given to the electron shells around the hydrogen atom. In that interpretation, the electron is said to exist here or there or somewhere else for a fleetingly brief period of time, but never everywhere all the time or at any one given time. In this work all sections of the wave patterns exist, not probably exist. Further they exist all the time and contribute to the overall whole all the time. Again this is the same sense that the legs of a table exist and exist all the time, not probably exist, even though their contribution to the total is small. Again the conclusion is reached that the ultimate final units of these equations are those of the real physical world, either relative and/or absolute, and that these equations do not result in numerical values in probabilistic space, momentum space, or other such mathematical conceptual oddities.

Finally the units resulting from these equations need to be briefly addressed. As noted earlier, the scaling factors C_L and C_P are actually conversion factors relating the Squigs system of absolute physics scales and the SI set of relative scales. In Analyses of Measurement Systems I, II, & III , the specific operations here have been shown to be derived from numeric quantities with universal or meta-units which are system independent. They are not just mere many decimal accurate coincidences of the SI relative set of units.

5.2 Static And Dynamic Descriptions

The equations in Tables 1 and 2 are static density equations. Although these equations are listed as ultimately containing temporal variables, they are still static in nature. The radial and angular integrations have effectively removed the temporal variables.

The key operative word for the first paragraph above is stabilized or stability. Both the photons and the leptons are stable waveforms. Just because the photons move and the leptons stay put does not diminish the stability of either form. One is an open ended waveform and the other closed or a "standing wave". The primary structural feature of both species' waveforms is a radial plane. Something about the nature of the forces which set up this plane must be self-balancing or stabilizing. Stated very generically, or simplistically, the expansive and contractive forces creating these radial structures must be counter-balancing. For these two most elementary electromagnetic waveforms there are only three forces involved in their structures or to which they respond; gravitational, electrical, and magnetic. Two of these forces can be seen create a 2 dimensional planar structure, while the third propels the wave pattern into space with time.

In the lepton report, the three factor appearance of the lepton radial equation was discussed. There the appearance of a driver, a shaper, and an attenuator were noted. These factors are mathematical representations of what must occur physically. Neglecting the shaping Laguerre polynomial factor, which might not apply to the photons, the double exponential appearance of the radial equation for the energy density of the particle was found to be:

$$D_{\text{wave form}}(r)/C_r = \int_0^{\infty} R_{csf} R_{esf} dt_r = \int_0^{\infty} e^{(R_c(t_r))} e^{(R_{eL}(t_r))} dt_r \tag{37}$$

Looking at the radial equations, $D_{\text{wave form}}(r)$ in particular, as seen above the contractive factor, R_{csf}, and the expansive factor, R_{esf}, are multiplied together as independent factors. The mathematical features of both of these radial factors are remarkable.

What are not found are dynamic, velocity, or first derivative descriptions. Assuming that these descriptive equations are representations of dynamic systems or stabilized wave patterns, then various questions can follow such as: How does the radial density change with the outward velocity of the wave? Is the outward velocity of the wave constant? Several similar such questions can be asked. This information is not addressed by these equations and is not seen directly, if it is available at all.

Taking the first derivative of $D_{\text{wave form}}(r)$ simply removes the integral from around R_{csf} and R_{esf}. This removes the summation of the physical properties under discussion. For these equations taking a first derivative means that they are then describing the wave patterns themselves at a specific single point in space. This is not the same as the physical property. The original integrated physical property for the leptons is a static quantity, mass. The un-integrated or first derivative quantity, itself becomes a derivative and should show a corresponding appearance in its measurement units. Whereas the physical property described by the integrated photon equation already has the appearance of a derivative. That is, some quantities of mass and distance divided by time. Taking the integration off the photon's equation leads to a second derivative type appearance in its measurement units. What is interesting then in this context of the original integrated equations and their derivatives is; since the leptons' physical property is static and the photons' has the appearance of a derivative, then the leptons and the photons could be thought of as mathematical-physical partners. This is in the same sense that the sin() and cos() form an original function and derivative function pair.

Taking the total second derivative of $D_{\text{wave form}}(r)$ the following is obtained:

$$d^2 D_{\text{wave form}}(r) = \frac{d^1(R_{csf})}{dt_r} R_{esf} + R_{csf} \frac{d^1(R_{esf})}{dt_r} \tag{38}$$

Examining the nature of the terms and factors of this expression for both species, some understanding can be approached of the interplay of the mass or energy density of these waveforms with the dynamics of the wave.

Another, yet very important, observation on the nature of these functions concerns their balancing of what could loosely be called "their tendency toward movement". Starting with the contractive radial factor, although t^2 is always positive, the negative sign preceding it as seen in the expression $e^{(-6t_r^2)}$ ensures that this radial factor or the force it represents is at the top of an energy hump. Spatially the internal balance of the wave is such that it would want to "roll" forward or backward. Since the particle is a stable "object", this "tendency toward movement" must be balanced by some other feature of the particle's energy representation. Here the leptons' expansive radial factor is in a well with very steep walls of $e^{(+at_r^{\sim 1})}$. The photon with a flat expansive radial factor of $e^{(+at_r^0)}$ has no such balancing action to prevent spatial movement. Looking at the angular functions, as is found the second function of these expressions is inner or implicit and cannot effectively balance either particle species' external spatial tendency to roll, rotate, or spin about their center.

5.3 Special Properties Of Some Of The Functions

Several valuable properties of the functions involved in the radial planar structures of the elementary electromagnetic waveforms are the self-normalization of their free standing initial conditions, and their implicit angular functions, as seen in Table 3.

Table 3 Integrals Of Free Standing Functions Used In e_p

FUNCTION	SYMBOLS	INTEGRATED EXPERSSION
Lepton radial initial condition	$I_{rL}(t_r)$	$\int_0^\infty \text{FHDif}[1.697, 525, 533 t_r^1] dt_r = 1.0$
Photon radial initial condition	$I_{rP}(t_r)$	$\int_0^\infty \text{FHDif}[1.980, 416, 377 t_r^{1/2}] dt_r = 1.0$
Lepton angular initial condition	$I_{\theta L}(t_\theta)$	$\int_0^{\pi/2} \cos(t_\theta) dt_\theta = 1.0$
Photon angular initial condition	$I_{\theta P}(t_\theta)$	$\int_0^{4/\pi} \cos(\pi^2/8\ t_\theta) dt_\theta = 1.0$
Lepton implicit angular function	$A_{iL}(t_\theta)$	$\int_0^{\pi/2} T_1^\dagger(\cos[n^{-1} t_\theta^1]) dt_\theta = 1.0$
Photon implicit angular function	$A_{iP}(t_\theta)$	$\int_0^{4/\pi} T_1^\dagger([1-(\pi/4\ t_\theta)^2]^{1/2}) dt_\theta = 1.0$

Referring back to the mathematical preliminaries, the application of the distance function, ds, was discussed as applying to a general weighted expanding "circular area", $Y_{area} = R_e(t_r) = (ak_{2n}^{-1/2}\ \pi t_r^{2n})$. As seen this "area" figure was producible from an initial curve $Y_{circ} = (a^{1/2}k_{2n}^{-1/4}t_r^{n})$.

Applying this same concept of a hidden, original, or precursor function to the implicit variables within the expansive radial functions, $R_e(t_r)$, the following are found. For the leptons, within $R_{eL}(t_r)$ the distance function is applied to $2\pi t_r^{2}/k_L^{1/2}$, which can be thought of as being the area of a circle produced from $2^{1/2}t_r^{1}/k_L^{1/4}$. For the photons, within $R_{eP}(t_r)$ the distance function is applied to $\pi t_r^{1}/k_P^{1/2}$, which can be thought of as being the area of a circle produced from $t_r^{1/2}/k_P^{1/4}$. What can be noted with these "behind the scenes" formulations is that the ultimate variables here have the same power relations as were found for the variables in the radial initial spatial conditions: $I_{rL}(t_r)$ with $\mathrm{FHDif}[k_L t_r^1]$ and $I_{rP}(t_r)$ with $\mathrm{FHDif}[k_P t_r^{1/2}]$.

Other subtle yet revealing features within $R_{eL}(t_r)$ and $R_{eP}(t_r)$ are the distinct appearances of $k_L^{1/2}$ and $k_P^{1/2}$ in the denominators. The appearance $1/k^{1/2}$ is a classic normalization appearance, where k is the norm. By norm the integral of some function of concern is meant, squared, over its natural range, when this function is free standing or not embedded in the current application. This is a common meaning, and is the mathematical usage when working with both probability functions, orthogonal polynomials, and expressions involving the distance function such as in differential geometries. Also the $1/k^{1/2}$ can be backed out from under the squared term of the distance function, ds, to produce simple the appearance of 1/k. This is also a legitimate appearance for a normalization factor, when working with functions that are not squared, such as the Fraunhofer diffraction integrals above.

For the angular functions there is also some subtle interplay between the various functions and their resulting numerical values. As shown in Table 2 the lepton initial angular constant $C_{\theta L}$ evaluated to 0.890,365,284.... For the photon for a free standing or more general form of the angular function the following properties are found:

$$\int_0^1 \sin(\pi/2\ \ A_{iP}(t_\theta))dt_\theta = \int_0^1 \sin(\pi/2\ \ [1-(t_\theta)^2]^{1/2})dt_\theta = 0.890,365,284\ldots \quad (39)$$

Further, letting a = 0.890,365,284..., b be any constant, and the upper limit u = b/a, then for all:

$$\int_0^u \sin(\pi/2\ \ [1-(t_\theta/u)^2]^{1/2})dt_\theta = b \quad (40)$$

The integral of the photon angular function self normalizes when

u = 1/a = 1/0.890365,284... = 1.123,134,536...

or in effect is linearly scaling. This property can be very useful.

5.4 Parity

Before examining the key differences between the equations which give rise to the observed physical properties of the photons and the leptons, one other important topic, parity, should be examined. Without structural equations describing the various particles, physics has proceeded by observing what occurs to the various particles when time is reversed, when space is reflected, et cetera. Physics then makes a tabulation of these observations and notes where expected reflection outcomes are violated. Without a knowledge of structure though, there can be no general mathematically based explanation for the various outcomes.

Here what to expect when the basic human measuring devices of time and space are negated or reflected can directly be seen. Tables 4 and 5 summarize these results.

Table 4 Result Of Sign Variations Of Equation Parameters For Leptons

FUNCTION	EXPRESSION	t_r, t_θ positive	t_r, t_θ negative
FHDif[F(t_r)]	FHDif[$k_L t_r^1$]	+	+
R_{csf}	$e^{(-6t_r^2)}$	+	+
precursor to ds argument	$\left(\frac{2^{1/2} t_\tau^1}{k_L^{1/4}}\right)$	+	-
ds argument	$\left(\frac{2\pi t_\tau^2}{k_L^{1/2}}\right)$	+	+
ds in $R_{eL}(t_r)$	$\left[1 + \left(\frac{4\pi t_r^1}{k_L^{1/2}}\right)^2\right]^{\frac{1}{2}} dt_r$	+	+
R_{esfL}	$e^{\left[\left(\frac{\pi}{2}\right)^{1/2} \text{x above}\right]}$	+	NA
$A_{iL}(t_\theta)$	$T_1^\dagger(\cos[n^{-1} t_\theta])$	+	+
A_{osfL}	$T_1^\dagger(\sin[\pi/2\ A_{iL}(t_\theta)])$	+	NA

Table 5 Result Of Sign Variations Of Equation Parameters For Photons

FUNCTION	EXPRESSION	t_r, t_θ positive	t_r, t_θ negative
FHDif[F(t_r)]	FHDif[$k_P t_r^{1/2}$]	+	+
R_{csf}	$e^{(-6t_r^2)}$	+	+
precursor to ds argument	$\left(\frac{t_\tau^{1/2}}{k_P^{1/4}}\right)$	+	imaginary
ds argument	$\left(\frac{\pi t_\tau^1}{k_P^{1/2}}\right)$	+	-
ds in $R_{eP}(t_r)$	$\left[1 + \left(\frac{\pi t_r^0}{k_P^{1/2}}\right)^2\right]^{\frac{1}{2}} dt_r$	+ constant	+ constant
R_{esfP}	$e^{\left[\left(\frac{\pi}{2}\right)^{1/2} \text{x above}\right]}$	+	NA
$A_{iP}(t_\theta)$	$T_1^\dagger([1 - (\pi/4\ t_\theta)^2]^{1/2})$	+	+
A_{osfP}	$T_1^\dagger(\sin[\pi/2\ A_{iP}(t_\theta)])$	+	NA

When looking at the effects of parameter sign variation, what is found is that, with only one exception of $R_{csf,}$ the internal or implicit variables and not the external functions drive the nature of these responses.

As indicated in Section 2.1 the ultimate implicit variables for both the radial and angular equations could be viewed equally well as representing either spatial or temporal parameters. That is r and θ or equally the two dimensional set t_r and t_θ. These mass density equations give no guarantee as to which is the actual or "correct" view. In this report, the view was chosen that the implicit parameters are temporal quantities. The distinct appearances of radial time t_r and angular time t_θ have been used. If this temporal interpretation is to be a permitted interpretation of these variables or is even "the correct view", then this mean the implicit variables here are more in keeping with how implicit variables are typically presented in college textbooks. There particularly with the discussions of vectors, implicit variables are typically shown as representing time. This is also in keeping with the discussions concerning the nature of time and space, in Appendix 2, Time & Space. In those discussions time was viewed as an internal or embedded phenomenon, and further was always the reference or driver from which the external spatial phenomenon flowed.

Here is where a conceptual difference between the mathematical and the physical occurs. When thinking of trigonometric functions, sin() and cos(), and a cylindrical helical structure, typically something physically cycling, making a complete rotation or revolution is visualized. Mathematically though for the angular functions as seen here there is a pattern which oscillates or alternates direction, rather than one which continually proceeds in the same direction and repeats itself. The implicit angular functions are both "even", as shown in the tables. As long as the implicit variable t_θ is within its valid range, whether it is positive or negative the result is the same. The external angular function A_{osf} always receives a positive argument. Therefore the Chebyshev T_n polynomial only covers a range of 0 to 1, i.e. is a shifted $T_n^\dagger$ polynomial and not a full range, -1 to +1, T_n polynomial. For example with the lepton, as the implicit variable t_θ progresses from $-\pi/2$ to 0 then the external sin() function increases from 0 to 1. Then as the t_θ continues from 0 to $\pi/2$ the sin() function decreases from 1 back to 0. Nowhere does this exterior sin() function produce a negative value. One mathematical way to produce a continuous cycling pattern rather than an oscillating one is by introduction of ±cos() for the leptons and $\pm\sqrt{}$ for the photons as the implicit functions $A_i(t_\theta)$ and appropriate adjustment of the equation symmetric multipliers. Unfortunately such a maneuver results in the integral of the angular function neutralizing itself to zero. A means to solve this difficulty may be by the introduction of i, the $\sqrt{-1}$ then squaring various quantities, but such procedures, besides being of dubious benefit, would violate the principle of "keep it simple". Another means of resolving this apparent anomaly is experimental. Designing an experiment to detect whether a single photon makes a complete rotation or oscillates back and forth should be relatively simple with the modern high tech instruments now available and the advent of the ultra low temperature sodium light traps. This apparent anomaly between the mathematical and the consensus physical remains an open question yet to be addressed.

What is not found here is what occurs when the path or direction of propagation of the photon is reversed. Likewise, what is not known is what happens when the path or direction of revolution of the lepton is reversed. These parameters, which could be called X distance covered for the photon and φ angle progressed for the lepton, are not addressed by the gravitational structures or their radial planar equations. The vector expressions for the electromagnetic structures of these waveforms are needed to make sense of the dynamics of this particular parameter.

6 Structural Contrasts

6.1 Mass : Masslessness

Examining the radial function for the photon and leptons in Tables 1 and 2 the key difference between the two species is found to be their mass density or gravitational relationship to their ultimate internal variables. With this use of these variables as representing temporal phenomena an amazing discovery is found. Here again the focus is on the expansive radial factors R_{esf} of the energy density equations. As found in the discussions in Section 5.3, paragraph 3 above, the outer most level of time or consensus time on which the distance function ds is operating, could be conceived of as having some inner, hidden, or preliminary function of time. These interior relations with time of the two species have the same power relationships as those of their initial spatial Fraunhofer Diffraction conditions.

These expansive radial factors R_{esf} supply one of the factors producing the ultimate mathematical values for these expressions. In this temporal view the expansive factor of the leptons on the outer most level would have a pseudo first-order relation to time.

$$R_{eL}(t_r) \propto t^1 \text{or} \left[1+\left(\frac{4\pi t_r^1}{k_L^{1/2}}\right)^2\right]^{\frac{1}{2}}$$

Whereas on the outer most level the photon is related to time as

$$R_{eP}(t_r) \propto t^0 \text{or} \left[1+\left(\frac{\pi t_r^0}{k_P^{1/2}}\right)^2\right]^{\frac{1}{2}}$$

or is mathematically independent of time. This does not mean that the photon has no relationship to consensus time, or a null or zero relationship. Rather the photon has a very special relationship, that of a constant value of 2.446,139,955, if just considering the mathematical segment of R_{esfP} shown. If the entire function $D_P(r)$ is considered then of course this value is 48.445,551,845 as shown in Table 1. This is probably the mathematical reason why the photon has no mass and why it travels at the speed of light, c, as the speed of electromagnetic energy transfer is called. This is in keeping with relativity which indicates any "object" moving at the speed of light effectively does not age or experiences an unchanging relationship with time, at least from the viewpoint of an external or stationary observer.

From the inner most or precursor view, the photon was found to still be related to time as $t_r^{1/2}$. An assumption can be made that this is necessary for the photon to exist. Referring to the discussions of the metaphysical assumptions of this work in Chapter 4.1, Methodology and the discussions of time and space in Appendix 2, a generalization is. All "objects" that exist must have a form or structure. That structure must have internal references to time just as they have external references to space.

6.2 Charge : Chargelessness

Another required discussion is why the photon has no charge, or at least why it displays no measurable charge. Absolutely proving a negative, the absence of something, why something does not exist, or at least does not appear to exist is always hard or even impossible. Nevertheless, the following explanation is offered for discussion.

The one useful item of information that is available for this topic is the mathematical nature of the charge of the leptons discussed in the lepton report. There the charge of the leptons was shown to probably relate to the square of the curvature of a vector formulation for the electromagnetic (e/m) structure of the leptons.

The vector formulation of a broadly generalized "cylindrical spiral" is:

$$\mathbf{R}(t) = a\cos[F(t)]\,\mathbf{i} + a\sin[F(t)]\,\mathbf{j} + bG(t)\,\mathbf{k} \tag{41}$$

With the one constraint that G'(t) = F'(t) then the curvature and torsion of this figure are:

$$\text{curvature } k = \frac{|\mathbf{R}'(t)\times\mathbf{R}''(t)|}{|\mathbf{R}'(t)|^3} = \frac{a}{a^2+b^2} \tag{42}$$

$$\text{torsion } \tau = \frac{|\mathbf{R}'(t)\times\mathbf{R}''(t)\bullet\mathbf{R}'''(t)|}{|\mathbf{R}'(t)\times\mathbf{R}''(t)|^2} = \frac{b}{a^2+b^2} \tag{43}$$

These are invariant numerical constants, independent of the variable t and the function F(t). This invariance was shown to be key in producing the invariant vector based charge of the leptons. The equation which calculated to produce the charge of the leptons as, e_{calc} = 1.602,177,29 x 10^{-19} C based off the cylindrical helix form:

$$\mathbf{R}(t)_{e/mL} = a\ T_n^{\dagger}(\cos[F(t)])\ \mathbf{i} + a\ T_n^{\dagger}(\sin[F(t)])\ \mathbf{j} + bG(t)\ \mathbf{k} \qquad (44)$$

Where $G(t) = F(t) = \pi/2\ A_{iL}(t)$.
Where $A_{iL}(t) = T_n^{\dagger}(\cos[n^{-1}t^1])$ with n = 1,3,5 for electron, muon, and tau respectively.
Within $\mathbf{R}(t)$ a = 6 and b/n = 1.

Here the function F(t), of the unsubscripted form of t, did not need to be explicitly stated or even known at the time. Later in the radial angular mass density equations, gravitational structure, this angular function was found to be as shown.

Relating the curvature κ to charge and the torsion τ to hand, as was done in the lepton report, then by varying the signs of a and b in **R**(t) four combinations of charge and hand can be produced for the leptons and their anti-particles. Allowing space to freely rotate about the particle reduces this to two combinations of charge and hand in a free floating environment.

Although not directly provable by the mathematics of this work, the photon probably violates the one constraint that G'(t) = F'(t). By analogy to the leptons, the electromagnetic (e/m) vector formulation for the photon is likely to be something similar to the following:

$$\mathbf{R}(t)_{e/mP} = a\cos[F(t)]\ \mathbf{i} + a\sin[F(t)]\ \mathbf{j} + bG(t)\ \mathbf{k} \qquad (45)$$

Where G(t) = t and $F(t) = \pi/2\ A_{iP}(t)$; ie. $G(t) \neq F(t)$.
Where $A_{iP}(t) = [1 - (n^{-1}t)^2]^{1/2}$ with $n = \pi/4$.
Probably within $\mathbf{R}(t)$ a = 6 and b/n = 1 as with the leptons.

Again with these references to the charge expressions for the waveforms the unsubscripted form of t is used. For **R**(t) which violate the constraint G'(t) = F'(t) the curvature κ and torsion τ still mathematically exist, but they form messy and irreducible expressions in both G(t) and F(t), and ultimately of the variable t.

The photon may spin or rotate clockwise or counterclockwise about the centerline of its axis of progression, related to the sign of a, and it may progress in a forward or backward sense, related to the sign of b. Even so, the photon does not have a charge because it does not have a fixed invariant curvature nor likewise a fixed invariant torsion. What is found is that the photon does in fact have a charge, but mathematically this is constantly changing probably at some "infinitely" fast speed. The photon does not display a charge, a fixed constant physical property which is measurable. To humans and their instruments the photons appear chargeless at the gross time scale upon which humans operate.

This unsubscripted variable t in the vector helix expression of the photon relates to position along the wavelength or flight path of the photon. By analogy, the variable t for the lepton relates to position along the donut coil flight path of the lepton. In the lepton report this coil length t was found to be independent from both the radial and angular expressions of time, t_r and t_θ, for the rotating or spinning mass density planar pattern. Assuming that in the final expressions in Tables 1 and 2, of (ML)(L/T) for the photon, and mass for the leptons, relate to time at the outer most or exterior level then there are some other possibilities to unscrambling this mystery of charge, or lack of it. Referring to the discussions of time and space in Appendix 2, Time & Space, another key could be in determining to what level of time the t in their respective electromagnetic structures or their vector cylindrical expressions is related.

Examining these vector expressions for the electromagnetic structures of these waveforms, the parameter of propagation can be addressed. This was left open by their radial planar gravitational structures. Imagine what would be seen if humans had both the conceptual and physical apparatus to see into time. Further imagine that an observer had shrunk down to the size of a photon and watched one move across his front. Assume that as he is standing focusing forward, that he sees the present in time. With the specified propagation vector of bG(t)**k** and G(t) = t for the photon, what would he see as time progressed or regressed? If he looks to the left in time, call this the past, he sees the photon moving in one direction. If he looks to the right in time, call this the future, he sees the photon moving in the other direction. Since this variable in time is so simple and immediately translates into the spatial parameter, the observer would find the same situation in space. If he looks to the left, negative values in space, he finds the photon progressing toward him. If he looks to the right in space, positive values, he finds the photon receding away from him. This is all very simple and also as expected.

What is not simple is this same analysis for the lepton circulation around the donut. With the lepton parameter of $G(t) = \pi/2\ T_n^{\dagger}(\cos[n^{-1}t^1])$ the progression dynamics are very different. Here a reminder is needed that the variable t and the parameter $\cos[n^{-1}t^1]$ are limited by the valid range of the $T_n^{\dagger}$ function. This is assuming that the electromagnetic T_n function of the leptons is a $T_n^{\dagger}$ function the same as the gravitational one, and that it is not a more general full range T_n function. As discussed in Section 5.4 concerning parity for the gravitational structure variables, under this restriction the cos() function always produces a positive value. As the observer views the curved rim of the lepton donut, to his left or to his right, he finds the energy pattern progressing in the same direction. Whether this is negative toward him or positive away from him doesn't matter. It is doing the same on both sides of him in time. Again, since this temporal variable is so simple, movements here in either the past or future immediately translate to their spatial analogues.

How can the radial planar pattern progress either away from the observer in both directions or toward him from both directions? This is possible under one set of conditions, that there are two waves, one forward and one reverse, and that they pass through each other at the point of the observer. This is a plausible possibility since the specification was made that the observer is at the present, the zero point in time, which corresponds to the zero angle in space. This probably means that at any other point in time the observer cannot see equally well into the past and into the future. Also his view to the right and left in space is not identical, but probably is two mirror images.

Additionally as discussed in Section 4.3 and in the lepton report, the lepton has two orthogonal angular forms. These must exist simultaneously since their contributions show up numerically in the lepton's symmetric multiplication factors. These two orthogonal angular forms could account for the necessary two opposite directional waves. Whereas the photon one has a single angular form. This explanation may help alleviate some of the apparent anomaly that was raised in Section 5.3

6.3 Spin

As to spin or intrinsic angular momentum, again there is not much information with which to work. Nevertheless making a few simple assumptions allows for progress. The first assumption is that angular momentum has to do with the radial planar form of the gravitational structures presented in Tables 1 and 2. Second, assume that these radial planar forms are spinning in a sense similar to the turning of the blades on a windmill. Third, assume the angular momentum of these waveforms has nothing to do with or is independent of the forward propagation of the figures, a straight line for the photon and a circular donut for the lepton. In terms of the electromagnetic vector expressions, the angular momentum would be related to the **i** and **j** vectors and would be independent of the **k** vector. Fourth, assume that angular momentum has only to do with the angular equations of these structures and is not related to the radial equations. Fifth choose a very simplistic or basic concept for the mathematical meaning of momentum. That is assume that momentum is related to velocity or the first derivative of a function of time. Specifically the following generic expression for momentum is chosen:

$$\text{Momentum, } MO[F(r,t)] = F(r,t) \times d^1F(r,t) \qquad (46)$$

Looking at the angular mass density equations, A_{osf}, in Equation 25 and Tables 1 and 2, both waveforms are seen to have the same form, as follows:

$A_{osf} = T_n^\dagger(\sin[\pi/2\ A_i(t_\theta)])$

Further for n = 1, $d^1A_{osf} = \pi/2 \cos[\pi/2\ A_i(t_\theta)]\ d^1A_i(t_\theta)$
Ultimately the result produced is:

$$\text{Momentum} = A_{osf}\ \text{x}\ d^1A_{osf} = \pi/2 \sin[\pi/2\ A_i(t_\theta)] \cos[\pi/2\ A_i(t_\theta)]\ d^1A_i(t_\theta) \quad (47)$$

This information is of course identical with what would be obtained if the starting point had been the **j** vector of the electromagnetic formulation. Since the waveforms, the photons and leptons, are bosons and fermions respectively, then they must have a spin angular momentum ratio of 2:1. Examination of Equation (47) and the individual particle's angular expressions with their embedded implicit functions and derivatives, $A_i(t_\theta)$ and $d^1A_i(t_\theta)$, rapidly reveals that they are sufficiently complicated and substantially different enough that production of this simple 2:1 ratio is not possible no matter what manipulations are tried.

A second deeper look is needed. While initially this next step appears difficult to rationalize, upon examination though this is found to be the only proper logical choice. A sixth and final assumption is needed that the angular momentum of these particles relates only to the inner, implicit, or embedded functions of time. This last step or assumption produces the desired simplification and the ultimate desired result. Why can this last choice be made excluding the greater or exterior angular equations as the starting point for the computations? Upon examination the answer is obvious. The external angular equations are functions of θ and space and are not functions of time. As part of the definition of momentum the decision was made that an expression representing momentum must be a function of time. Examining the inner, implicit, or temporal angular expressions for the particles the information in Table 6 is obtained. For both species k = π/2.

TABLE 6 Spin Of Leptons And Photons

FUNCTION	LEPTONS	PHOTONS
$A_i(t_\theta)$	$kT_n^\dagger[\cos(n^{-1}t_\theta)]$	$k[1 - (t_\theta/u)^2]^{1/2}$
$d^1A_i(t_\theta)$	$-k\sin(t_\theta)$	$-k(t_\theta/u^2)\ [1 - (t_\theta/u)^2]^{-1/2}$
$A_i(t_\theta)$ x $d^1A_i(t_\theta)$	$-k^2\cos(t_\theta)\sin(t_\theta)$	$-k^2(t_\theta/u^2)$
Equivalent	$-k/2\ \sin(2t_\theta)$	
Integrated	$-k/4\ \cos(2t_\theta)$	$-k^2/2\ (t_\theta^2/u^2)$
Upper limit	π/4	4/π
Eval @ upr	0	$-k^2/2$
Eval @ 0	$-k^2/4$	0
Net	$k^2/4$	$-k^2/2$

Here a reminder is needed that the lepton angular equations are most probably the solutions to second order differential equations and are the trigonometrically substituted $T_n^\dagger$ orthogonal polynomials. Therefore the upper limit on the lepton integral of $A_i(t_\theta)$ and $d^1A_i(t_\theta)$, is such that it gives the full range of the $T_n^\dagger$ polynomials, 0 to 1, and here is specifically set at π/4.

Here the value of u in the photon equations does not need to be specified, but only needs to be set equal to the upper limit of integration. From the photon's angular equation this upper limit is probably equal 4/π.

The simplification of the lepton presentation, which shows only the electron information, still needs to be addressed. The use of higher member odd $T_n^{\dagger}$ polynomials was neglected here because of possible complications which could be added by the auxiliary shells of the higher members of the lepton series.

Finally, the above presentation of course only shows how to obtain values related to angular momentum which have the correct numerical ratio. There is no scaling factor or correlation constant which converts these values to real world measured quantities. These manipulations can be thought of as only a relative correlation, and are not truly equations from which spin angular momentum can be calculated. The importance of this proposed explanation, though, again lies in verifying that the equations developed for both waveforms, not only meet their respective numerical requirements, they also meet key conceptual requirements. That is, they can potentially explain obvious physical property differences between the two species.

7 Conclusions

Summarizing, mathematical models for describing geometric structures for the elementary electromagnetic waveforms, "particles", leptons and photons have been discovered. These wave structures are simple and easily understood by anyone familiar with second semester calculus. These structures base on two dimensional radial-angular energy density patterns, in scalar mathematics, to describe the gravitational picture of these particles. Additionally rectilinear vector mathematical descriptions of the same structures give the electromagnetic picture of the waveforms.

An amazing amount of information has been obtained. Once equations have been developed-discovered for the gravitational structure of these two species, leptons and photons, then indeed these equations explain much more than just the observed mass or (ML)(L/T) for the waveforms. Each factor in these equations offers interesting insights into other observed physical properties of the waveforms.

1 The implicit function of the expansive radial factor R_{esf}, when stated as temporal, $R_e(t_r)$, offers a plausible explanation for a waveform's mass : masslessness.
2 The implicit function of the angular equation, $A_i(t_\theta)$, offers a clear explanation for a waveform's spin angular momentum.
3 The relation between the implicit temporal functions, F(t) and G(t), of the vector description of the waveform's electromagnetic structure offers an explanation for the waveform's charge : chargelessness.
4 Varying the sign of the several inner or implicit functions gives results for the external radial and angular functions in agreement with the observed parity requirements for these waveforms.

In short the temporal functions or factors in the waveform's structural equations are what provide insight into many of the major observed physical properties of the waveforms.

Through continued analysis of mathematical statements for the structures of both the photon and the leptons appears to be how the definitive answers to other physical property questions can be found. Questions such as: what gives rise to or is responsible for the magnetic moment of the leptons?

Likewise, for the neutrinos, gravitons, quarks, gluons, et cetera, structural equations (gravitational, electromagnetic, and color where applicable) must be developed before much further progress can be made in particle physics. Assuming structureless particles such as mathematical points or mathematical lines with no cross section (strings), and possibly even mathematical sheets with no thickness (membranes), will probably never yield physical property information. Further several decades of history have shown that assuming anything, such as a starting hypothesis, has little likelihood of ever connecting with the real world. The most effective means of developing mathematical descriptions for physical property information for the subatomic waveforms appears to be by starting with the data and letting the data guide the way to the appropriate equation forms, and not by imposing human intellectual ideas down upon the particles. As this collected work has shown, this is true even when the original starting data consisted of only three data points; m_e 9.109,389,7 x 10^{-31} kg, m_μ 1.883,532,7 x 10^{-2}kg, and m_τ 3.167,88 x 10^{-27}kg, numeric values with units.

Lastly connecting to other schools of thought in particle physics two other final but amazing conclusions can be made.

5 Since the geometric forms found for both the leptons and the photons consist of spinning radial-angular plane waves, which propagate into 3-space with time, a person could say that these two elementary energy forms are membranes of the string-membrane school of hypothetical particle physics.

6 Likewise, since ultimately both the leptons and photons consist of primary radial "energy" density patterns which could be said to rotate angularly, a person could say that these two elementary energy forms are strings of the string-membrane school of hypothetical particle physics.

CHAPTER 1.3 A MODEL FOR DETERMINING PHYSICAL PROPERTIES IV: CHARGE OF THE QUARKS

1 Introduction

1.1 Abstract

Differential geometry formulas have been found which correctly describe or correspond with the ±2/3 and ±1/3 charge of the quarks. These formulations extend the previous work discussed in Chapters 1.1 and 1.2 that showed how the ± 1 charge of the leptons could be explained as arising from the fixed curvature of certain mathematical vectors traveling in 3 spatial dimensions or $\mathbb{R}^3$. This current work describes how the charge of the quarks is directly related to the fixed curvature of certain geometric vector curves in 4 spatial dimensions or $\mathbb{R}^4$.

1.2 Objective & Scope

The specific objective of this report is to show how the charge of the quarks ±2/3 and ±1/3 times the numerical value of the charge of the electron, e = 1.602,177,33 x 10^{-19} C, can be exactly determined by using the fixed curvature of a small group of curves in 4 spatial dimensional. Additional objectives are to compare / contrast the equations found for charge of the quarks as being curves in 4 spatial dimensions with those found for the leptons as being curves in 3 spatial dimension.

Lastly this report also considers some of the many mathematical and physical implications of there being physical particles, the quarks, residing in a 4th spatial dimension.

The formulas discovered for the charge of the quarks are simply presented here. Their parts (factors) are described and analyzed to ensure understanding of their correct calculation and use. This work is best described as a mathematical analysis of some very limited and exact data which has resulted in an applied mathematical model. This presentation should be thought of as only a demonstration. The descriptive equations found are not proven nor derived in the usual formal or rigorous mathematical sense of those words.

This model of the quarks as being represented by mathematical curves in 4 spatial dimensions was not derived from a hypothesis but was the result of using the hints and ideas provided by the previous work with the leptons. The model presented here does not start with a hypothetical platform, does not specifically support any particular hypothesis, has not been embedded into any preexisting hypothesis, nor has a hypothesis been constructed around it. While there may be implications concerning particle hypotheses indirectly supported by this work, only those conclusions directly supported by the mathematics of the equations that were found are reported. Implications of this work for the multitude of current grand particle hypotheses are beyond the scope of this work.

1.3 Short History of Quarks

In the late 1950's and on into the 1960's the number of elementary particles mushroomed out of control into a whole zoo of exotic species. In 1964 Murry Gell-Mann and independently George Zweig proposed the existence of the quarks to bring some order and simplification to the world of sub-sub-atomic particles. Thru the use of having these ±2/3 and ±1/3 charged subspecies they could reduce most of the 100+ "elementary" particles down to two major categories. Mesons were proposed to be binary composites and hadrons as composed of ternary arrangements of these more basic quarks.

The now long established quark family has 6 members formed by 3 sets of 2 pairs each. Of these only the lowest energy members up (u) with charge +2/3 and down (d) with charge -1/3 have any relevance to the common material world.

The first experimental evidence for the existence of the quarks came in 1968 with the deep inelastic scattering experiments at the Stanford Linear Accelerator Center. The last theorized member of the quark family to be observed was the top quark at Fermilab in 1995.

The mesons are now accepted to be binary composites and the baryons as ternary composites, analogous to the di and tri atomic molecules of the elements of chemistry. For example the π^0 meson = ($u\bar{u}$ or $d\bar{d}$) can be thought of as analogous to H_2 or N_2. The π^+ meson = $u\bar{d}$ or π^- meson= $d\bar{u}$ could be considered as analogous to HCl or NaCl. Likewise the stable proton can now be understood to be composed of u_2d analogous to H_2O. Whereas the somewhat unstable neutron is composed of ud_2 and is analogous to HO_2, which is so unstable as to be nonexistent.

Ever since their proposal and experimental verification or validation the quarks have presented numerous difficulties for the particle physics community, both experimentalists and theoretcians alike.

First these particles behave as if they are somehow "hidden". That is, they are never seen directly or in isolation by themselves.

They appear to exhibit some force or strength properties which are completely at odds with the other known elementary forces, such as gravity and electro-magnetism. The nature of the forces of the first three basic or primary forces, the unary gravity, and the binary set electrical and magnetic, all have inverse square decays in space. The nature of the set of the three color forces, blue, green, and red, associated with the quarks exhibit the property of rapidly increasing in strength with distance. This ternary color force set is also called the strong forces. Physicists have never been able to determine numerical values for what might called $Blue_o$, $Green_o$, and Red_o, because these forces have never been found in isolation.

For these and other reasons physicists have concluded that the quarks are somehow "confined", "hidden", or "rolled up" or other such choice of words.

If the quarks are though of as "stationary", "long life", the "originators" and "receivers" of the color or strong forces, then they have associated with them the gluons. The gluons are thought of as "moving", "temporary short lived", carriers or transmitters of the strong forces. The gluons like their counterpart quarks tend to give the the particle physicists fits and present what has appeared to be unacceptable inpenetratable challenges.

So for all these reasons there appears to be a need for a new approach or re-think of the basic assumptions that are being used to describe these particular sub-sub-atomic species. The first major and required step in this rethinking is dropping the human imposed and limiting idea that the quarks are species composed of and residing in 3 spatial dimensionals. A new paradigm in which the quarks are 4 dimensional in nature could solve many of the physicist's current puzzles concerning these species.

2 The Work

2.1 Mathematical Preliminaries - Clarifications

Some clarification of the terminology used in the discussions in Chapters 1.1 and 1.2 is needed. In these chapters, especially Chapter 1.1, the word figure was used to refer to the geometric form being discussed. Both the words figure and form do not sit well in formal mathematical settings, in that they are somewhat indeterminate and probably do not construe the meaning as used in these chapters.

Likewise in both Chapters 1.1 and 1.2 the leptons and photons were referred to as structures or as having structures, specifically radial planar energy density patterns which are propelled into the 3rd dimension with time. While this choice of a word served a beneficial purpose for the discussions there, again in the world of mathematics it is not technically correct. What was being discussed particularly in reference to the determination of the charge for the leptons and the lack thereof for the photons was 3 dimensional mathematical vectors in rectilinear coordinates.

Mathematically the final vectors $\mathbf{R}(t) = a\cos[F(t)]\mathbf{i} + a\sin[F(t)]\mathbf{j} + bF(t)\mathbf{k}$ used in the discussions concerning charge in Chapters 1.1 and 1.2 are curves moving thru 3 dimensional space with time. They just make the pencil point outlines in empty space. They do not represent anything "solid" or objects. They may outline a figure, form, or structure, but there is nothing "solid" there. This is an important distinction and recalibration of the thinking needed then moving on to the 4 dimensional vector descriptions for the charge of the quarks.

Another clairification is needed concerning the derivation of the form of the vector for the generalized cylindrical figure used to show the relation of the charge of the leptons to the curvature of this curve in 3 spatial dimensions. A general cylindrical figure of vector

$$\mathbf{R}(t) = a\cos[F(t)]\,\mathbf{i} + a\sin[F(t)]\,\mathbf{j} + bG(t)\,\mathbf{k} \qquad (01)$$

was used as the starting form. As a result of the derivations for the curvature κ and torsion τ the necessity of the requirement that G'(t) = F'(t) became apparent. In Chapter 1.1, Section 3 it is stated that "with this one restriction, that the implicit function G'(t) = F'(t), then the curvature and the torsion of this generalized cylindrical figure are found to be numerical constants, independent of the implicit variable t and all functions of F(t)." This is a true statement, but somewhat deceiving. Behind the scenes in this derivation there are other requirements placed upon the functions F(t) and/or G(t). These are that to calculate the curvature κ in this general form, then the 1st and 2nd derivatives of F(t) and/or G(t) need to exist. And further to determine torsion τ for this general form, the 3rd derivative needs to exist. While this requirement may seem trivial this ensures that the curve being dealt with is truly a 3 dimensional form or a curve in 3 spatial dimensions or R^3. The extension of this requirement is directly involved in the derivations which follow for mathematical curvature of curves in 4 spatial dimensions or $\mathbb{R}^4$.

Finally a reminder is needed concerning the use of the word dimensions as used in this report and entire book. In this work, as should be obvious, the word dimension is reserved for the mathematical concept of spatial or temporal dimensions. This is not the same as using the word for; arguments of functions, constants, measurement units, parameters, variables… Engineers and other technical people, and yes physicists, love to talk about dimensions and dimensional analysis. They especially love dimensionless groupings in which various quantities have been intentionally ratioed together in such a manner so as to purposely throw away all the measurement units and only leave the numeric portion of a quantity.

In a hold over from early mathematical perspectives on engineering, discussions and even chapters are found in first semester engineering texts and reference books with the title of "Dimensional Analysis". For example in one current engineering reference book under the section on mathematics there is a listing of 16 dimensionless groups. In a later chapter on fluid and particle mechanics there is a listing of 20 dimensionless groupings, with an overlap of only 3 from the previous listing. There are yet the chapters on heat transfer and mass transfer which have their own such listings. To show the promulgation and plethora of such unitless groupings, the reference section in the back of one equipment manufacturer's catalogue lists 140 such ratio blocks under the title of dimensionless numbers.

In these engineering and physics contexts what is meant by the use of the words dimensional analysis is an analysis of the parameters involves in a scenario and of their measurement units and how they associate when formed into particular grouped engineering quantities. Further and maybe far worse yet, physicists have a habit of automatically equating or associating every new parameter in a calculation or scenario to being a new mathematical spatial dimension. This usage of referring to measurement units or parameters in a mathematical expression or equation as dimensions is not what is meant throughout this work.

2.2 Mathematical Vectors in n-Dimensions

To approach the concept of a 4th dimensional vector consider the following. First consider a curve in a plane, 2 spatial dimensions. The word curvature κ describes a measure of the deviation of the path of the vector from being in a straight line.

Now what if the curve turns upwards out of the confines of the plane into the 3rd spatial dimension? This would be like cutting a flat circle and pulling up on one end. This then is the energy pattern of the photons and leptons. Now the word and concept for curvature κ still remain. A new word torsion τ and concept for which it stands describes the deviation of the path of the vector from being in a plane.

Now consider what if the curve again turned "upwards" into the 4th spatial dimension. Nothing prohibits this mathematically. Only the lack of human imagination causes difficulties. At this point of generalization mathematicians specializing in differential geometries just use the symbols k_1 for curvature κ, k_2 for torsion τ, and k_3 for the new concept describing the deviation of the curve from being in 3 spatial dimensions.

What are now needed are equations or the means to develop such equations for these k_1, k_2, k_3 in 4 space or $\mathbb{R}^4$. Such formulas were developed by Professor Jeanne Nielsen Clelland of the University of Colorado Boulder durning the second week of May 2019. This was done as a favor outside of other job and family requirements in a mere 5 days. This was because of her love of the beautiful subject $\mathbb{R}^4$. The exact derivation of these formulas as transmitted to the author by email are in Appendix 7.

The author wishes to make it abundantly clear that while Professor Clelland developed the necessary equations and gave advise about their usage, she was not responsible for the numerics which were put into them. So any discussions about the actual application of these formulas as found here, their numerics, possible errors, or other such matters should be directed to the author. Professor Clelland is not responsible for usages or abusages of these differential geometric formulas.

There are several restrictions or restraints put upon the curves in $\mathbb{R}^4$ to allow for the development of the necessary simplistic closed form formulas.

1 The curves must move at unit speed.

2 For the determination of k_1 and k_2 the first three derivatives of the curve with the implicit or independent variable need to exist. For the determination of k_3 additionally the 4th derivative of the curve is required to exist.

3 In the specific derivation or the special case worked up k_1, k_2, and k_3 were all assumed to be non-zero and constant. Specifically in the originating form below, $k_1(s) > 0$, $k_2(s) > 0$, and $k_3(s) \neq 0$.

These restrictions result in a set of linear ordinary differential equations which are solvable to give the desired formulas. This solution then also points towards the form or apparence of the originating curve.

Starting with the originating form or curve, in terms of the parameter curve length, s

$$\gamma(s) = (\ r_1 \cos(as);\ r_1 \sin(as);\ r_2 \cos(bs);\ r_2 \sin(bs)\ ) \qquad (02)$$

This can be restated in terms of a variable of time, t

$$\gamma(t) = (\ r_1 \cos(At);\ r_1 \sin(At);\ r_2 \cos(Bt);\ r_2 \sin(Bt)\ ) \qquad (03)$$

The restrictions that $a \neq b$ and $A \neq B$ are required here.

Going thru various derivations, whose objectives are to determine; k_1 "curvature", k_2 "torsion", and k_3 all to be constants simoultaneously, or to have fixed values independent of time, t, gives the following.

$$k_1 = [\ a^2 \cos^2(\alpha) + b^2 \sin^2(\alpha)\ ]^{1/2} \qquad (04)$$

$$k_2 = [\ a^2 - b^2 \sin(\alpha) \cos(\alpha)\ ] / k_1 \qquad (05)$$

$k_3 = ab / k_1$ (06)

Where α is the angle between the two curves or circles having r_1 and r_2.
Again for these discussions the curve is required to have unit speed or

$$v = [\ (Ar_1)^2 + (Br_2)^2\ ]^{1/2}. \quad (07)$$

This $[\ (Ar_1)^2 + (Br_2)^2\ ]^{1/2}$ is used as a Normalizer or normalizing constant in all further calculations.
Therefore; a = A / Normalizer and b = B / Normalizer
This unit speed also requires
$\cos(\alpha) = A\, r_1$ / Normalizer and $\sin(\alpha) = B\, r_2$ / Normalizer
Therefore; $\alpha = \text{acos}[\ A\, r_1 / \text{Normalizer}\ ]$, or equally, $\alpha = \text{asin}[\ Br_2 / \text{Normalizer}\ ]$

Looking at the setup and definitions there appears to be 5 variables or parameters which can be or need to be specified A, B, r_1, r_2, and α. But there can only be 3 independent and chosen parameters for the three fixed objectives k_1, k_2, k_3. Where:

A determines frequency or cycle length of 1 pair of the angular trigonometric arguments.

B determines frequency or cycle length of 1 pair of the angular trigonometric arguments.

The radii of the two circles r_1 and r_2 obviously are important in determining the shape or structure of this 4 dimensional curve-figure.

Finally the angle α is not really of any importance to the nature of the calculations, but is just a mathematical device to keep tract of the requirement for unit speed of the curve.

There are several items to note here of the development of these formulas.

First the requirement that $k_1(s) > 0$, $k_2(s) > 0$, and $k_3(s) \neq 0$ means that this curve is in fact 4th dimensional in space. It is not some 3 dimensional curve pretending to be 4th dimensional while hiding out in 4 spatial dimensions.

A person could think of a build-it-yourself set of book shelves. They come a flat box and are assembled to make a true 3 dimensional set of shelves. Or a person could think of a book of mica found in areas with pegmatite geology. These are thick books of the mineral material composed of thin, 2 dimensional like, sheets. These ideas are not how to think of 4th dimensional vector curve being analyzed. Again it is truly 4th dimensional in space, not just a bunch of originating 3 dimensional "objects" stacked up.

Second is the requirement that the various derivatives exist. Besides the mathematical implications, this requirement has profound implications for applications in engineering, such as mechanical engineering, and for physics. All classical mechanics recognize and heavily rely upon the following.

velocity	dL / dT	the change in position with time
acceleration	dL / dT^2	the change in velocity with time

here L is a generic, meta, or place holder for distance-length,
and T is a generic, meta, or place holder for duration-time.

What is rarely used or even recognized as having any relevance is the higher order derivatives of distance with time.

jerk	dL / dT^3	the change of acceleration with time
jounce	dL / dT^4	the change of jerk with time

While although appearing to be exotic concepts, these could have applications in specialized fields of engineering, such as in the lift-off of satellites in their initial launch from earth.

The fact that with the 4 dimensional space curves, to whose curvature the charge of the quarks is to be related, require the existence of jerk and jounce could have a multitude of ripple effects in the realm of sub-sub-atomic calculations. Speculations concerning this and other such topics are presented in the discussions in 5.2 below.

2.3 Review of 3 Dimensional Work for Charge of the Leptons

A brief review is needed of the work done and reported in Chapter 1.1 for the 3 dimensional determination of the charge of the leptons. This is needed because it is the starting point or reference for the search for a formula for the charge of the quarks.

The search for a formula to describe or match the charge of the leptons started with the curve,

$$\mathbf{R}(t) = a \cos[F(t)]\, \mathbf{i} + a \sin[F(t)\,]\, \mathbf{j} + bG(t)\, \mathbf{k} \tag{01}$$

This was restricted by requiring $G'(t) = F'(t)$. This then gives the fixed invariant curvature and torsion as follows. Curvature, $\kappa = a / (a^2 + b^2)$ and Torsion, $\tau = b / (a^2 + b^2)$

This mathematical appearance was then used to find the charge of the leptons as follows.

$$e = [\, \mu_o (G\, \varepsilon_o)^{1/2}\,]\, A, \text{ measurement units of Q relative, Coulombs} \tag{08}$$

$$A = (2\pi)^{-3/2} \times (\pi\, \rho^2) = 1/2(2\pi)^{-1/2}\, \rho^2 \tag{09}$$

Reviewing the definitions of the absolute units in the Introduction to Part 1, $[\, \mu_o (G\, \varepsilon_o)^{1/2}\,]$ is easily seen to have the numerical value of $3.054,438,950 \times 10^{-17}$ and the units of L_absolute / Q_relative or specifically l_{Sgs}/C. Here A is a geometric constant, with the mixed relative per absolute physics units $C^2\, l_{Sgs}^{-1}$.

From the form or generic appearance of κ and τ above

$$\rho = \kappa = 6 / (6^2 + 1^2) \text{ when } A = 6, B = 1; \text{ or } \rho = \tau \text{ when } A = 1, B = 6 \tag{10}$$

This produces the numerical values of ;

$$\rho = 6/37 = 0.162,162,162\ldots \tag{11}$$

$$\rho^2 = 0.026,296,567\ldots \tag{12}$$

$$e_{calc} = 1.602,177,29 \times 10^{-19}\ C \tag{13}$$

$$e_{measured} = 1.602,177,33 \times 10^{-19} \pm 4.9 \times 10^{-26}\ C \tag{14}$$

$$\text{Ratio } e_{calc} / e_{measured} = 1.000,000,024 \tag{15}$$

Physically the measured charge of the quarks = ± 2/3 and ± 1/3 charge of the leptons.

Assume that the origination or derivation of the charge of the quarks is the same, the curvature of a n-dimensional curve, except this time in 4 spatial dimensions.

Assume all the front end numerical scaling constants $(2\pi)^{-3/2} \times (\pi)$ and the conversion factor of physical constants $[\, \mu_o (G\, \varepsilon_o)^{1/2}\,]$ remain the same.

Then the charge of the quarks is;

$$= 2/3\, \rho^2 = 2/3\, (0.026,296,567\ldots) = 0.017,531,045\ldots \tag{16}$$

$$\text{and } = 1/3\, \rho^2 = 1/3\, (0.026,296,567\ldots) = 0.008,765,522\ldots \tag{17}$$

3 Charge of the Quarks

For a curve in 4 space the formula for the a fixed curvature k_1 was given above. Repeated here.

$k_1 = [\, a^2 \cos^2(\alpha) + b^2 \sin^2(\alpha)\,]^{1/2}$ (04)

This curvature as derived is a fixed invariant constant whose values depend on the constants selected for 3 of the 4 parameters; A, B, r_1, and r_2. Several cases were found which matched ρ^2 or $(k_1)^2$ with the above numerical values for the ± 2/3 and ± 1/3 charge of the quarks.

Red Series, Radii $r = r_1 = r_2$

A series was found with $r = r_1 = r_2$. Setting $r_1 = r_2$ eliminates one degree of freedom as necessary.
This is called the radii or RED series for the RED quarks.

Cases were found with A : B as follows:

2 : sqrt(2); 3 : sqrt(3); 5 : sqrt(5)

These cases worked equally as well when A and B were uniformly multiplied by n, such as:

2n : n x sqrt(2); 3n : n x sqrt(3); 5n : n x sqrt(5)

In these scaled cases n can be any value and does not need to be an integer or even a rational number.
These scaled cases require a multiplying factor within the $r = r_1 = r_2$ of n^2.
Reversing the positions of A and B made no difference in the resulting k_1, but only flipped the calculated angle α from being < 45° to its 90° compliment of being > 45°. But this choice does flip the sign of k_2 from positive to negative.

The required radii of the curves $r = r_1 = r_2$ was found to be composed of or the product of 4 factors, as follows:

$r = r_1 = r_2 =$ (1) Scaling factor for multiples of A and B
x (2) angle factor
x (3) constant base factor
x (4) 3D / 4D ratio factor

(1) A scaling factor for multiples of A and B .
If the "standard" cases of A : B is scaled to be nA and nB, then this factor is n^2.
(2) A factor which is called an "angle" factor or a trigonometric factor.
This factor is very specific to each case and best illustrated in Tables 1 and 2 below. This factor was the result of trial and error, and is "artsy" and highly irrational.
Note, this factor is the only input which changes between the cases for the ± 2/3 and ± 1/3 charges.
(3) There is a constant factor = sqrt(5) = $(1^2 + 2^2)^{1/2}$
This can be thought of as being related to a base reference case which uses the form $(a^2 + b^2)^{1/2}$. This is the simplist possible case where a and b are integers and a ≠ b. That is a = 1 and b = 2 and this constant factor = sqrt(5).
(4) A 3D to 4D scaling factor.
From the matching of the charge of the leptons to the curvature of cylindrical spiral in 3D the form a / $(a^2 + b^2)$ appeared. This was made specific to match the charge of the leptons as $6 / (6^2 + 1^2)$. The form of importance here is simply the $(1^2 + 6^2) = 37$, which is used in the numerator of a ratio. The denomenator of the this ratio is related to the specific 4D cased being calculated, as follows.
Dimensional scaling Factor, 3D / 4D = 37 / $(A^2 + B^2)$

Note the factors for the radii (3), and (4) both can also be seen as just expressions of the distance function ds or ds^2 or combinations / ratios of various ds^2. For example $(A^2 + B^2)$ can be seen as $(1 + (B/A)^2)$ and since for these cases B = sqrt(A), then this = $(1 + (A/A^2))$.

Several cases are demonstrated in Tables 1 & 2. These should be sufficient to build a quite broad generalization. Going back to the original curve as a function of time,

$$\gamma(t) = (r_1 \cos(At); r_1 \sin(At); r_2 \cos(Bt); r_2 \sin(Bt)) \qquad (03)$$

what has been built is a foundation for working with A = 2, 3, 5, the first 3 prime numbers, and with B = sqrt(A).

Considering the property that nA and nB can be used, then a wide variety of possibilities are opened up, but this can be tricky and deceptive. For examples:

If a person decided for some reason that they want $A = 1 / (2\pi)$, this could be thought of as originating from two immediate sources.

First from the base case A = 2, now scaled to $A = 1 / (2\pi) = 2 / (4\pi)$ and $B = \mathrm{sqrt}(2) / (4\pi)$
This has the scaling factor for $r = (1 / (4\pi))^2$ and an angular factor = sqrt(6) / 6.
The resulting r = 5.629371, angle $\alpha = 35.2644^o$, and $k_1^2 = 0.017531$ as desired to match the value needed for the ± 2/3 quarks.

Equally from the base case A = 3, now scaled to $A = 1 / (2\pi) = 3 / (6\pi)$ and $B = \mathrm{sqrt}(3) / (6\pi)$
This has the scaling factor for $r = (1 / (6\pi))^2$ and an angular factor = sqrt(3) / 2.
The resulting r = 5.970849, angle $\alpha = 30^o$, and $k_1^2 = 0.017531$ as desired to match the value needed for the ± 2/3 quarks.

3.1 Blue and Green Solutions

Solutions have been found to match the required numerical targets of $\rho^2 = 0.017531$ and 0.0087655 with r_1 and r_2 being chosen as 2 separate parameters. To do this then either A, called G for GREEN here, or B for BLUE must not be specified. The input numerical values of one of these must be allowed to float according to the formulas;

Green = $\cos(\alpha) / r_1$ x Normalizer, or equally, g or green x Normalizer
Blue = $\sin(\alpha) / r_2$ x Normalizer, or equally, b or blue x Normalizer

There is one immediate "problem" or at least a calculational difficulty to note with these choices of parameters r_1, r_2, and either Green or Blue. The beginning parameter of the calculations which must float is self refential thru the Normalizer. This cannot be avoided. But modern spreadsheets are easily capable of handling this situation, and the operator can specify the number of iterations and the of decimals of accuracy.

Logically if these Green and Blue solutions follow the examples of the Red Series, then there should be series for these solutions, not just one point snap shots. There are two requirements here. First any proposed additional solutions must match 8 decimal place numerics. Secondly there must be recognizable closed form algebraic expressions driving these numerics.

At this point after at least 48 trial and error attempts, the author has not been able to find any more cases than the ones listed in Table 3 following. This does not mean that such additional cases forming a discrete series do not exist. This only means that it is extremely difficult to discern any closed form algebraic expressions giving rise to such possible members. The author leaves it to some aspiring graduate student to earn their PhD discovering additional case solutions.

In Table 3 following, the matching of the results of the trial forms to the target values are purposely shown to 9 decimals. This is to emphasize the difficulty of what is being done here with a self refential loop involved in the calculational pathway. Ultimately what the result of the calculation is being compared to is the numerical value of the charge of the electron, e = 1.602,177,33 x 10^{-19} C. The reader has the right to see the accuracy of the trial and error test form.

In these Blue and Green solution forms again the distance function like appearances play a heavy part. Again there is nothing wrong with this appearance and it might even be expected. After all what is being formulated and calculated is again radii of circles or distances.

Also what is found in these solutions is that the radii have an interplay with each other. They have a square and square root base relationship, similar to the A's and B's of the Red Radii solutions. For example in the Blue solutions R1 and R2, $(1+\pi^4)$^(1/2) and $(1+\pi^4)$^(1/4), obviously are squares or square roots of each other, with maybe external multipliers of 2 throw in.

Another interesting feature is with Blue solutions the miscellaneous multipliers, 2 in these cases, are external to the quantities in parenthesis which are being raised to either the 1/2 or 1/4 power. Whereas with the Green solutions there is a 2 multiplier internally in the parenthesis directly multiplying the π^4.

The appearance of the many occurances of π^4 within the distance function expressions need some further explaination, because they appear excessive or not normally physically attainable. Within a distance function such as $ds = [\ 1 + (d(\gamma(t))/dt)^2\ ]^{1/2}$, if $d(\gamma(t))/dt$ is simply a numeric without any expression of time then this appearance is possible. Further if this $d(\gamma(t))/dt$ is again parsed as constant times what is left of what was a function of time originally to the first power, what is found is as follows; $\pi^4 = [\ (\pi/2) \times (2\pi)\ ]^2$. In these all cases the $(\pi/2)$ is the λ parameter of either the Blue or Green circle which is open for specifying. The 2π is simply the derivative of the circumference of a expanding circle in time or $d(2\pi\ \gamma(t))/dt$ when $\gamma(t)$ just equals t or even more simply is just the circumference of a circle with a radius = 1. Even more interesting this π^4 has the appearance found for the square of the π constants within both the surface and volume formulas for 4 dimensional spheres when R = 1. Volume of 4D sphere = 1/2 π^2 and Surface of 4D sphere = $2\pi^2$.

3.2 Limiting Solutions

In the search for Blue and Green solutions to match the ± 2/3 and ± 1/3 charge of the quarks two limiting solutions or limiting cases were found. These are shown in Table 4. These solutions violate the one primary constraint of the entire 4D curvature formula derivation, that G ≠ B. In these cases $\lambda_1 = \lambda_2$ or G = B and also the radii of the curves $r_1 = r_2 = r$. If G, B, r_1, and r_2 are considered to be specified then the solutions are over specified. Or if setting G = B is considered as removing one degree of freedom and $r_1 = r_2$ as removing another degree of freedom, then the solutions are under specified.

Never-the-less these limiting cases are informative and appear to be bases from which the Red or Radii cases arise, or else devolve to as their limiting forms.

One other minor but still interesting mathematical feature in these limiting solution cases is that the $r = r_1 = r_2 = (37/4)$ is easily recognizable as (9 + 1/4).

Table 1 Summary Of Case Results, Red Series, For Quark's ± 2/3 Charge

NUMERICAL TARGET for ± 2/3 charged quarks, ρ^2 = 0.017531

INPUT

A	2	4	3	6	5	10
B	sqrt(2)	2x sqrt(2)	sqrt(3)	2x sqrt(3)	sqrt(5)	2x sqrt(5)
numerical	1.414214	2.828427	1.732051	3.464102	2.236068	4.47213595
Factors for r						
Scaling Factor	1	4	1	4	1	4
Angle Factor	sqrt(6)/6	sqrt(6)/6	sqrt(3)/2	sqrt(3)/2	sqrt(5)x((12+1)/12)$^{1/2}$	
numerical	0.408248	0.408248	0.866025	0.866025	2.327373	2.327373
Constant Factor	sqrt(5)	sqrt(5)	sqrt(5)	sqrt(5)	sqrt(5)	sqrt(5)
numerical	2.236068	2.236068	2.236068	2.236068	2.236068	2.236068
3D / 4D Factor	37/(4+2)	37/(16+8)	37/(9+3)	37/(36+12)	37/(25+5)	37/(100+10)
numerical	6.166667	1.541667	3.083333	0.770833	1.233333	0.308333
Final r value	5.629371	5.629371	5.970849	5.970849	6.418470	6.418470

CALCULATIONS

Normalizer ($A^2 r_1^2 + B^2 r_2^2$)$^{1/2}$						
numerical	13.7891	27.5782	20.6836	41.3673	35.1554	70.3108
a = A / Normalizer	0.145042	0.145042	0.145042	0.145042	0.142226	0.142226
b = B / Normalizer	0.102560	0.102560	0.083740	0.083740	0.063605	0.063605
α =ACOS(a r_1)	0.615480	0.615480	0.523599	0.523599	0.420534	0.420534
α = ASIN(b r_2)	0.615480	0.615480	0.523599	0.523599	0.420534	0.420534
α, degrees	35.2644	35.2644	30	30	24.0948	24.0948
cos(α)	0.816497	0.816497	0.866025	0.866025	0.912871	0.912871
sin(α)	0.577350	0.577350	0.5	0.5	0.408248	0.408248
$a^2 \cos^2(\alpha)$	0.014025	0.014025	0.015778	0.015778	0.016857	0.016857
$b^2 \sin^2(\alpha)$	0.003506	0.003506	0.001753	0.001753	0.000674	0.000674
$K1^2$ = sum(above)	0.017531	0.017531	0.017531	0.017531	0.017531	0.017531
Compare to target	1	1	1	1	1	1
K1 = sqrt($K1^2$)	0.132405	0.132405	0.132405	0.132405	0.132405	0.132405
K2 = [(a^2 - b^2) x sin(α) cos(α)] / K1	0.037450	0.037450	0.045866	0.045866	0.045549	0.045549
K3 = ab / K1	0.112349	0.112349	0.091733	0.091733	0.068323	0.068323
K1 / K2	3.535534	3.535534	2.886751	2.886751	2.906888	2.906888
K2 / K3	0.333333	0.333333	0.5	0.5	0.666667	0.666667

Table 2 Summary Of Case Results, Red Series, For Quark's ± 1/3 Charge

NUMERICAL TARGET for ± 1/3 charged quarks, ρ^2 = 0.0087655

INPUT						
A	2	4	3	6	5	10
B	sqrt(2)	2x sqrt(2)	sqrt(3)	2x sqrt(3)	sqrt(5)	2x sqrt(5)
numerical	1.414214	2.828427	1.732051	3.464102	2.236068	4.47213595
Factors for r						
Scaling Factor	1	4	1	4	1	4
Angle Factor	sqrt(3)/3	sqrt(3)/3	sqrt(6)/2	sqrt(6)/2	$\text{sqrt}(5)\text{x}((12+1)/6)^{1/2}$	
numerical	0.577350	0.577350	1.224745	1.224745	3.291403	3.291403
Constant Factor	sqrt(5)	sqrt(5)	sqrt(5)	sqrt(5)	sqrt(5)	sqrt(5)
numerical	2.236068	2.236068	2.236068	2.236068	2.236068	2.236068
3D / 4D Factor	37/(4+2)	37/(16+8)	37/(9+3)	37/(36+12)	37/(25+5)	37/(100+10)
numerical	6.166667	1.541667	3.083333	0.770833	1.233333	0.308333
Final r value	7.961132	7.961132	8.444056	8.444056	9.077088	9.077088
CALCULATIONS						
Normalizer $(A^2 r_1{}^2 + B^2 r_2{}^2)^{1/2}$						
numerical	19.5007	39.0014	29.2511	58.5021	49.7173	99.4345
a = A / Normalizer	0.102560	0.102560	0.102560	0.102560	0.100569	0.100569
b = B / Normalizer	0.072521	0.072521	0.059213	0.059213	0.044976	0.044976
α =ACOS(a r_1)	0.615480	0.615480	0.523599	0.523599	0.420534	0.420534
α = ASIN(b r_2)	0.615480	0.615480	0.523599	0.523599	0.420534	0.420534
α, degrees	35.2644	35.2644	30	30	24.0948	24.0948
cos(α)	0.816497	0.816497	0.866025	0.866025	0.912871	0.912871
sin(α)	0.577350	0.577350	0.5	0.5	0.408248	0.408248
$a^2 \cos^2(\alpha)$	0.007012	0.007012	0.007889	0.007889	0.008428	0.008428
$b^2 \sin^2(\alpha)$	0.001753	0.001753	0.000877	0.000877	0.000337	0.000337
$K1^2$ = sum(above)	0.0087655	0.0087655	0.0087655	0.0087655	0.008766	0.008766
Compare to target	1	1	1	1	1	1
K1 = sqrt($K1^2$)	0.093624	0.093624	0.093624	0.093624	0.093624	0.093624
K2 = [$(a^2 - b^2)$ x sin(α) cos(α)] / K1	0.026481	0.026481	0.032432	0.032432	0.032208	0.032208
K3 = ab / K1	0.079443	0.079443	0.064865	0.064865	0.048312	0.048312
K1 / K2	3.535534	3.535534	2.886751	2.886751	2.906888	2.906888
K2 / K3	0.333333	0.333333	0.5	0.5	0.666667	0.666667

Table 3 Summary Of Case Results, Blue And Green Solutions

NUMERICAL TARGET for ± 2/3 charged quarks, ρ^2 =	0.017531045
NUMERICAL TARGET for ± 1/3 charged quarks, ρ^2 =	0.008765522

CASE TARGET **Free Driver**	± 2/3 quarks Blue λ	± 1/3 quarks Blue λ	± 2/3 quarks Green λ	± 1/3 quarks Green λ
G, Green λ	Forced not Free	Forced not Free	π /2	π /2
numerical	0.456580795	0.728307313	1.570796327	1.570796327
B, Blue λ	π /2	π /2	Forced not Free	Forced not Free
numerical	1.570796327	1.570796327	7.988601590	0.568927007
Raduis 1	$(1+\pi^4)$^(1/2)	$2(1+\pi^4)$^(1/4) or 2 x sqrt(R2)	$(1+2\pi^4)$^(1/2)	sqrt(R2) x $(1+\pi^1)$^(1/2)
numerical	9.920135636	6.299249363	13.993504996	7.612844244
Radius 2	$2(1+\pi^4)$^(1/4) or 2 x sqrt(R1)	$(1+\pi^4)$^(1/2)	(π/24) x sqrt(R1) x $(1+\pi^1)$^(1/2)	$(1+2\pi^4)$^(1/2)
numerical	6.299249363	9.920135636	0.996518981	13.993504996

CALCULATIONS

Normalizer $(G^2r_1^2 + B^2r_2^2)^{1/2}$				
numerical	10.882222	16.243845	23.378114	14.365975
green = G / Norm	0.041957	0.044836	0.067191	0.109341
blue = B / Norm	0.144345	0.096701	0.341713	0.039602
α =ACOS(a r_1)	1.141518	1.284467	0.347473	0.587373
α = ASIN(b r_2)	1.141518	1.284467	0.347473	0.587373
α, degrees	65.404151	73.594564	19.908761	33.653999
cos(α)	0.416215	0.282432	0.940236	0.832399
sin(α)	0.909266	0.959287	0.340523	0.554176
$a^2 \cos^2(\alpha)$	0.000305	0.000160	0.003991	0.008284
$b^2 \sin^2(\alpha)$	0.017226	0.008605	0.013540	0.000482
$K1^2$ = sum(above)	0.017531045	0.008765522	0.017531045	0.008765522
Compare to target	1.000000000	1.000000000	1.000000001	0.999999999
K1 = sqrt($K1^2$)	0.132405	0.093624	0.132405	0.093624
K2 = [(a^2 - b^2) x sin(α) cos(α)] / K1	-0.054522	-0.021243	-0.271443	0.051179
K3 = ab / K1	0.045740	0.046309	0.173407	0.046251
K1 / K2	-2.428458	-4.407268	-0.487782	1.829362
K2 / K3	-1.191996	-0.458724	-1.565345	1.106552

Table 4 Summary Of Case Results, Limiting Solutions

NUMERICAL TARGET for ± 2/3 charged quarks, ρ^2 =	0.017531045
NUMERICAL TARGET for ± 1/3 charged quarks, ρ^2 =	0.008765522

CASE TARGET **Free Driver**	± 2/3 quarks Blue λ	± 1/3 quarks Green λ
G, Green λ	Forced not Free	sqrt(3/2)/(1+(2/3)^2)
numerical	0.847900296	0.847900296
B, Blue λ	sqrt(3/2)/(1+(2/3)^2)	Forced not Free
numerical	0.847900296	0.847900296
Raduis 1	(37/4) x sqrt(1/3)	(37/4) x sqrt(2/3)
numerical	5.340489990	7.552593374
Radius 2	(37/4) x sqrt(1/3)	(37/4) x sqrt(2/3)
numerical	5.340489990	7.552593374

CALCULATIONS

Normalizer $(G^2r_1^2 + B^2r_2^2)^{1/2}$		
numerical	6.403846	9.056406
green = G / Norm	0.132405	0.093624
blue = B / Norm	0.132405	0.093624
α =ACOS(a r_1)	0.785398	0.785398
α = ASIN(b r_2)	0.785398	0.785398
α, degrees	45.000000	45.000000
$\cos(\alpha)$	0.707107	0.707107
$\sin(\alpha)$	0.707107	0.707107
$a^2 \cos^2(\alpha)$	0.008766	0.004383
$b^2 \sin^2(\alpha)$	0.008766	0.004383
$K1^2$ = sum(above)	0.017531045	0.008765522
Compare to target	1.000000000	1.000000000
$K1 = \mathrm{sqrt}(K1^2)$	0.132405	0.093624
$K2 = [(a^2 - b^2)$ x $\sin(\alpha) \cos(\alpha)] / K1$	0.000000	0.000000
K3 = ab / K1	0.132405	0.093624

4 Discussion

The first obvious question is, why bother with this whole procedure. Why not just stick with the demonstrated geometry for the charged leptons, the electron and positron families, and the numerical scaling constants and conversion factor of physical constants found there. Just multiplying this by ± 2/3 and ± 1/3 would have worked. The equally obvious answer is, because then nothing would have been learned about the geometric nature of the quarks.

Likewise, why bother with trying to match the exact target value for ρ^2 for the ± 1/3 quarks. In the Red solutions this requires the development of a different "angular" factor embedded in $r = r_1 = r_2$. Why not just use the pattern developed for the ± 2/3 quarks and divide by 2. Again because nothing would have been learned about the geometric differences between the two species of quarks.

Referring again to the 3 dimensional model whose fixed curvature corresponded with the charge of the leptons, some comparison and contracts are needed.

For the leptons the curved vector form $\mathbf{R}(t) = a \cos[F(t)]\ \mathbf{i} + a \sin[F(t)]\ \mathbf{j} + bF(t)\ \mathbf{k}$ was used after making the restrictions required by the differential geometric analysis that $G'(t) = F'(t)$. What was important to note there is that the embedded or implicit function F(t) is completely unspecified. Of course its first 3 derivatives must exist as already discussed. When this appearance was used in the mass equations for the leptons, specifically the angular mass equations, it took on the appearance of an outer trigonometric with an inner or embedded trigonometric function. For examples $\sin[\pi/2\ A_i(t_\theta)]$ where $A_i(t_\theta) = \sin[n^{-1}t_\theta]$ or equally $A_i(t_\theta) = \cos[n^{-1}t_\theta]$.

With this work for the quarks the function F(t) is not completely wide open. Of necessity in the setup of the original vector form, the trigonometric appearances had to be cos(At), sin(At), cos(Bt), sin(Bt). Additionally as found by trial-and-error the constants for the Red or Radii Series, A and B, giving the frequency, f, and cycle length, λ, of the two interlinked circles were required to have a ratio of $N : N^{1/2}$ or $\lambda : \lambda^{1/2}$.

Another important note is that r as calculated in these formulas is just a human black board conceptual device. If a person wants to know the real physical world value of r, then this black board r needs to be scaled appropriately to the sub-sub-atomic quantum scale referencing the absolute Stoney scales. This is of course without the use of his assumed number of dimensions and their corresponding π constants. Such modified Stoney Scales are called Squigs Scales in this work. See the introduction to Part 1.

Appearance

All this is good and well, but what do these little buggers actually look like? As seen in the original setup to produce fixed curvatures k_1, k_2, k_3, the objective here, then the space curves were required to be as in Equations (02) and (03), repeated here.

$$\gamma(s) = (\ r_1 \cos(as);\ r_1 \sin(as);\ r_2 \cos(bs);\ r_2 \sin(bs)\ ) \qquad (02)$$

$$\gamma(t) = (\ r_1 \cos(At);\ r_1 \sin(At);\ r_2 \cos(Bt);\ r_2 \sin(Bt)\ ) \qquad (03)$$

Repeating verbatim from the email to the author from Professor Clelland should clarify this picture.

"Geometrically, this curve is traversing a circle of radius r_1 in the (x1, x2) plane with angular speed a, while simultaneously traversing a circle of radius r_2 in the (x3, x4) plane with angular speed b. The entire curve is contained in the surface given by the Cartesian product of these two circles, which is topologically a torus. (This is analogous to the fact that a circular helix in $\mathbb{R}^3$ is contained in a cylindrical surface.) But unlike the case of a helix in $\mathbb{R}^3$, there are two very different possibilities for the global behavior of this curve:

1. If the ratio a / b is equal to a rational number p / q (in reduced form) with p and q integers, then the curve is closed and periodic, with period equal to 2π times the least common multiple of p and q.

2. If the ratio a / b is irrational, then the curve is quasi-periodic and never closes up, but rather fills out a dense subset of the toroidal surface on which it lives."

As found above after the numerics were added by the author the ratios of a / b have a square / square root ratio for the Red or Radii solutions. The ratios of a / b for the Blue and Green solutions are likewise highly irrational.

These are of course just the vector curve pictures of the which give rise to the quarks' charge or at least a descriptive model for it. The mass picture of the quarks is probably analogous to those of the leptons as being 2 linked simultaneous circular "sheets" or radial planar forms which then move and fill out their 4 spatial dimensions according to the description given above in 2.

Open Ended Number of Red Cases: While the requirements on the form of the trigonometrics are restrictive, they actually do not limit the number of possible applications in determining the nature of k_1 of the curves. That is curves which result in the the desired ρ^2 values. Actually there probably are an infinite number of combinations which work. The only restraint is that most combinations beyond those demonstrated in Tables 1 and 2 are highly complicated and not really worth messing with. Seeing the rapid increase in the complication of the "angular" factor within r, between the two base cases of A = 2 and A = 3 and that of the next prime number base case A = 5 a person can imagine what the next "angle" factor looks like.

Additionally a person needs to be aware of the balance of $a^2 \cos^2(\alpha)$ and $b^2 \sin^2(\alpha)$ contributing to the final resulting curvature $K1^2$. The further in this prime number series of inputs, 2, 3, 5… the more out of balance or the "worse" the balance of these two contributors. Ultimately what happens is that one contributor does all the work and the other does not contribute anything significant to the final result.

Quantum Nature of Red Quarks: An important and obvious observation is in order here. The cases found and presented here for the Red calculational route are steps in a series. These base Red or Radii series are clearly discrete and appear to be "harmonics" or standing wave patterns. This could be similar to only observing the peaks of say a Fraunhofer Diffraction pattern. This is a very real consideration in that phenomena such as Fraunhofer Diffraction patterns are a planar or flat 2 dimensional radial phenomena. Here what is being described is two circles attached to each other as a unit figure, form, or curve.

4.1 Again a Little Human Technical History

Humans have long since mastered the concept and applications of energy as electromagnetic energy. In 1767, 250 years ago, Joseph Priestley of England proposed the inverse square law for the drop off of the electrical force. Much work was done for the next century expanding the human knowledge of the electrical and magnetic fields and forces. This work culminated in 1861-1862, 157 years ago, when James Clerk Maxwell of Scotland published what is now known as Maxwell's Equations describing electric and magnetic fields. He also proposed that light is an electromagnetic phenomena.

All of these endeavors helped to condense and rigidfy the concept of energy, both electromagnetic and mechanical energy, as being described as ML^2T^{-2}. This energy comes in 2 forms. Kinetic energy is thought of and expressed as mass x a velocity squared, or $M(L/T)^2$. Potential energy is thought of and expressed as a mass x a distance x an acceleration, $ML(L/T^2)$. Both are defined in the human scale metric system as Joules.

Futher humans even use the parametric grouping ML^2T^{-3}. But this grouping has only been thought of as power, the amount of energy inparted to or work done on an object in a given amount of time, $M(L/T)^2/T$. The defined unit for power, the Watt, was named after the Scottish mechanical engineer James Watt who first revolutionized the steam engine design with his invention in 1776.

A long time has passed and humans are still stuck with this limited vision that the only useful parametric groupings as being energy ML^2T^{-2} and power ML^2T^{-3}. Likewise they are stuck with the limiting concepts that the only useful relationships between distance and time are the first derivative, velocity (L/T), and the second derivative, acceleration (L/T^2). The importance of this human mental stuckness is seen shortly.

4.2 Disclaimers

Nowhere in this report does the author claim:

1 That the quarks exist or do not exist. That is, other than that they are an accepted and established part of the Standard Model for listing sub-atomic and sub-sub-atomic particles and other such observed and measured phenomena of the real physical universe.

2 That a 4th spatial dimension in fact exists or does not exist.

3 That a 4th spatial dimension in fact exists or can exist without quarks creating it.

4 That the words 4th spatial dimension as discussed in this report refer to "The" 4th dimension, rather that "a" 4th spatial dimension.

5 That a 4th spatial dimension existing at a scale 36 orders of magnitude smaller in distance than humans and 44 orders of magnitude smaller than the human invented second has any application or relevance to objects and phenomena at the human realm scale.

6 Anything about the nature of the 4th spatial dimension being discussed here, such as it being Euclidian, ultra-spherical, elliptical, hyperbolic…, or anything other than what is required and assumed by the mathematics of the applied differential geometry used here.

7 That what the author has found and presented here about the charge of the quarks, and about the quarks themselves, is The Way and The Truth, The Only Way and The Only Truth. If other researchers want to knock themselves out for 28 years finding other mathematical-geometric explanations for the charge of the leptons and quarks, the author won't stand in their way.

Never-the-less, regardless of these disclaimers there are many speculations, not backed by any mathematics or decimal places, which are too obvious and just waiting to be mentioned.

5 Speculations

While what is said next may be considered speculation what has been shown in this chapter so far is a definitive application of differential geometry. In this work what has been demonstrated is that the ± 2/3 and ± 1/3 charge of the quarks exactly correlates with curvature of a specific series of vector curves in 4 spatial dimensions.

5.1 The Confinement of The Quarks

Given what has been demonstrated here, the quarks are confined all right! They are confined to 4 spatial dimensions! They are truly creatures existing only as 4 dimensional forms and live in a 4D space. To try to down grade the quarks and force them into the 3 spatial dimensions destroys the quarkiness of the quarks.

This is a profound concept. What has been demonstrated here could help alleviate or mitigate many of the puzzles that both the experimental and theoretical physicists have faced when dealing with these strong force species. For example, the quarks have a new physical property not seen with the leptons, which have or know mass and charge. The quarks have the additional property of color; blue, green, and red. They now appear to have an additional degree of freedom due to their existing as forms in 4D space. Further the matching of the curvature k_1 of the curves with the ± 2/3 and ± 1/3 charge of the quarks can be done by three calculational routes, according to which three input parameters or four are chosen to be independent.

This work with the charge of the quarks has shown that the prior human ascribing or assigning the mathematics of the quarks as being 3D in nature is yet another example of human impositions, interpretations, and misunderstandings of what they are dealing with. The whole procedural approach of attempting to ascertain the true nature of particles as free independent species in and of or by themselves by examining the extremely short lived smoking garbage resulting from insanely high speed particle wrecks has in the case of the quarks been a failure. Consider even at the human scale realm such after-the-fact investigations are often useless. As an example of this futility consider attempting to describe what was the true nature of the front seat of a car by examining the burnt out wreckage after it has been hit by a hellfire missle. Hypothetical physics needs a re-think and new paradigm as to how it does its business.

Finally continuing to use the mathematics of 3D geometry to represent what is truly 4 dimensional in space will not lead to correct calculations. Using human assumptions as to the geometric form of "particles" in a size realm 36 orders of magnitude smaller in distance and 44 orders of magnitude smaller in time than humans is a seriously risky propisition. George Johnstone Stoney and Max Karl Ernst Ludwig Planck both already tried this. Further they threw in various assumed π constants in their "natural" or absolute units for distance-length, duration-time, mass, and charge for this scale realm of forms of existence. Again a re-think and new conceptual paradigm is in order for the quarks.

5.2 Senergy

This 4D spatial geometry immediately opens several concepts which can be applied in the Stoney scale realm of sub-sub-atomic physics calculations. These are the use of the third derivative of distance with time, jerk, and the fourth derivative, jounce. Just like their counterparts, the first derivative velociry and the second derivative acceleration, these higher member derivatives do not have to just be linear-radial. They can also be and highly probably are angular and rotational.

Further a person can ask questions about the interplay between jerk and jounce and the size of scale. At the astronomical scale jerk and jounce may not be of any relevance to the motion and interaction of bodies. Likewise at the human or "macro" scale. But the assumption should not be made that that jerk and jounce have no relevance at a scale 36 orders of magnitude smaller in distance than humans and 44 orders of magnitude smaller than the human invented second.

Additionally "objects" or "bodies" in these size realms are essentially just energy forms with no material substance. They cannot be expected to respond in any manner at all similar or analagous to how human scale objects and bodies respond to changes in velocity, acceleration, etc. The classical mechanical descriptions of how bodies behave and interact at the human or "macro" scale probably have very little to no relevance in this Stoney scale realm.

Now the parametric grouping for power can be thought of and expressed as a mass x jerk, $M(L/T^3)$. Additionally, besides being stuck the 2 dimensional like appearance of L^2 in all human size realm calculations dealing with energy, power, etc., now the volumetric appearance L^3 is available when calculating quantities dealing with sub-sub-atomic particles that are 4th dimensional in form..

A new concept name is needed here. This is called "**senergy**". Maybe in the Stoney scale realm of the quarks, besides just having energy, $E = mc^2$, there can be **senergy,** $\mathbf{S = mc^3}$. This probably should be stated as $\mathbf{S = mV_g^3}$ for the velocity of the gluons.

This could be reason why the color forces are so "powerful". They may be representations of the third derivative L/T^3 and they may also be representations of 3D volumetric quantities L^3, not just the planar L^2.

5.3 Super-luminal Information Transfer

A possibility is opened or at least the question needs to be asked, why does a truly 4th dimensional form living in 4 spatial dimensions have to abide by the rules for 3 dimensional photonic energy transfer, ie. the speed of light, c?

Yes, all 3D photons may not go faster that c, the speed of light. In the 3 dimensional subatomic scale realm those particles containing or representing matter, the leptons, defer to those who do not contain or represent matter, the photons, for their speed limit. And accordingly, as is well know, all matter starting with the atomic scale and on up may not go faster than c.

But quarks are not classical matter embedded in 3 spatial dimensions. Being 4th dimensional and living in a world of 4 spatial dimensions, quarks and/or gluons may have their own speed limit, V_g for the velocity of the gluons.

Following this line of thought, no matter at what speed of senergy transfer the quarks and gluons may traverse their home realm, they probably can not, do not "see" anything of the 3D world until they are stopped. Until these 4D creatures are stopped in their tracks, the 3D world may be just a blurr. Only when they are stopped can they "see" 3D phenomena and the "stars come out".

5.4 "Dark Matter" and "Dark Energy"

If as demonstrated here, the quarks are truly "closed" 4 dimensional forms living in a 4D space, then obviously so are the gluons and probably likewise many other open ended or moving 4D forms. These possibilites open un-counted means for describing and investigating "dark matter" and "dark energy".

5.5 Speculative Physics and Hypotheses

If a 4th spatial dimension exists as a universe, then within it there can be an infinite number of sub 3 dimensional universes. This gives intellectual plausibility to such physics hypothesis as those about "multiverses". Likewise this gives intellectual plausibility to discussions concerning "worm holes".

5.6 Oddball Time and Psychic Phenomena

Science in general and physics in particular have always cringed at and poo-poo-ed the idea of many well observed and documented psychic phenomena. This is because there has been no way to rationally explain these events in terns of the known 3D universe. Specifically events where there are what appears to be dislocations and/or discontinuities in time or space utterly raise science's hackles. Events such as forseeing the future, precognition, remote viewing in either or both time and space have never been accepted by the scientific community. And further these events can never be accepted by scientists, if the observed 3D universe with its limitations of the speed of light is all that exists. The existence of a 4th dimensional universe at the human scale, would allow for such events which appear to be non-local in either or both time and space.

While this report does not prove, demonstrate, or even show the existence of any of these events, nor mechanisms for them to occurr, what has been shown in this report does give food for the intellect to chew upon and calm down a little in relation to these psychic phenomena.

CHAPTER 1.4 A MODEL FOR DETERMINING PHYSICAL PROPERTIES IV: TERNARY FORCE INTERACTION CONSTANT

1 Introduction

1.1 Abstract

An equation has been discovered which explains the universal Ternary Force Interaction constant, $(G/\varepsilon_o)^{0.5}/\mu_o$ = 2.184,555,091 x 10^{+6} (C/kg relative)(L/T absolute)^2. Previous work discovered equations which described the mass densities of the leptons and the (ML)(L/T) for the photons in terms of G, ε_o, and μ_o. The equation found for the interaction amongst the three basic forces uses two of the same generic mathematical forms found with these elementary electromagnetic waveforms. Two of the equation's specific factors are identical, unchanged factors straight from these classes of waveforms. The resulting value calculated from this equation matches the decimal accuracy of the least accurately measured force constant, G. This value and the definitive mathematical-geometric explanations, with their links to the elementary electromagnetic wave form, are far superior to any hypothesis to date which attempts to specify the inter-relation of the three basic forces gravitational, electrical, and magnetic.

1.2 Objective & Scope

The objective of this report is to find a mathematical derivation for the universal Ternary Force Interaction (TFI) constant, $(G/\varepsilon_o)^{0.5}/\mu_o$ = 2.184,555,091 x 10^{+6} (C/kg relative)(L/T absolute)^2. The equation to be discovered should give strong hints of how the three basic forces gravitational, electrical, and magnetic become encapsulated or stabilized to form the elementary electromagnetic waveforms, "particles", the leptons and the photons. At the same time, the preference is that the actual equation to be found will represent the free space forces with no involvement of the particles or their properties. This equation also might conceptually point towards how the unary force of gravity "disproportionates" to become a binary radial-angular pair, electromagnetism.

The importance of the work in the previous reports on the leptons and photons [1-2] is paramount to the work here. In this work comparisons are made with the equations previously which were found which describe the mass density structures of the leptons and the structure for the (ML)(L/T) of the photons. An understanding of these previous equations is necessary, since many of their parts (factors) are embedded within the formula for the TFI constant. Some space is devoted here to highlighting the key findings of this prior work which are relevant and necessary to the formula discovered for the TFI constant.

Attention is also given to the background research analyses of measurement systems which shows that this TFI constant has several absolute units and these ultimately lead to its universality, numerical independent of any specific system of measurement. That is, this constant to be studied and its numerical value express a universal mathematic-calculus-geometric relationship amongst the three basic forces. That is even though it must represented with units from some systems of scales, both absolute and relative, for it to have any meaning or use within the context of the world experience of humans or any other species of inquiring beings.

The equation discovered is simply presented here. Its parts (factors) are described and analyzed to ensure understanding of their correct calculation and use. This work is best described as a mathematical analysis of some very limited and exact data which has resulted in an applied mathematical formula. This presentation should be thought of as only a demonstration. The descriptive equation found is not proven nor derived in the formal or rigorous mathematical sense of those words

The formula discovered was not derived from a hypothesis but was the result of a correlative approach. The formula presented here does not start with a hypothetical platform, does not specifically support any particular hypothesis, has not been embedded into any preexisting hypothesis, nor has a hypothesis been constructed around it. While there may be implications concerning particle hypotheses indirectly supported by this work, only those conclusions directly supported by the mathematics of the equation that was found are reported. Implications of this work for the multitude of current grand particle hypotheses are beyond the scope of this work.

1.3 Nomenclature

The following nomenclature is used in this report. It is included here for ease of understanding. For a more complete listing, see Table 1 in the introductory pages at the beginning of Part 1.

m, s, kg, C — The human versions of measurement units, placed on the relative or common SI metric scales

l_{Sgs}, t_{Sgs}, m_{Sgs}, q_{Sgs} — The human SI based versions of absolute units, placed on the absolute physics Squigs scales. These Squigs scales are based upon the measurement units put forth by George Johnstone Stoney in 1874, except have had his assumed 2 or 3 dimensional π constants removed.

L, T, M, Q — Generic, meta, or universal place holders for absolute or relative units of; distance-length, duration-time, mass, and charge

B_o, G_o, R_o — The color force constants, yet to be measured; blue, green, red

Table 1 Definition Of Absolute Physics Measurement Units

		Input -- Exponents				Derived -- Exponents				1 Squigs or Absolute Unit	
Quantity	Symbol	of Unit Combinations				of Force Constants				= n Common or Relative Units	
		L	T	M	Q	G	ε_o	μ_o	e	n	reciprocal
Length	l_{Sgs}	**1**				**0.5**	**0.5**	**1**	**1**	4.893753×10^{-36}	$2.043422 \times 10^{+35}$
Time	t_{Sgs}		**1**			**0.5**	**1**	**1.5**	**1**	1.632380×10^{-44}	$6.126024 \times 10^{+43}$
Mass	m_{Sgs}			**1**		**-0.5**	**-0.5**		**1**	6.591572×10^{-09}	$1.517089 \times 10^{+08}$
Charge	q_{Sgs}				**1**				**1**	1.602177×10^{-19}	$6.241506 \times 10^{+18}$

1.4 What Is An Interaction Constant?

An interaction constant needs to be defined. Most people appear to have never heard of a ternary force interaction (TFI) constant. They wonder just what it is that is the subject of the discussion of this whole report.

A good start would be making an analogy to another very well understood physical property constant. The gas law constant 82.0575 (atm cm^3)/(gmole K) describes the quaternary relationship of how the 4 parameters of pressure, volume, temperature, and molar quantity all interact to form a body of gas. This constant could be called a quaternary parameter interaction constant.

In the same view of generalized or generic naming, this report discusses how the basic three forces gravitational, electrical, and magnetic interact as they assemble to form the electromagnetic particles. The particles, both the leptons and the photons alike, can be thought of as assemblies of the three forces just as a body of gas is an assembly of four parameters. The constant $(G/\varepsilon_o)^{0.5}/\mu_o$ could be called the three force assembly constant. Additionally there is an emphasis here that this constant is more than just about three forces. It represents something about the three basic forces as an indivisible collective whole.

1.5 Prior Research & Where To Next

To the author's knowledge, academic hypothetical physicists have made no investigations into this area of research what-so-ever. Much calculational and investigative effort is made to predict at what energy the basic forces ultimately unify but none appear to be made which show just how they interact to form the elementary particles.

In the last two reports the mathematical-geometric structures of the basic electromagnetic waveforms, "particles", the leptons and the photons were investigated. In the lepton report [1], a solution was proposed for the puzzle of the masses of the leptons, the most basic particles responding to the electromagnetic forces. In the photon report [2] a solution was proposed to the mystery or origins of the (ML)(L/T) for the photon, the "carrier" of the electromagnetic forces.

Before an investigation is even begun into a possible fixed constant relationship amongst the three most basic forces, (gravitational, electrical, and magnetic) the question needs to be asked is this endeavor the next most logical step. What needs to be considered is not what physicists might like to investigate or have answers to next, but what are the next most logically important puzzle pieces. Most academic physicists probably want to continue investigating the properties of the quarks and gluons. This is not a good choice. This would be a choice based on desire not on logical priority. There are still several very basic puzzles pieces needed to get a well rounded or "complete" understanding of the most elementary waveforms that respond to and are constructed from the electromagnetic and gravitational forces, leptons and photons. One question to be answered is, how do the two basic constants ε_o and μ_o relate to each other? Another is, how does the unary force gravity, represented by G, relate to this binary pair?

First consider some obvious preliminary questions. How do ε_o and μ_o relate to each other. Specifically why are the two values ε_o = 8.854,187,817 x 10^{-12} $C^2/(kg\ m) \times (s/m)^2$ and μ_o = 1.256,637,061 x 10^{-6} $(kg\ m)/C^2$ what they are? Well known is that $(\varepsilon_o\ \mu_o)^{-1/2}$ = c, the "speed of light", but this reveals nothing. Well known is that $(\mu_o/\varepsilon_o)^{1/2}$ = 376.730,313,475 numerically and is important when used in relation to the Planck constant h or the fine structure constant α. Again this reveals nothing. While there are many equations in electrical engineering relating the electrical and magnetic forces as derivatives of each other, these still do not reveal why either is what it is. Using 1 / c as a fulcrum, μ_o and ε_o could be thought of as a balance beam, seesaw, or a teeter totter. Why then are the length of the arms of balance what they are? An infinite number of other combinations of other numerical values could have produced this same numerical value for c, by the same $(\varepsilon_o\ \mu_o)^{-1/2}$ mechanism. The value 376. 730... could have been double, half, the square, or the square root of what it is. For example the numerical sets (1.770,837 x 10^{-11} and 6.474,326 x 10^{-8}) and (2.350,272 x 10^{-14} and 4.734,132 x 10^{-4}) would have produced the same "speed of light". Examining this puzzle for a short time, a conclusion is quickly reached that this teeter totter is not linear, but is a logarithmic or exponential one. This still does not help much. Something inherent in the "structure" of these two forces or this force pair results in their particular numerical values. This remains an open question. If there is no understanding of the binary force set, why it is the way it is, how come its members have the values that they do, then there is not any hope of beginning any realistic conjectures about the ternary force set B_o, G_o, R_o ?

Next how does G relate to the force pair ε_o and μ_o. The question is simply, what is the mathematical geometric relation between the unary force set and the binary force set. Or what would be even better, how does G relate to the individual members of this binary set? If there is no means to get from the unary force set to the binary set as a collective whole or to its individual members, then again there is no hope of jumping out to the correct starting point to discuss the ternary set?

Before going off the spacy end into Grand Unified Hypotheses, a break and reconsideration are needed. The questions asked are not and do not require some grandiose speculation about how the current general state of the physical universe got to be the way it is. These questions do not require a Hypothesis Of Everything. Grand unified hypotheses are not necessary. Super symmetric partners are not needed to answer these questions. More mighty particle smashing machines are not necessary. More data is not even necessary. The necessary values of the three basic force constants G, ε_o, and μ_o have been around for many decades, even a century. Again, a correlation or model just needs to be built of the three of them.

Now that the process of becoming space cadets has been stopped and the idea of returning to common obtainable mathematics and real world verifiable physics phenomena has again arisen, serious consideration is needed of where this list of "obvious" questions has lead. These "obvious" questions are found to be bogus from the start. There were hidden assumptions behind all these questions. Every question was asked about a binary relationship; between ε_o and μ_o, between G and the set (ε_o, μ_o), between G and ε_o, or between G and μ_o. The assumption was that questions involving binary relations could be answered. In over eagerness to dissect relationships amongst these three most basic forces consideration was not given to some of the critical constraints placed by analyses of measurement systems upon any mathematical usages of these three forces.

The entire Part 3 is dedicated to very detailed analyses of measurement systems. Specifically Analyses of Measurement Systems I, II, & III show the role these three forces play in the absolute physics scales. These three basic forces are the source of the intertwined absolute scientific or particle physics Squigs scales. These Squigs scales are based upon the measurement units put forth by George Johnstone Stoney in 1874. Except the Squigs scales have had his assumed 2 or 3 dimensional π constants removed. Statements of relationships involving only two of these three force constants are not definitive or measurement system independent. There is no hope of ever finding a universal, measurement system independent, nor even a SI analogous measurement set independent, mathematical value involving a binary relationship such as these questions asked. A return to the idea of investigating the three forces as the next priority is needed. This still needs to be done as the next most logically required puzzle piece. Now though the need for an investigation the three basic forces G, ε_o, μ_o as a collective indivisible ternary whole is seen.

1.6 Structural Features Of Elementary Electromagnetic Waveforms, "Particles"

As detailed in the lepton and photon reports both species of the elementary electromagnetic waveforms, the leptons and the photons, were found to have many mathematical-geometric features in common. These are as follows.

1 A radial (2 dimensional, planar) energy density equation designated in the photon report as, $D_L(r)$ for the leptons and $D_P(r)$ for the photons.

The masses of the members of the lepton family were found to base upon the even numbered members of the Laguerre orthogonal polynomials. A series of shells were found for the higher members of the series, based on Laguerre polynomial even numbered derivatives. Note when used within the lepton report, $D_L(r)$ appears as $D_{pk}(r)$, where p designated the particle (electron, muon, or tau) and k designated the different energy shells for that particle. While the appearance of these mathematical patterns for the leptons family members is interesting in its own right, this is not of relevance here. The much simplified appearance of the electron's mathematics, which is based on the 0^{th} Laguerre polynomial, is used here. Both the Laguerre premultiplication factor, one in this case, and the normalizing factor, also one, become invisible.

2 Within the radial equations there is a Contractive Radial Spatial Factor R_{csf} which is identical for the two species leptons and photons. This contractive factor uses the argument $R_c(t_r)$, a function of the ultimate radial implicit variable t_r. Specifically; $R_{csf} = F(R_c(t_r)) = e^{(R_c(t_r))} = e^{(-6t_r^2)}$

3 Within the radial equations there is an Expansive Radial Spatial Factor R_{esfL} for the leptons and R_{esfP} for the photons. These two have the identical generic appearance or form R_{esf} seen later.

4 Within the Expansive Radial Spatial Factor there is an embedded or implicit variable $R_e(t_r)$.

This is distinct for the two species and was the subject of much discussion in the photon report. Note in the photon report this variable was labeled $R_{eL}(t_r)$ and $R_{eP}(t_r)$ when being applied to the specific species. As with the lepton family radial energy shells mentioned above, this mathematical distinction between the two species is interesting in its own right, but is not of relevance here. Again, the appearance of the electron's mathematics is used.

5 An angular (a single angle) energy density equation, $D_L(\theta)$ for the leptons and $D_P(\theta)$ for the photons. Note when used within the lepton report, $D_L(\theta)$ appears as $D_{pk}(\theta)$, where p designated the specific particle and k designated the different energy shells for that particle. Again, while working with the inter-relation of the free forces, these additional complexities of the stabilized or encapsulated forces, "particles", is not necessary.

6 The angular equation $D(\theta)$ has an Outer or exterior Spatial Functional appearance A_{osfL} for the leptons and A_{osfP} for the photons. These two have the identical generic appearance or form A_{osf}. This function is based on the odd numbered Chebyshev $T^{\dagger}$ orthogonal polynomials.

7 The angular equation $D(\theta)$ has an Inner or implicit Functional appearance $A_{iL}(t_\theta)$ and $A_{iP}(t_\theta)$ as the argument of the Outer Spatial Function. These are again identical in generic appearance or form $A_i(t_\theta)$. What was important to note for both particle species is that the ultimate angular implicit temporal variables t_θ and the radial implicit temporal variables t_r are completely independent of each other. While the internal details of these angular appearances are interesting and important to both the waveforms "particles" and to the free forces, the angular equations are used here in-tact as final integrated entities. Repeated explanations and discussions of their parts are not necessary here.

8 With the particles, stabilized force waveforms, there are initial temporal conditions for both the radial and angular equations, which lead to initial multiplying factors or constants. In the lepton report, these are I(r) which leads to the factor C_{rpk} and $I(\theta)$ which leads to the factor $C_{\theta pk}$. In the photon report these were designated $I_{rL}(t_r)$, $I_{rP}(t_r)$, leading to C_{rL}, C_{rP}, respectively, and $I_{\theta L}(t_\theta)$, $I_{\theta P}(t_\theta)$, leading to $C_{\theta L}$ and $C_{\theta P}$. As is discusses later, the lepton (electron) radial initial condition is still needed here, but not the photon's radial initial condition, nor either of the waveforms' angular initial conditions.

9 The mass and energy equations for the particles required a general scale factor or correlation constant. These were composed of basic a-priori measured physical constants, mainly the values of the free space force constants with their units. This appears as C_g in the lepton report, and were designated C_L and C_P in the photon report. Here the focus is on an interrelationship of the pure forces as represented by the most basic measured physical constants. Derivable physical properties of the waveforms is not investigated as in the previous reports. Nor is this report explaining or describing some other physical phenomena. There is no physical feature which needs to be scaled to the size realm of the common or relative human measurement systems. Never-the-less, as is seen in Section 3.4, the TFI constant is expanded into two parts; a universal mathematical-geometric constant and a factor with which to embed this constant into the absolute physics Squigs scales, or the absolute scales of any other inquiring species of beings.

10 Finally for the particles there is an overall equation combining the radial equations, angular equations, and the final scale factors as multipliers. This appeared as m_p in the lepton report, and describes the mass of the particle in kg. In the photon report this quantity was designated as e_p to describe the "energy" of the elementary electromagnetic wave form. Here again a final equation or formula combining the various pieces (factors, divisors) into a coherent whole is used.

Several mathematical-geometric complications which applied to the forces when stabilized or encapsulated as particles, is not necessary here when discussing the inter-relation of the 'free' forces. Eliminating these extra mathematical features the following remain:

1 The form or appearance of the complete radial equation for the electron is needed, including the form for its radial initial constant. The temporal constant from the photons radial equation is needed to go with these electron forms.
2 The complete integrated angular equations for both the electron and the photon is needed, in-tact, without their initial constants.
3 One other major feature to be borrowed from the physical properties of the waveforms is needed. This is the explanation found which describes the fixed charge of the leptons and the lack of a fixed charge, or the display of one, for the photons. This explanation was detailed at length in both the lepton and photon reports, so are only briefly summarized here.

2 The Ternary Force Interaction Constant

2.1 Logical Expectations

Is it reasonable to expect any commonality at all between the equations describing the properties of the leptons and photons and that to be found describing the ternary force inter-relationship? The answer is yes. As stated in Section 1.6 these classes of objects were described in the previous work as stabilized waveforms of the three basic forces.

Is there any help to be gotten from the other elementary waveforms, "particles", towards explaining a TFI constant? None is available. The neutrinos and their possible open ended wave counterpart the gravitons do not respond to electrical nor magnetic fields and presumable do not contain either internally. The values of the various physical properties of the neutrinos, primarily mass, have not been measured yet and the gravitons have not even been definitively shown to exist. The quarks and their moving form counterparts the gluons have the severe added complication of interacting with the color forces. The color forces themselves have not been thoroughly defined/mapped/explained nor have their values been determined.

In the lepton and photon reports both the fermions (leptons) and bosons (photons) were postulated to be utterly dependent upon the forces for their existence. Somehow between the three forces of concern, they set up the stable radial planar structures at the heart of the waveforms. These flat structures then rotate angularly and are driven forwards in flight in time. Of course the well known flight path of the photons being a straight line and the leptons' was found to be a circle. In these reports this force-to-particle relationship was treated as if it was a one way dependency street. This over simplified view kept the focus there on the waveforms and served a useful purpose but in fact is not correct.

The fact is the forces do not exist in isolation or somehow in a vacuum by themselves. Without the stable waveforms of the "carrier" particles, the photons, and without the "originator and receiver" particles, the leptons, then electromagnetism would not exist. Gravity could get along with the neutrinos, and possibly the missing gravitational "carriers", the gravitons, as its simplest expressions. Never-the-less, without origination and termination particles, none of the forces could display themselves. This is more than just a subtle philosophical point. The particles give form to the forces just as much as the forces give form to the particles. In fact the particle-force relationship is a total bi-directional dependency. The forces are utterly dependent on the waveforms, "particles", for their existence, just as the particles are dependent upon the forces.

Humans cannot even demonstrate the existence of the basic forces without there being some source material, matter, energy, instruments, etc and/or some termination material, matter, energy, detectors, etc. Since the forces can only definitively be found in the immediate vicinity of waveforms, either fermions and/or bosons, then finding similar mathematics involved for both these stabilized waveforms and the forces would be logical.

In Section 1.6 detailing the particles' mathematical features many items were found that would not be needed when describing the forces. The logic on one such feature should be examined for further clarity. Most of the initial conditions which applied to the waveforms are not needed here with the forces. Why is this so? While although maybe not a rigid mathematical proof, the logic for this is simple. The particles, both the 'fixed' fermions (leptons) and the moving bosons (photons), have been described as some form of standing waves of energy. Particularly for the fermions, this stabilized or encapsulated energy appears to humans as the property of mass. In science, nuclear and particle physics, mass or mass-energy is described as or thought of as a second derivative. Static or potential energy is also associated with position and acceleration, a second derivative phenomenon.

In mathematics, engineering, and science to resolve a differential equation back to its original equation, a boundary condition is needed, typically either in time or space. This boundary condition makes a general solution to the differential equation specific to the particular problem being solved. Usually for every derivative level (1st, 2nd, 3rd, etc) one such specifier is needed to make up for information which is 'lost' as the derivatives are taken. In pure mathematics such conditions are called constants of integration.

The mathematics found for the waveforms, that both the radial and angular equations, appeared to be the solutions to second order differential equations. The requirement of initial conditions there was to be expected. What was somewhat surprising was that only one such integration or initial-boundary constant was needed for each spatial-temporal frame, at least explicitly. A second condition, such as the radial equations going to zero at infinite distance was implied but never stated. Such conditions were obviously necessary if the integrations of the equations were to be bounded and to converge to describable values.

Here the discussion is about an inter-relationship or property of forces. In basic science and physics, force is thought of as first derivative phenomenon, with its materialization being momentum. When finding an equation (non-differential) which mathematically describes a property of the forces, the appearance of an initial conditions would not be surprising. Viewed from the equation descriptions of the waveforms, "particles", requiring less initial conditions here also is to be expected.

What mathematical features should not be expected to be found in the equation to be discovered? First a reminder that what is being sought is a mathematical explanation for a proposed ternary relationship amongst the three forces. Seeking to explain properties of any of the three forces by themselves is not the objective. There is no expectation that an inverse square law appearance of $(1/d)^2$ might be found nor necessarily any other property feature which is specific to any of the three forces. For example, gravity and electricity are most often thought of as an interaction between two separated point sources, ie a radial distance phenomenon. Whereas a magnetic field is most often depicted with a planar cross section that shows strong angular dependence. None of these concepts about the individual forces are relevant here.

2.2 Selection Of Candidate

Next various combinations of the bases of the four absolute scientific or physics Squigs scales are examined. A focus is on using G which is intentionally dismissed in most particle physics work. The intention of this work is to find a relationship amongst the three pure or free space elementary forces, if this is possible, without the properties of any particles (encapsulated or stabilized forces) involved in the mix. The elementary electron charge e is not included in the combinations.

Since ternary combinations of three of the four bases for the absolute physics Squigs system of scales are used, then there is assurance that all the numerical values that are found are independent or universal, for SI analogous sets of units. Or as is found with the constant for chosen combination, can easily lead to this universality. This is similar to the key constants used in the pervious reports. This is regardless of the origin of the scales or names assigned to the parameters. This numerical universality needs to be remembered as the discussions proceed, especially when referring to the size of various values which are found.

Using analyses of measurement systems, likely combinations of G, ε_o, and μ_o are screened for candidate choices. All of the candidates are to be ternary combinations of these three free force constants. Binary combinations leaving out one of the forces are not used, nor quaternary ones involving e. All of the initial choices involve these three universal system bases raised to various combinations of powers of -1, -0.5, +0.5, and +1. See Table 2 below. After screening the 32 likely combinations, $(G/\varepsilon_o)^{0.5}/\mu_o = 2.184{,}555{,}091 \times 10^{+6}$ (C/kg)(L/T absolute)^2, is chosen for several reasons as follows.

1 Many of the combinations, 16, result in the the four measurement units of distance-length, duration-time, mass, and charge (L, T, M, Q) or (l_{Sgs}, t_{Sgs}, m_{Sgs}, q_{Sgs} on the absolute scales) being raised to fractional powers, 1/2's. For many of these, seven, the value of the powers of distance, time, and/or charge are already so large that squaring the combinations to remove these 1/2 powers is unreasonable. For others, nine, their numerical values, or that of their reciprocals, are so large that there is no hope of finding a simple mathematical-geometric explanation. Some combinations have both flaws. These combinations and their reciprocals have been eliminated.
2 Now there are remaining 16 candidate combinations of the three force constants which lead to integer powers of the four measurement units (L, T, M, Q) _absolute or (l_{Sgs}, t_{Sgs}, m_{Sgs}, q_{Sgs}). Again many of these, seven, have hopeless numerical values for which simple geometric explanations can not be achievable. Additionally one combination results in unreasonably large powers of the basic measurement parameters. The final list has been down selected to 8 candidates for further examination.
3 The final 8 combinations are examined for likely physically achievable or meaningful combinations of the four basic measurement parameters. Particular attention is paid to blocking out groups of $(L/T)^{\pm n}$. The candidate $(G/\varepsilon_0)^{0.5}/\mu_0$ has been chosen because of the appearance of its measurement unit groupings (Q/M relative)^1(L/T absolute)^2 or (C/kg)$(l_{Sgs}/t_{Sgs})^2$ as being likely. Not only is this candidate the most likely, in fact it is unique and the only choice, as seen following.

In terms of the four measurement parameters distance-length, duration-time, mass, and charge (L, T, M, Q) this candidate is very special. The form of the two variables experienced by humans (L & T), here $(L/T)^n$, with n <= 2 is rare. Of the 16 combinations of the parameters for the three forces raised to the ±1/2 and ±1 powers, which produce integer powers for the four measurement parameters, only this one gives the appearance of $(L/T)^{\pm 2}$, and only two give the appearance of $(L/T)^{\pm 1}$. That is not counting reciprocals. The remainder either totally lack time or can be resolved as $(L/T)^{\pm n}$, where n >= 3. Both of the combinations which produced $(L/T)^{\pm 1}$ have numerical values past any simply achievable geometric range. Even more unsettling, all of these 16 combinations, except these three, have one or more L's which can not be paired with any T's. All these 16 combinations produced total summed powers for (L, T, M, Q) in the numerator positions equal to the total summed powers in the denominator positions. Any L's which are extra after pairing with available T's, need to be paired with either M or Q. That is if they are to placed in derivative like appearances. Pairing, ratioing, making a derivative out of this conceptual quantity of humans, distance, with a quantity intrinsic to the particles, mass or charge, somehow seems unsavory.

The broader list of 16 needs to be examined from another criterion, selecting those forms which have equal powers for mass and charge. Add the further criterion that mass and charge need to be in opposed numerator/denominator positions. The importance of these criteria are seen later in Section 2.4. Only three such combinations meet these two criteria, those which were already found to produce the groupings of $(L/T)^{\pm 1}$ or $(L/T)^{\pm 2}$. Again the chosen candidate form is very special and unique.

By this process of elimination using the several criteria, this one candidate is left by default.

Table 2 Selection Of A Candidate Constant From Combinations Of Force Constants

Input -- Exponents of Force Constants				Derived -- Exponents of Unit Combinations				1 Absolute Unit Combination = n Equivalent Relative Units		Special Feature	Reason To Discard
G	ε_o	μ_o	e	L	T	M	Q	n	reciprocal		
1				3	-2	-1		6.672590×10^{-11}	$1.498668 \times 10^{+10}$	Gravitational Const	
	1			-3	2	-1	2	8.854188×10^{-12}	$1.129409 \times 10^{+11}$	Electrical Constant	
		1		1		1	-2	1.256637×10^{-06}	$7.957747 \times 10^{+05}$	Magnetic Constant	
			1				1	1.602177×10^{-19}	$6.241506 \times 10^{+18}$	Elementary Charge	
1	1	1		1	0	-1	0	7.424258×10^{-28}	$1.346936 \times 10^{+27}$		3
1	1	-1		-1	0	-3	4	4.701466×10^{-16}	$2.126996 \times 10^{+15}$		3
1	-1	1		7	-4	1	-4	9.470122×10^{-06}	$1.055953 \times 10^{+05}$		2
-1	1	1		-5	4	1	0	1.667494×10^{-07}	$5.997025 \times 10^{+06}$	Final 8	4
0.5	1	1		-0.5	1	-0.5	0	9.088785×10^{-23}	$1.100257 \times 10^{+22}$		1, 3
0.5	-1	1		5.5	-3	1.5	-4	$1.159333 \times 10^{+00}$	8.625646×10^{-01}		1, 2
0.5	1	-1		-2.5	1	-2.5	4	5.755540×10^{-11}	$1.737456 \times 10^{+10}$		1, 3
0.5	-1	-1		3.5	-3	-0.5	0	$7.341565 \times 10^{+11}$	1.362107×10^{-12}		1, 2, 3
1	0.5	1		2.5	-1	-0.5	-1	2.495047×10^{-22}	$4.007941 \times 10^{+21}$		1, 3
1	0.5	-1		0.5	-1	-2.5	3	1.580007×10^{-10}	$6.329087 \times 10^{+09}$		1, 3
-1	0.5	1		-3.5	3	1.5	-1	5.603892×10^{-02}	$1.784474 \times 10^{+01}$		1, 2
-1	0.5	-1		-5.5	3	-0.5	3	$3.548706 \times 10^{+10}$	2.817928×10^{-11}		1, 2, 3
1	1	0.5		0.5	0	-1.5	1	6.622899×10^{-25}	$1.509913 \times 10^{+24}$		1, 3
1	-1	0.5		6.5	-4	0.5	-3	8.447937×10^{-03}	$1.183721 \times 10^{+02}$		1, 2
-1	1	0.5		-5.5	4	0.5	1	1.487508×10^{-04}	$6.722654 \times 10^{+03}$		1, 2
-1	-1	0.5		0.5	0	2.5	-3	$1.897413 \times 10^{+18}$	5.270335×10^{-19}		1, 3
1	0.5	0.5		2	-1	-1	0	2.225736×10^{-19}	$4.492895 \times 10^{+18}$		3
1	0.5	-0.5		1	-1	-2	2	1.771185×10^{-13}	$5.645938 \times 10^{+12}$	(L/T)	3
1	-0.5	0.5		5	-3	0	-2	2.513767×10^{-08}	$3.978094 \times 10^{+07}$	Final 8	4
1	-0.5	-0.5		4	-3	-1	0	2.000392×10^{-02}	$4.999020 \times 10^{+01}$	Final 8	4
0.5	1	0.5		-1	1	-1	1	8.107760×10^{-20}	$1.233386 \times 10^{+19}$	$(L/T)^{-1}$	3
0.5	1	-0.5		-2	1	-2	3	6.451950×10^{-14}	$1.549919 \times 10^{+13}$		3
-0.5	1	0.5		-4	3	0	1	1.215084×10^{-09}	$8.229881 \times 10^{+0}$	Final 8	4
-0.5	1	-0.5		-5	3	-1	3	9.669334×10^{-04}	$1.034197 \times 10^{+03}$	Final 8	4
0.5	0.5	1		1	0	0	-1	3.054439×10^{-17}	$3.273924 \times 10^{+16}$		3
0.5	-0.5	1		4	-2	1	-3	3.449711×10^{-06}	$2.898794 \times 10^{+05}$	Final 8	4
-0.5	0.5	1		-2	2	1	-1	4.577591×10^{-07}	$2.184555 \times 10^{+06}$	$(L/T)^{-2}$	Selected
-0.5	-0.5	1		1	0	2	-3	$5.169973 \times 10^{+04}$	1.934246×10^{-05}	Final 8	4
0.5	0.5	0.5		0.5	0	-0.5	0	2.724749×10^{-14}	$3.670063 \times 10^{+13}$		1, 3
0.5	0.5	-0.5		-0.5	0	-1.5	2	2.168286×10^{-08}	$4.611937 \times 10^{+07}$		1
0.5	-0.5	0.5		3.5	-2	0.5	-2	3.077356×10^{-03}	$3.249542 \times 10^{+02}$		1, 2
-0.5	0.5	0.5		-2.5	2	0.5	0	4.083495×10^{-04}	$2.448882 \times 10^{+03}$		1

1 MUTFP, measurement units to fractional powers

2 MUPTH, measurement unit power to high, to square

3 NVOOR, numerical value out of range, to be easily geometrically achievable

4 LUPT, L's unpaired with T's

2.3 Ternary Force Interaction Constant – Examination Of General Form

Now that the candidate combination of the three force constants has been chosen, further examination of its form is merited to see what what else can be learned.

Of course the appearances of both $(L/T)^1$ and of $(L/T)^2$ are central in science. The velocity of an object or wave is represented as $(L/T)^1$, which is understood as the first derivative of position with time rather than just a simple one point ratio. $(L/T)^2$ can represent either a distance times the second derivative of position with time (acceleration) or the first derivative (velocity) squared. When mass is included as a factor with these appearances $(L/T)^1$ becomes associated with momentum and $(L/T)^2$ with either potential energy or kinetic energy. What makes these appearances even more significant here is that forces are being discussed and also particles that are understood as manifestations of energy, stabilized waveforms.

What should not be surprising is if the mathematics which is found contains a factor which has the appearance of a second derivative or a first derivative squared or maybe just that of a simple ratio squared. Of course upon beginning there are no assurances what mathematics holds this form or give materialization to it, or even if anything does. One known is that the mathematical-geometric forms found for both the leptons and the photons contained functions of distance, where distance is in tern the outer appearance of an internal variable of time. The leptons had a radial implicit temporal variable and both the leptons and photons had angular implicit temporal variables.

Looking at the other two measurement units involved here with the form of this chosen candidate for the ternary relationship amongst the three basic forces a first derivative appearance is found, $(Q/M)^1$. Again since what mathematics will ultimately be found is unknown, also makes unknown whether this is in fact a first derivative, a simple ratio, or something else entirely. What is known is that the curvature or torsion for the vector based representation of the leptons appeared to give rise to their fixed charge. Additionally the temporal variable within the leptons' radial equation distinguished them from the photons' radial equation, and is probably responsible for the mass of the leptons and lack of it with the photons. There should be no surprise if the appearance of these lepton charge giving vector features or that of their mass giving radial equation are found to be part of the TFI constant's equation.

In the lepton report the formula which related the elementary charge e with the three measured force constants and mathematical-geometric constants was first developed. This was:

$$e = [\ \mu_o (G\ \varepsilon_o)^{1/2}\ ]\ A, \text{ units of Q relative, C} \qquad (01)$$

where A is a mathematical-geometric constant, with mixed relative per absolute physics Squigs units of $C^2 l_{Sgs}^{-1}$. A can be calculated or decomposed as follows:

$$A = (2\pi)^{-3/2} \text{ x } (\pi\ \rho^2) = 1/2(2\pi)^{-1/2}\ \rho \qquad (02)$$

To which factor the units $C^2 l_{Sgs}^{-1}$ are attached was not self evident.
Here numerically ρ is a constant which was found to be:

$$\rho = 6\ /\ (6^2 + 1^2) \qquad (03)$$

This has the generic geometric form of $a/(a^2+b^2)$. This same generic form was seen for the Series Member Factor, F_{mp}, in the equations for the leptons' masses, where there a = 6 and $b = (n-1)^{1/2}$, and n is the number of the particle in the series.

Rearranging Equation (01), $e = F_1(G, \varepsilon_o, \mu_o, \text{geometry})$ to make G the independent variable and flipping around produces

$$F_2(e, \varepsilon_o, \mu_o, \text{geometry}) = G^{1/2}, \text{ specifically: } \quad e/(\varepsilon_o^{0.5}\mu_o A) = G^{0.5} \qquad (04)$$

The expression on the left is a relation between three of the four necessary bases for the absolute physics Squigs scale system; e, ε_o, μ_o. This expression equals a constant, with units, which is meaningful and just happens to be the universal gravitational constant. Note that now A is in the denominator, the appearance (a^2+b^2) takes position in the numerator.

Examining this chosen candidate ternary force relation, we have $G^{0.5}/(\varepsilon_o^{0.5}\mu_o)$ = a numerical constant, with absolute units. By analogy with Equation (04), the only difference is e in the left hand side numerator in one and G is in the other. Some similarity might be expected for the mathematical factors which are to be found.

A final appearance is worth noting, that of the arrangement of the three forces in the chosen candidate $(G/\varepsilon_o)^{0.5}/\mu_o$. The two forces most commonly thought of as being radial phenomena between two point sources, gravitational and electrical, are ratioed and both to the half power. This "unit" is then compared or referenced to the force, magnetic, most often depicted as having a strong angular appearance, requiring at least a 2 dimensional appearance.

2.4 Resolution Of The Universality Of The Constant To Be Found

Before proceeding, the issue of the universality of the numerical value of the chosen candidate ternary force combination needs to be resolved. This was intentionally left open until the form was seen that this constant was going to take.

The obvious question is; just because the three force constants are the bases of the absolute physics system of Squigs scales, what makes these constants or their scales special? Specifically, why should the numerical value of this chosen candidate combination be expected to be universal, for SI analogous sets of units? Such an assumption can be argued to automatically imply that the numerical result of an analogous combination from some other equally arbitrary set of absolute scientific scales invented by some other species of inquiring beings is not numerically universal. This claim needs to be examined.

In Measurement Systems Bases the unit of the coulomb is found to not in fact be an independent unit, but has been derived for the SI set of scales as a calculated quantity in terms of the other basic quantities of L, T, and M. This unit for the quantity of electrical charge underlies or is embedded into all four of the absolute physics Squigs scales that are being used. As found in Measurement Systems Bases

$$1 \text{ coulomb} = 1.0\ (\text{kgm})^{1/2} \qquad (05)$$

Substituting this expression into the candidate ternary force combination, $(G/\varepsilon_0)^{0.5}/\mu_0 = 2.184{,}555{,}091 \times 10^{+6}$ (C/kg)(L/T absolute)^2 or (C/kg)$(l_{Sgs}/t_{Sgs})^2$, then becomes:

$$(G/\varepsilon_0)^{0.5}/\mu_0 = 2.184{,}555{,}091 \times 10^{+6}\ (\text{L/T absolute})^2 \times 1.0\ (\text{m/kg})^{1/2} \qquad (06)$$

Secondly there is reminder in Measurement Systems Bases that one of the key bases underlying the entire metric system was the relationship tying 1 unit of mass to n units of distance cubed; as in 1 gram = 1 ml = 1 cm^3 or equally 1 kg = 10^{-3} m^3. Ultimately in terms of the most basic units the TFI constant is found to be:

$$(G/\varepsilon_0)^{0.5}/\mu_0 = 2.184{,}555{,}091 \times 10^{+6}\ (\text{L/T absolute})^2 \times 10^{3/2}\ \text{m}^{-1} \qquad (07)$$

Two major transformations have occurred which do make this system of absolute physics Squigs scales and this chosen ternary force combination of these scales very special.

First the measurement unit of charge and mass have gone away and all that is left are distance and time. This is a good thing. Consider the following statements or views upon the world. The only two parameters experienced and maybe understood by humans are distance and time. Mass, charge, and color are just verbal descriptions for composite-complicated groupings that scientists do not know how to explain or otherwise describe, and are probably not independent, inherent, or intrinsic properties of basic "physical" reality. In fact as demonstrated in the lepton report, charge and mass of the leptons can be explained in terms of math, geometry, and an absolute unit of length. This TFI constant has been resolved back to the two fundamental dimensions, distance and time, as demonstrated by G. E. A. Matsas, et al in their article "The number of dimensional fundamental constants" [3].

Second a second numerical value has been introduced with units, 1.0 $(m/kg)^{1/2}$ or equally 1 x $10^{3/2}$ 1/m. This is also a good development, in that now the TFI constant can be resolved into two factors. First there is the original proposed mathematical-geometric constant, with simplified units, 2.184,555,091 x 10^{+6} $(L/T \text{ absolute})^2$ and the new factor, a completely arbitrary human scaling factor 1.0 $(m/kg)^{1/2}$ or 1 x $10^{3/2}$ 1/m. This now has the same general form of all the three previous constants which were developed in the lepton and photon reports, a universal mathematical-geometric factor with meta-units and a human SI unit set scaling factor. See Tables 3 and 4.

Table 3 Equations For Elementary Physical Properties, Symbolic With Units

PHYSICAL CONSTANT	FORM OF CONSTANT	SCALING MULTIPLIER for System of Units	UNIVERSAL MATH-GEO CONSTANT SI & Squigs Units
Electron Properties			
Electron Charge	e, C	$\mu_0 (G/\varepsilon_0)^{0.5}$, L/C	A, C^2 / L
Electron Mass	m_e, kg	$C_g = e\mu_o(G\varepsilon_o)^{1/2}$, L	$D(r) \times D(\theta)$ = radial-ang prod
	or energy$_{lepton}$	or C_L in Photon Report	$D_p = D_L(r) \times D_L(\theta)$, kg/L
Photon Properties			
Planck Constant	*h*, (kgm)(m/s)	$C_P = e^2(\mu_o/\varepsilon_o)^{1/2}$,	$D(r) \times D(\theta)$ = radial-ang prod
	or (m·l)(l/t)$\_{photon}$	(kgm)(m/s) / (ML)(L/T)	$1/(2\alpha) = D_P(r) x D_P(\theta)$, (ML)(L/T)
Force Properties			
Ternary Force Const	$1/\mu_0 (G/\varepsilon_0)^{0.5}$	From amp-coulomb def'n	TFIC = $1/\mu_0 (G/\varepsilon_0)^{0.5}$
	$(L/T)^2$	C/kg = 1 x $(m/kg)^{1/2}$	$(C/kg)(L/T)^2$

Table 4 Equations For Elementary Physical Properties, Numerical

PHYSICAL CONSTANT	VALUE OF CONSTANT	SCALING MULTIPLIER for System of Units	UNIVERSAL MATH-GEO CONSTANT Radial x Angular Products
Electron Properties			
e	1.602,177,33 x 10^{-19}	3.054,438,950 x 10^{-17}	5.245,406,17 x 10^{-3}
m_e	9.109,389,7 x 10^{-31}	4.893,752,842 x 10^{-36}	1.861,432,180 x 10^{+5}
Photon Properties			
h	6.626,075,5 x 10^{-34}	9.670,562,404 x 10^{-36}	6.851,799,475 x 10^{+1}
Force Properties			
TFIC	2.184,555,091 x 10^{+6}	1.0	2.184,555,091 x 10^{+6}

There is a difference from the pervious wave form, "particle", investigations and what is being done here. Previously the objective quantity, column two in Tables 3 and 4, was the product of a scaling multiplier and a mathematical-geometric value with some form of universal place holder units. Here the objective quantity is just the mathematical-geometric value, with its absolute parameters of distance and time. There is no concern here what the constant looks like when it has been multiplied by the scaling factor and embedded into either the human absolute or relative scale systems.

Repeating for clarity in Table 3, L, T, M, Q stand for absolute or universal measurement units, SI analogous based, of distance-length, duration-time, mass, and charge and m, s, kg, C are human relative SI units of measurement.

As seen before, the scaling factor although it is based upon totally arbitrary human inventions, it is a known and fixed constant. As has been demonstrated this expanded TFI constant can now be exported to any absolute scientific scaling system of any species of inquiring beings. The only criterion was that one of the four scales, that for the static quantity of charge be mathematically linked to the other three scales. The original numerical value 2.184,555,091 x 10^{+6}, just now with the simplified units of absolute velocity squared, is indeed numerically universal. So the path has been cleared to show the objective of this research, the mathematical-geometric nature of this constant.

2.5 Actual Parts (Factors) Used In The Equation

Now that expectations have been set of what would be desirable to see in the ternary force equation, it is time to see what was actually found. In the previous photon report the subscripts of "L" was used to designate mathematical expressions associated with the leptons and "P" for those associated with the photons. Here the subscript "TFI" is added to designate expressions related to the Ternary Force Interaction constant.

1 First the actual radial mass density equation for the electron is used, except with a slight change to the implicit function within the expansive radial spatial factor $R_{eL}(t_r)$ to become $R_{eTFI}(t_r)$. The first factor in the TFI constant is.

$$D_{TFI}(r) = \int_0^\infty R_{csf} R_{esfTFI} dt_r \qquad (08)$$

This is $D_{electron}(r)$ without its initial or normalizing constant premultiplier.

The Contractive Radial Spatial Factor: $R_{csf} = F(R_c(t_r)) = e^{R_c(t_r)} = e^{-6t_r^2}$ (09)

The Expansive Radial Spatial Factor: $R_{esfTFI} = F(R_e(t_r)) = e^{R_{eTFI}(t_r)}$ (10)

The Expansive Radial Implicit Function: $R_{eTFI}(t_r) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}} ds\left(\frac{2\pi t_r^2}{k_P^{1/2}}\right) = \left(\frac{\pi}{2}\right)^{\frac{1}{2}}\left[1+\left(\frac{4\pi t_r^1}{k_P^{1/2}}\right)^2\right]^{\frac{1}{2}} dt_r$ (11)

where $k_P = 1.980,416,377\ldots = [\int_0^\infty \text{FHDif}(1.000,000,\ldots t_r^{1/2}) dt_r]^{1/2}$, the Fraunhofer Diffraction Constant as is used with the photon. Obviously ds is the two dimensional rectilinear distance function,

$$\frac{ds}{dt} = \left[1+\left(\frac{dy}{dt}\right)^2\right]^{1/2} \qquad (12)$$

and as used here represents the instantaneous change in the area of an expanding circle in time weighted by $2/k_p^{1/2}$.

Putting together these contractive and expansive factors as a final integrated equation, the result is.

$$D_{TFI}(r) = \int_0^{\infty} e^{-6t_r^2} e^{\left(\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\left[1+\left(\frac{4\pi t_r^{1}}{k_P^{1/2}}\right)^2\right]^{\frac{1}{2}}\right)} dt_r = 1.456,945,488 \times 10^2 \quad (13)$$

Note this is identical with the electron's Radial Equation as seen in the lepton report, except for the electron $k = k_L = \mathrm{FHDif}(r^1) = 1.697,525\ldots$ which of course leads to an entirely different integrated value.

Here FHDif() is the Fraunhofer Diffraction Function as described in both the lepton and photon reports. Mathematically it was defined as

$$\mathrm{FHDif}[F(r)] = \left[\frac{2J_1[F(r)]}{F(r)}\right]^2 \quad (14)$$

Typically the simple monomial (kr^p) is used for the function F(r), with J_1 being the Bessel function of the first Kind, Order 1. The Fraunhofer Diffraction Constant was defined as the numerical value $k_n = [\int_0^{\infty} \mathrm{FHDif}\,(1.000{,}000, \ldots r^n)dr]^n$.

2 This complete integrated radial equation needs to be multiplied by a Radial Initial Condition
$I_{rTFI}(t_r) = \mathrm{FHDif}(kt_r^p)$

which for the leptons is usually based on k_L= 1.697,525,53 and p =1. Except now $\mathrm{FHDif}(k_P t_r^{1/2})$ is used, which leads to the Radial Initial Constant

$$C_{rTFI} = \int_0^{\infty} \mathrm{FHDif}[k_P t_r^{1/2}] e^{-6t_r^2} e^{\left(\left(\frac{\pi}{2}\right)^{\frac{1}{2}}\left[1+\left(\frac{4\pi t_r^{1}}{k_P^{1/2}}\right)^2\right]^{\frac{1}{2}}\right)} dt_r = 5.699,931,642 \times 10^1 \quad (15)$$

3 Multiplying the initial or normalizing constant and radial equation together as a final form representing or holding the place of mass, the numerical result is.

$$(5.699,931,642 \times 10^1) \times (1.456,945,488 \times 10^2) = 8.304,489,686 \times 10^3. \quad (16)$$

4 Next the actual angular energy density equations from both the leptons and the photon is used. The paramters D(θ), $D_L(\theta)$ for the leptons and $D_P(\theta)$ for the photons are introduced, which have the form of A_{osf} the Outside Angular Spatial Function.

$$A_{osf} = \int \mathrm{Sin}(\pi/2\ A_i(t_\theta)\ dt_\theta \quad (17)$$

The Angular Inner or Temporal Implicit Functions: $A_{iL}(t_\theta) = [1 - f_L^2(t_\theta)]^{1/2}$ (18)

Where $f_L(t_\theta) = \sin(n^{-1}t_\theta)$, with n = 1 for the electron. Ultimately giving

$$A_{osfL} = \int_0^{\pi/2} \sin[\left(\frac{\pi}{2}\right)\cos(t_\theta)]dt_\theta = 1.180,580,071 \quad (19)$$

For the photons, again

$A_{iP}(t_\theta) = [1 - f_P^2(t_\theta)]^{1/2}$ (20)

Where $f_P(t_\theta) = 1/u\ t_\theta$, and u is the upper limit of the integration. Ultimately giving

$$A_{osfP} = \int_0^{4/\pi} \sin[(\pi/2)(1 - (t_\theta/u)^2)^{1/2}]dt_\theta = 1.133{,}648{,}187 \quad (21)$$

Where $u = 4 / \pi$.

5 These integrated angular equations now need to be ratioed and then this ratio squared to give

$(A_{osfL} / A_{osfP})^2 = (1.041{,}398{,}984)^2 = 1.084{,}511{,}844$ (22)

This then holds the place of the form $(L/T)^2$ in the TFI constant. Finally, this composite needs to be multiplied by a symmetric factor of 4 and the Chebyshev $T_1^\dagger$ normalizing factor of $(\pi/2)^{-1/2}$. The angular factors of the equations borrowed from the particles, stabilized waveforms, ultimately produce the value here;

$1.084{,}511{,}844 \times 4\ (\pi/2)^{-1/2} = 3.461{,}261{,}030$ (23)

6 A final factor needs to be borrowed from the lepton equations. This is the relationship which correlates the charge of the leptons with the curvature or torsion of a generalized cylindrical spiral. For a generalized cylindrical spiral, in three dimensional rectilinear vector notation, of the implicit variable t,

$\mathbf{R}(t) = a \operatorname{Cos}[F(t)]\ \mathbf{i} + a \operatorname{Sin}[F(t)]\ \mathbf{j} + bG(t)\ \mathbf{k}$ (24)

For such a vector the curvature and the torsion are rigorously calculated as

$$\text{curvature } \kappa = \frac{|\mathbf{R}'(t) \times \mathbf{R}''(t)|}{|R'(t)|^3} \quad (25)$$

$$\text{torsion } \tau = \frac{[\mathbf{R}'(t) \times \mathbf{R}''(t) \bullet R'''(t)]}{|\mathbf{R}'(t) \times \mathbf{R}''(t)|^2} \quad (26)$$

and as such both are scalar quantities. To obtain a usable analytical form for these conceptual parameters the simplifying assumption $G'(t) = F'(t)$ is required. Then two simple forms, numerical constants free from the implicit variable and all functions of it, result.

$$\kappa = \frac{a}{a^2+b^2} \quad (27)$$

$$\tau = \frac{b}{a^2+b^2} \quad (28)$$

This form $b / (a^2 + b^2)$ is used to hold the place of charge in the TFI constant, where a = 6 and b = $\sqrt{2}$. The resulting constant $\sqrt{2} / 38$ is found. As noted in the last preliminary expectation above, the simple analogy in Equation (04) this form takes the reciprocal position $(a^2 + b^2) / b$ or $38 / \sqrt{2}$.

7 As a final miscellaneous or geometric factor is needed. Numerically this fourth factor = 2 x sqrt(2) or equally sqrt(8). If rolled into the third factor, this "trivial" but necessary fourth factor, would disappear. It is a necessary factor and as such the author would like to assign a plausible origin or meaning. But this obviously would only be the supporting of one story out of dozens which could be made up. This fourth factor is left alone to stand as it is.

With these final four factors the results in Table 5 are found. Considering the measured numerical accuracy of one of the key force constants, G, becomes questionable in the 4th decimal, this is an acceptable result for this research endeavor.

Table 5 Decomposition Factors Of Ternary Force Interaction Constant

Factor	Source	Numerical Value
Mass Related	Radial, $C_{rTFI}D_{TFI}(r)$	8.304,489,686 x 10^{+3}
Space-Time Related	Angular, $(D_L(\theta) / D_P(\theta))^2$ x $4(\pi/2)^{-1/2}$	3.461,261,030
Charge Related	$1/\tau = 38 / \sqrt{2}$	2.678,005,768 x 10^{+1}
Geometric-Numerical	2 x $\sqrt{2}$	2.848,427,125
Project Results	Product All Factors 2.184,544,496 x 10^{+6}	Numerical Target 2.184,555,091 x 10^{+6}
Comparison Ratios	product / target 0.999,995,150	target / product 1.000,004,850

3 Discussion

Now that the objective of finding an equation which meets the numerical requirements has been met, some explanations are required. At the start of this research, before even finding the equations which explained the charge and masses of the leptons and the (ML)(L/T) of the photons, a commitment was made to meet several strict logical requirements. The second one being that any equations developed must be explainable. Every mathematical factor in any such equation must be identifiable as having some plausible origin.

A reminder is needed that the objective is not just to find, by brute force, a numerical value which matches the objective constant. Much more than this is required. A mathematical-geometric model of what occurs when the three basic forces interact is being developed. To be of benefit, this model must reflect as accurately as possible what actually physically occurs. Hopefully, this model gives hints of how the radial force gravity "disproportionates" under suitable conditions or resolves itself into a much more powerful perpendicular radial-angular binary pair, electro-magnetic.

In selecting a candidate ternary force combination the form of the measurement units involved were focused upon. To develop the actual factors which would make up the TFI constant, analogies to factors within the lepton and photon equations were the focus, again based upon their perceived functionality in those equations in terms of measurement units. Here the conceptual focus needs to shift and to place the various mathematical-geometric factors within a physical context. Within a space-time framework, what positions are held by the factors that were found?

3.1 Radial Factor

The first factor found was previously related to the mass of the leptons and lack of mass of the photons. It was also clearly the solution to a radial energy density pattern. Here this factor represents a radial force density pattern in three spatial dimensions, and probably models the gravitational force behavior in the ternary force assembly.

If this factor is interpreted in terms of the mass originating equations of the stable waveforms, "particles", then in terms of the ultimate implicit variables it would relate to t_r. As seen though in Equations (09) and (10) this temporal variable t_r is rapidly converted to the spatial analogues R_{csf} and R_{esfTFI}. Then the independent variables are ultimately eliminated into the overall dependent parameter by the surrounding outside integration. The final equation is time independent or shows a long term steady state relationship amongst the three basic forces.

This temporal independence is a good thing. Physically the radial decay of the three forces, (gravitational, electrical, and magnetic) is the well known inverse square law, but the rate of their temporal decay has not and probably can never be investigated. That is unless humans can measure events in range of the absolute duration-time scale of the t_{Sgs}, 1.632,380 x 10^{-44} seconds. Only then will humans be able to show in time how the forces come into being at initiation or decay at termination.

Again a reminder is needed that a ternary interrelation is being investigated and not the individual forces, even though this particular factor may give hints as to the temporal or spatial behavior of the forces in isolation.

Equally the integration gives a reminder that this interrelation is a distributed phenomenon and that all temporal-spatial regions contribute to the overall numerical value. All temporal-spatial regions exist, those near the origin and the final outwards extents. That is, the whole of this interrelation exists, not probably exists. Again the ultimate final units of these equations are seen to be those of the real physical world, either relative and/or absolute, and that these equations do not result in numerical values in probabilistic space, momentum space, or other such mathematical conceptual oddities.

3.2 Angular Factor

As found this factor is the ratio of two integrals, the integrated angular factors for the leptons and photons. What would have been more desirable is a single integral which was composed of the ratio of the un-integrated lepton and photon angular expressions. A short investigation shows this is impossible. An upper limit cannot be assigned to the integral of such a ratio. The integral for the lepton angular equation has an upper limit of $\pi/2$, and the integral for the photons angular expression has an upper limit of $4/\pi$. One fraction has an irrational numerator and the other an irrational denominator, there is no hope of finding a lowest common denominator or period at which to terminate an integral of their ratio that might be applicable or of interest. A simple ratio of their previously integrated values is the best that can be done. Then any applicable symmetry and normalizing factors are applied to this ratio, just as would have been done if the expression had been the result of a single integral.

What would give further meaning to the ratio of two different angular expressions? This could describe two angular waves rotating in opposite directions, passing thru each other repeatedly, and continuing from and to infinity. Hints were seen of exactly this wave behavior in the examination of the nature of the angular mathematics for the leptons, which was done in the photon report. The only difference was for the leptons, the forwards and reverse angular waves were identical in description and passed thru each other at regular fixed intervals. Here such waves would have a highly irregular way of passing thru each other.

This factor has been assigned as holding the position represented by the units of $(L/T)^2$. Since this factor derived from the angular equations for the stable waveforms, "particles", then in terms of the ultimate implicit variables it would relate to t_θ. Due to its angular source, this factor probably also models the behavior of the magnetic force in the ternary force assembly. This mathematical description here also hints as to why magnetic monopole have not been found, because the magnetic field may actually be composed of dual waves.

3.3 Third Factor

The third factor has been related to the appearance of the equation describing the charge of the leptons. This conceptual view is of no use here, where putting the various factors into spatial or temporal contexts is the objective. Referring back to the curvature or torsion of a generalized cylindrical spiral, this third factor is found to probably represent the behavior of the TFI constant in the third spatial dimension. In radial-angular coordinates this would be the second angle of discussion, typically called φ. Or if stated in terms of the ultimate implicit variables, this factor would be related to the unsubscripted variable t.

Mathematically what has been found is a pure number. Vector expressions for the curvature or torsion would give a spatial geometric context to this third factor. This curvature or torsion probably models the behavior of the electrical force in the ternary force assembly.

There are alternative means to accomplish this same mathematical goal. There are other mathematical expressions which lead to the objective form $b / (a^2 + b^2)$. For example;

$$\int_0^\infty e^{-ax^1}\sin(bx^1)x^0dx = b/(a^2 + b^2) \tag{29}$$
$$\int_0^\infty e^{-ax^2}\sin(bx^2)x^1dx = (\tfrac{1}{2})b/(a^2 + b^2) \tag{30}$$

Analogous cosine expressions are also available. The e^{-ax^2} decaying density appearance seen here was also found with the contractive radial (gravitational) factor $R_{csf} = e^{-6t_r^2}$. Also as with the original vector based curvature or torsion possibilities, these two integral forms also carry angular, or at least trigonometric, constituents.

The importance of these other possibilities here can be found by referring back to the original assumptions 1 and 2 in Chapter 4.1, Methodology. There the assumption was made that all elementary physical forms can be described by appropriate mathematical expressions. Here the converse possibility is intentionally raised. That is, that all conceptual mathematical expressions can have, or even do result in, some form of physical expression. The only constraints prohibiting this possibility are that most, a vast majority, of such mathematical expressions would result in forms which are either so nebulous or so unstable that humans cannot identify them with any practical force or energy manifestations. This does raise the possibility though that these alternative expressions leading to $b / (a^2 + b^2)$ may have real existence or physical expression. Further this raises the possibility that both mathematical forms, the original curvature-torsion interpretation and that of these alternatives, and their physical manifestations may exist simultaneously, leading to the ternary force complex.

3.4 Geometry

Some discussion is needed as to what all this means or how this ternary force collection appears in terms of 3 dimensional spatial geometry. While although not supported by mathematics with decimal places, the author feels that what follows is a logical possible representation or description of the geometric structure formed by the three basic forces as they meet and interact in 3 dimensional space.

When the three basic forces meet, it is not as an apex of a tetrahedron at the center of elementary electromagnetic particles, the leptons and the photons. This is not the correct concept or representation.

Rather gravity forms a tether to the center of the wave structure of the leptons and photons. At the other end of this tether, or vector, gravity meets the electromagnetic pair at the corner of a spherical triangle. The electric and magnetic vectors form the legs of a 90^o spherical triangle curving about the surface of the "object" of discussion. Further gravity does not have to meet this pair at a fixed 90^o angle.

Consider an analogy with the earth.

At the poles of the earth, the magnetic field lines "enter" the earth perpendicular to the surface. Gravity meets this basic force component in a parallel manner or at a 0^o angle. In a similar manner the cross section of the leptons could be considered an "infinitely" short bar magnet. The tethering gravity would meet the electromagnetic pair parallel to or off the point of their spherical triangle.

At the 45th latitudes of the earth, gravity meets the magnetic field lines at a 45^o angle. The analogy is offered, that this is how gravity meets the flight path of the leptons.

At the equator of the earth, gravity meets the magnetic field lines at a 90^o angle or perpendicularly. The analogy is offered, that this is how gravity meets the flight path of the photons, and also the cross section of the photons.

4 Summary - Conclusions

An equation has been discovered which explains the universal Ternary Force Interaction (TFI) constant, $(G/\varepsilon_o)^{0.5}/\mu_o = 2.184,555,091 \times 10^{+6}$ (C/kg)(L/T absolute)^2.

This constant could be decomposed into two parts;

1 One factor is a universal mathematical-geometric constant $2.184,555,091 \times 10^{+6}$ with measurement units of (L/T absolute)^2 or $(l_{Sgs}/t_{Sgs})^2$ in the system of the four interlocked, self consistent, and comprehensive absolute physics Squigs scales.

2 The other factor is an arbitrary scaling constant 1.0 $(m/kg)^{1/2}$ which places this universal value into the relative human SI set of units.

This same "trick" could be employed to place this numerical constant in the framework of interlocked absolute scientific scales from any time, place, or species of inquiring beings, with the provision that the scale for charge was in fact not independent but was derivable in terms of the scales for mass and distance. That is, this scaling must use a SI analogous set of units.

As decomposed the TFI constant was found to be a composite of four factors;

1 A double exponential factor analogous to the mass density factor for the electron. Here this factor probably represents a radial force density pattern and models the behavior of gravity in the three force interaction assembly.

2 A ratio of the angular wave patterns for the electrons and photons. Due to its angular nature, this factor was assigned to hold the place or to model the behavior of the magnetic force in the ternary force composite.

3 A vector based curvature or torsion factor analogous to the explanation for the arisel of the charge of the electron. This factor was assigned as modeling the electrical force behavior as the three forces interact.

4 A "pure" mathematical factor, for which the author has left open the intrepretation or meaning.

The strong overlaps here with the mathematics of this three force interaction assembly and those of both the photons and leptons show the strong relationship between the forces and the energy wave patterns of the elementary electromagnetic waveforms. Whether this is the open ended moving bosons (photons), or the closed form "fixed" fermions (leptons), the relationship is now seen between these encapsulated force structures (perceived as energy) and the entangled free space forces.

6 References

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