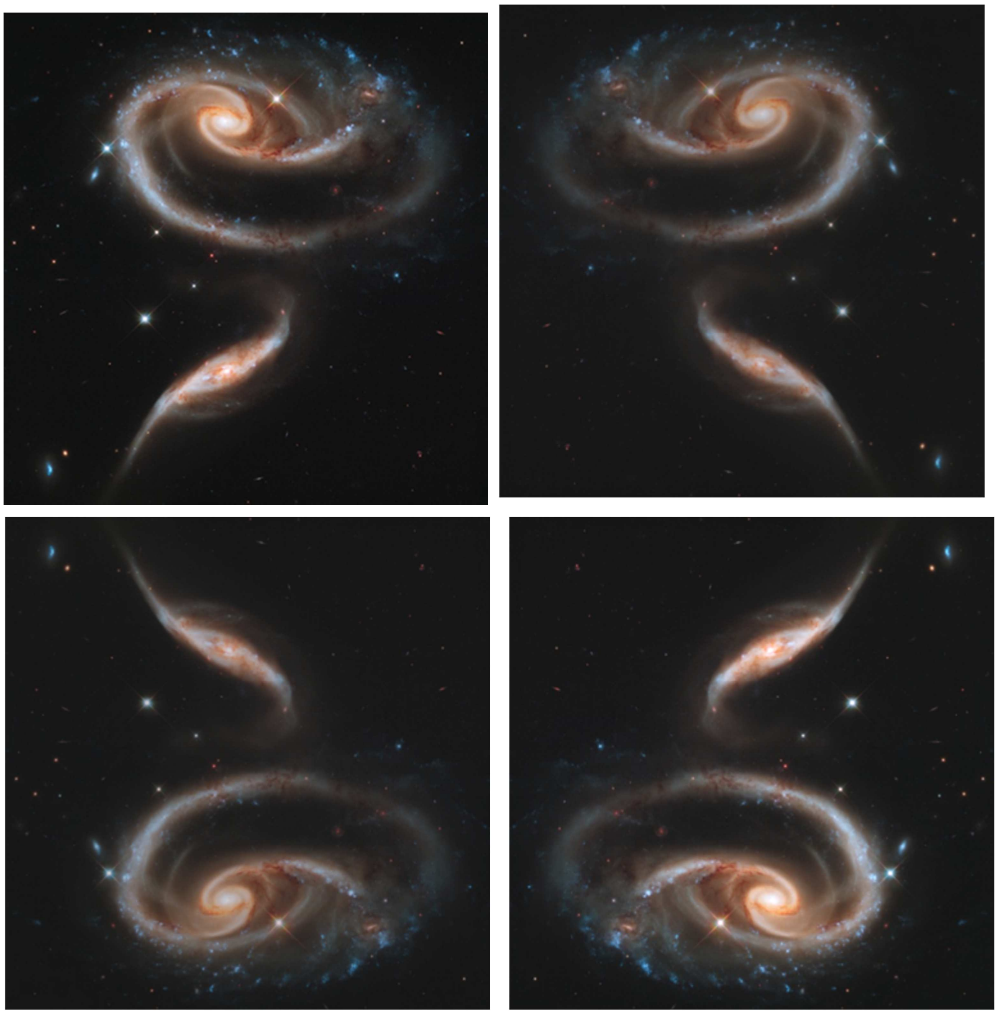
APPENDICES



List Of Appendices

Appendices List Of Figures

APPENDIX 1 PARTICLE PHYSICS PRIMER

1 Introduction

The objective of this appendix-report is to give a brief orientation on the topics of discussion, LEPTONS and PHOTONS, found in the other core reports of this body of work. What are these "objects" and how do they fit in with other scientific information of modern technological societies?

The research presented in the other reports of this overall work was pre-screened by persons with technical, scientific, and engineering backgrounds. While these people clearly had the necessary intellectual where-with-all and also had great knowledge concerning their own particular fields of specialty, most were only vaguely familiar with the terminology used in particle or subatomic physics. Occasionally after reading these reports and apparently easily following the mathematics and analyses there, several of them asked, "What is a lepton"? Oops, such a question makes reading these research reports a bit disconcerting, if someone doesn't know what the subject of the discussions is. Therefore this particle physics primer is presented so that technical persons who are not familiar with the current realm of subatomic physics can follow the presentations in these reports with more ease.

2 What Is A Lepton – The Short Version

Leptons and photons are elementary subatomic particles, waveforms. They are from the world and size realm of the physicists. This is many orders of magnitude smaller than the realm where the elements of chemistry are found. And that in tern is of course many orders of magnitude smaller that the consensus world realm which humans inhabit. The lepton series is comprised of the well known electron and its two bigger but highly unstable brothers, the muon and tau. The photon is of course a single or discrete electromagnetic wave. Persons in modern societies are familiar with visible photons seen when turning on a light switch or invisible ones when using a cell phone.

A general reminder is needed that the world size realm of George Johnstone Stoney and the particles, the electron family, is at a scale 36 orders of magnitude smaller in distance than humans and 44 orders of magnitude smaller than the human invented second. The electron is 33 orders of magnitude smaller in mass than a human and the quarks appear to inhabit a world of 4 spatial dimensions. Futher the little critters of investigation are really only just wave forms or energy bodies and do not really have any "solid" form. Assuming or trying to impose laws and physical property inter-relations upon them based upon the human world experience and mechanics is a seriously dubious proposition.

There are several good publicly available primers at web sites which describe the features found in this subatomic realm of the particle physicists. These can be found at the following two sites, plus many more.

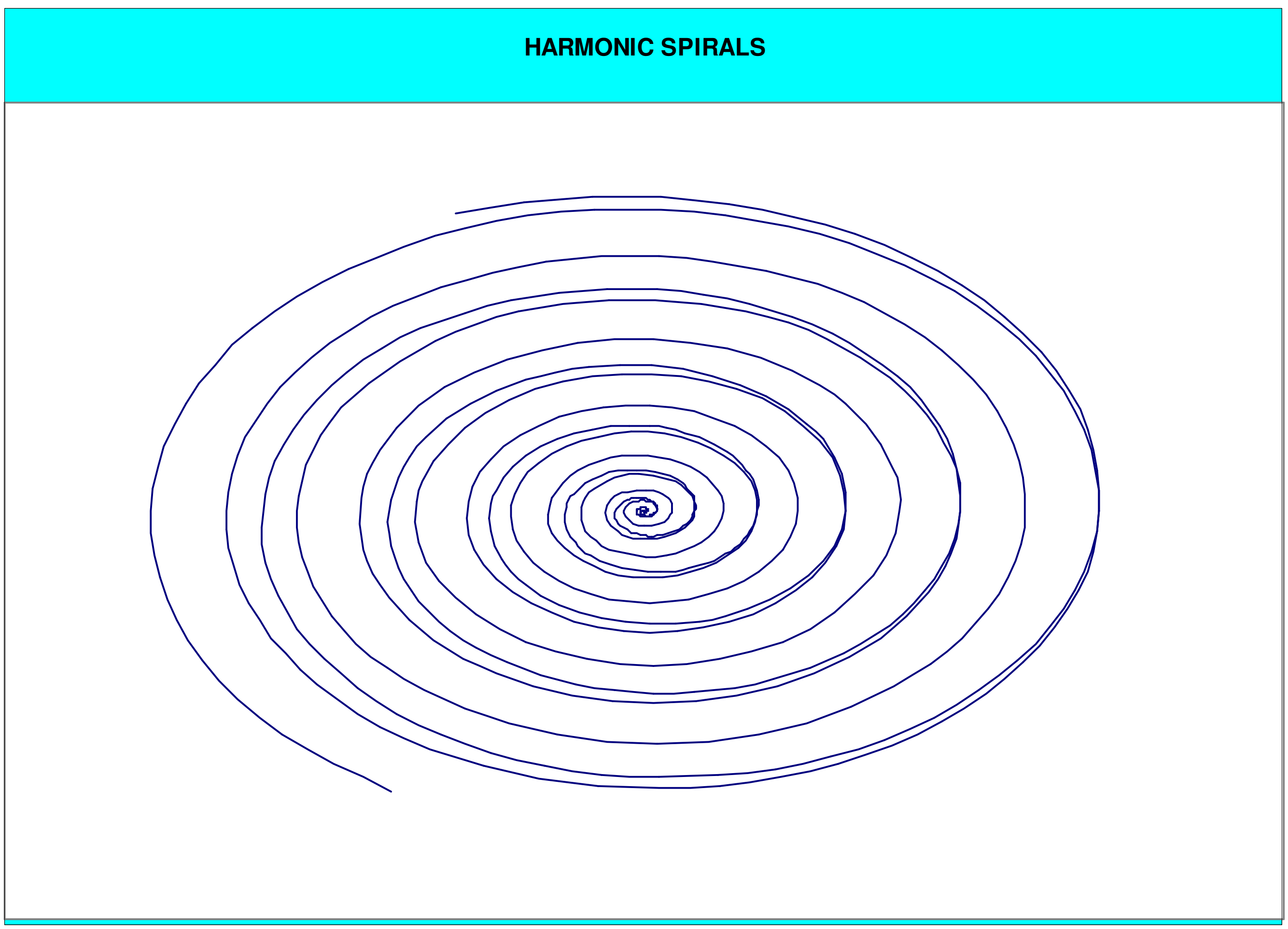
www.Wikipedia.org any and all of the search words; neutrinos, leptons, quarks, photons, elementary particle, etc at this site gives an essential part of the picture.
www.Particleadventure.org has a wonderful pictorial chart.

The explanations and organization of this subject matter given at these web sites is sufficient to orient the reader as to what is being discussed in the core reports of this body of work. Never-the-less, a different presentation is made here of the organization of this subject matter. For example in the web sites listed above, a person finds that particle physicists tend to lump the neutrinos together with the charged leptons, as if neither had any importance. Here the two groups or classes have been separated. The name lepton is only be ascribed to the electron, muon and tau.

As is found, the physicists tend to use their own vocabulary to segregate themselves from the other scientists, particularly from chemists and biologists. For example in the reports of this work, an individual type of elementary particle is simply called an elementary particle, a variety, or a species which is more in line with biological thinking. Whereas physicists call an individual type of elementary particle a flavor. Don't try to eat them. This will result in a stomach ache. Likewise a group of similar type particles, as might appear in a column of a periodic table, are referred to here as a class, a family, or a group as is done in chemistry. Physicists are more fascinated with what would be a row across a periodic table, calling it a generation. This follows from a misguided belief in physics which is held over from biology or social studies that the bigger, more massive in subatomic physics, always produces the next generation, the smaller or less massive.

Unlike the Periodic Table of the Elements of Chemistry (PTEC) which has over 100 elements, there are only a few basic or elementary particles in physics. According to how a person counts and categorizes these basic physics "objects" there are only from 24 to 34 of them. To orient the reader, a very small selected cut is taken from the PTEC and is shown in Table 1 below. The emphasis here is to refresh the reader about some of the key features of the grander PTEC, the hows and whys of its organization.

Below this in Table 2, an arrangement is seen of the elementary particles of physics. Table 2 is a proposed Periodic Table of the Elements of Physics (PTEP). The emphasis of Table 2 is of course to build on or bridge from the organization in Table 1. These two tables again are adequate to orient the reader as to the subject matter of this overall research work.



3 The Long Version

For those readers for whom college science classes might have been a long time ago, this brief refresher is offered. Those persons who do not need a long general overview of the history of chemistry and physics can skip directly to Section 5.

3.1 Chemistry – Molecules, Atoms, Nuclei, And Electrons

At a small enough scale, all material in the physical world consists of molecules. Molecules are the basic repeating building blocks of all large bodies of solid matter. For example; minerals within rocks, single strands of modern polymers, the smallest internal parts of cells, and the basic compounds within the atmosphere all can be identified as distinct molecules. Molecules in tern consist of distinct ordered combinations of the elements of chemistry. If proper precautions are taken, molecules can be isolated and their exact sequences of chemical elements can be identified. All molecules of the same type have the same physical properties and reaction behaviors. For reactions of the same set of molecules, conducted under the same conditions, the reaction products are always the same. For those reactions which produce a multitude of products, the ratios of the various products are always the same.

At yet a smaller scale the elements of chemistry are found. For example, the diameter of the Iron atom with all its electron shells in-tact is about 1.26×10^{-10} meters. The discussion concerning the molecules can just be repeated. These elements combine to make the molecules. The elements like the molecules can be isolated. Atoms of the same element all have the same physical properties and show the same reaction behaviors. For a long time during the development of the science of chemistry the individual atoms of the elements were viewed as the smallest distinguishable building blocks of the physical universe. From the scale of existence of humans this view is still true. The atoms of the elements of chemistry are the basic building blocks of any practical relevance.

Some of the chemical elements, such as gold, occur in nature in their elemental form and have been known by humans for thousands of years. Some can be isolated easily and some are readily accessible due to their abundant presence on the surface of the earth. Overall there are about 91 elements which occur naturally and are accessible to humans on or in the upper crust of the planet. Most of these were first isolated in the 1700 and 1800's with the development of the science of chemistry. There were a few holdouts due to either their scarcity or to their chemical similarity to other elements with which they are found. These required some of the more sophisticated techniques or modern hardware of the first part of the 20th century to be isolated and their existence verified. The remainder of the actual 100+ elements are radioactive and have long since decayed away in nature. The only existence of these radioactive elements now are those quantities produced by humans from nuclear reactions.

With the discovery of dozens of elements the obvious question arose. "How can this be", that the basic building blocks of the physical universe can come in dozens of different forms? Why do many of these supposedly most elementary of all forms come in varieties which essentially duplicate the properties of some of the others? Why do many of the elements have similar physical and/or chemical properties? Why do some combine with others one way but yet others apparently refuse to combine that way? Between 1868-1870 the Russian chemist Dmitri Mendeleyev proposed a solution to these mysteries.

Again size regression occurs. The atoms of the elements of chemistry obviously are not the most elementary or basic building blocks of the physical universe. After some high level disputes amongst the early chemists, the obvious reality was accepted that the atoms are divisible. The atoms have a core, nucleus, which retains or carries the modern identity of the atom. The nucleus of the atom is responsible for almost the entire mass of the atom and some of the other physical properties related to this "weight". There is a "surface" feature, the electrons, which can leave the atom and go off to combine with other atoms. The surface electrons which come and go determine most of the physical and chemical properties of the atom. Mendeleyev proposed a periodic table in which the various elements with similar properties were just repeating heavier members of the same families. Mathematicians and the early physicists helped show why these properties repeated.

Slowly a "full" picture developed. The nucleus of atoms consisted of positively charged particles, protons, and neutral particles, neutrons. The protons needed particles to balance the overall charge of the atoms. These were the flocks of electrons which "flew" around the outside. The PTEC was arranged based on the count of the number of the protons in the nucleus of the atoms. Although the presence of the neutrons helps in determining the mass of the atom, the distinctness of an atom in terms of physical and reaction properties is determined by the interplay of the charged particles, the protons and the electrons. The elements in the PTEC were named or distinguished according their number of protons. For example, calcium element number 20 has 20 protons and 20 electrons. The success of the PTEC was the 90+ elements could be reduced to combinations of 3 even more elementary subatomic particles, electrons, protons, and neutrons.

What the mathematicians helped show was how and why certain numbers of surface electrons behaved as they did. They developed the mathematics of the electrons as wave patterns or shells around the exterior of the atoms. How these energy patterns repeat as they do can directly be linked to or explained as discrete solutions to mathematical equations. These equations have been formulated to precisely model the nature of the different atoms with their varying number of protons and electrons. These mathematical models have been overwhelmingly demonstrated to be the correct models. Since these models are directly linked to mathematical expressions which have series of discrete solutions, the name quantum mechanics arose. This is in opposition to Newtonian mechanics which can explain physical phenomena of a continuous nature.

This knowledge is all that modern chemists and chemical engineers need. On a practical level, the repetition of certain chemical properties is the main concern as progress in made from row to row in the PTEC. To the chemist and chemical engineer, the understanding and manipulation of the elements to produce certain desired properties is what is important. This knowledge creates all the wonderful stuff which is familiar to everyone in modern societies.

In Table 1, a selected cut from the Periodic Table of The Elements of Chemistry (PTEC) is shown. Yes admittedly to chemists or even to chemistry students in high school the presentation in this table looks "goofy". The usual presentation of the PTEC has been drastically rearranged to emphasize analogies found for the leptons. Also shown are examples of what chemists clearly know to be compounds or composites of the basic elements. This is to again compare / contrast with how the physicists currently think about the basic building blocks with which they deal.

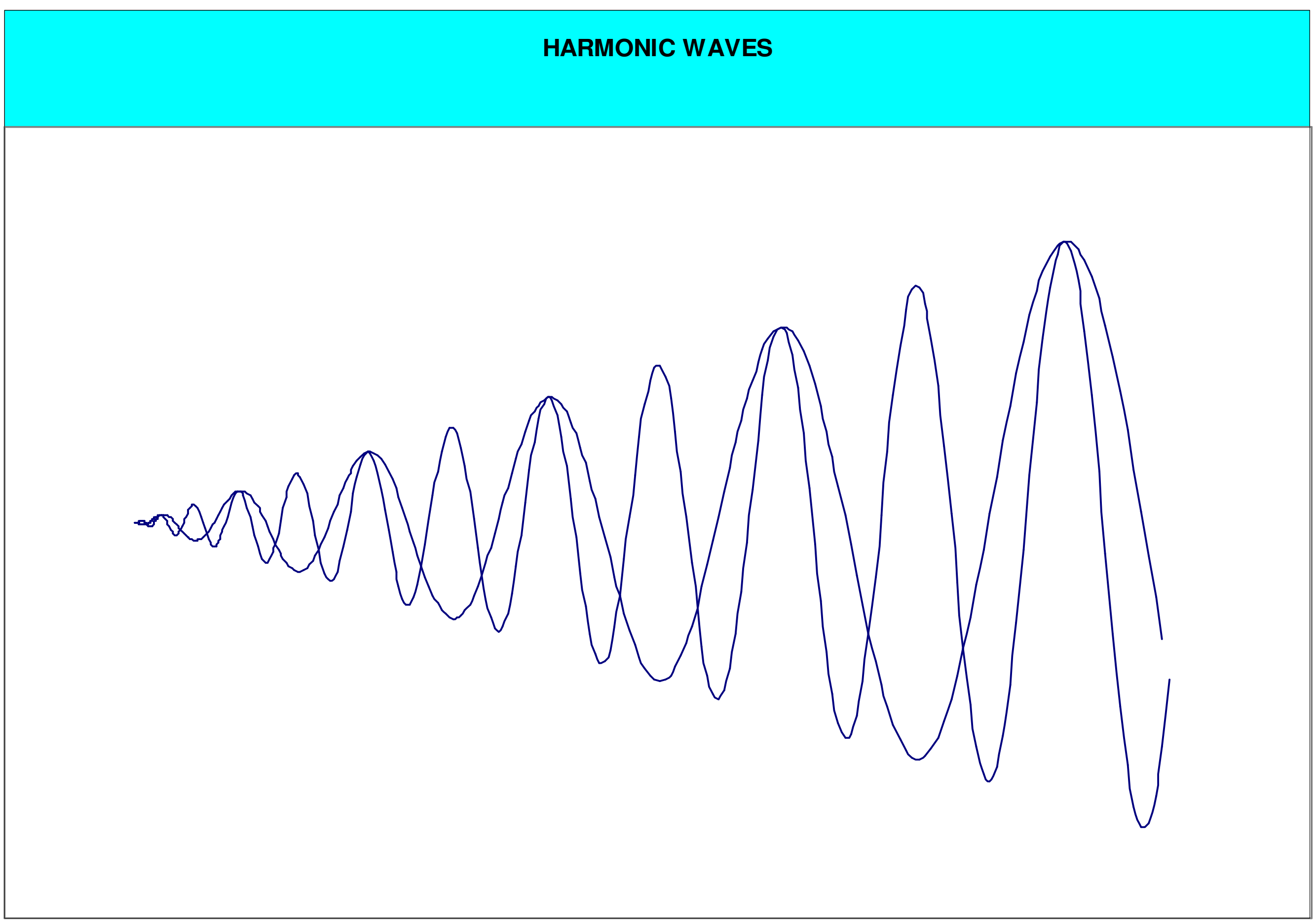
One of the main purposes of the presentation in Table 1 is to show the increasing mass of the elements with each row as progress is made upward in this particular re-arranged table. The numerical values listed with each element are their atomic weights. As just discussed, this increasing mass is almost totally determined by the nucleus of the element, and has little to no dependence on their exterior electron shells.

But the electron shells which give the elements their chemical properties are what are of importance to chemists and chemical engineers. The compressed notation for these electron shells and their mathematical formulations usually appear in most PTEC's. Here on the left of this abbreviated version, Table 1, only the key or highest S shell is shown. All the underlying shells are identical and do not need to be repeated. The other upper shells such as P and D shells, which have 3d angular dependencies, are intentionally not shown here. The mathematics of the S electron shells are such that these shells are described radially outwards by their corresponding Laguerre $L_n(r)$ orthogonal polynomial. That is the 3S electron shell is mathematically described by the L_3 orthogonal polynomial.

Table 1 Periodic Table Of The Elements Of Chemistry (PTEC) -- Abridged

Selected Elements – Rearranged Order							
Group	O	I	VII	II	VI	III	V
Key Valence	**0**	**+1**	**-1**	**+2**	**-2**	**+3**	**-3**
Key Shell 4s	Krypton 83.80	Potassium 39.102	Bromine 79.909	Calcium 40.08	Selenium 78.96	Gallium 69.72	Arsenic 74.992
Key Shell 3s	Argon 39.948	Sodium 22.990	Chlorine 35.453	Magnesium 24.312	Sulfur 32.064	Aluminum 26.9815	Phosphorus 30.9738
Key Shell 2s	Neon 20.183	Lithium 6.939	Fluorine 18.9984	Beryllium 9.0122	Oxygen 15.9994	Boron 10.811	Nitrogen 14.0067

Examples Of Composites (Molecules)	
Binary Compounds – Homogenous	H_2, O_2, N_2
Binary Compounds – Heterogeneous	HF, NaCl, KI, CO
Ternary Compounds – Stable	H_2O, CO_2, N_2O, HCN
Ternary Compounds – Unstable, nonexistent	HO_2, C_2O, NCN
Complex Forms – Stable in isolation	NH_3, CH_4, V_2O_5, CH_3-CH_2OH
Complex Forms – Unbalanced cannot be isolated	NO_3-, NH_4+, CO_3-
Temporary, Reaction intermediary, High energy	CH_3O-, $CH_3C(OH)_2$+, $C(CH_3)_3$+



The importance of this whole layout is to show how the mathematics of this PTEC, specifically that of the S electron shells and their descriptor $L_n(r)$ orthogonal polynomials, increase "coincidentally" with the increasing mass of the elements. This same relationship was found for the column of the three leptons, as is discussed later in Section 5.

3.2 Nuclear Chemistry – Isotopes, Their Reactions, And Radioactivity

The number of neutrons in the nucleus of atoms can vary since these particles "don't do anything" except keep the protons from getting in each others way. Lower down in the PTEC the number of neutrons never varies beyond some number slightly greater than or slightly less than that of the number of protons. In general across the whole PTEC the ratio of the neutrons to the number of protons in an element's nucleus is approximately 1.4 neutrons to 1 proton.

A quick definition is needed. An isotope refers to the total number of protons plus neutron found in the nuclear core of an element. For example, element number 92, Uranium has 92 protons. But it can also have several isotopes due to different number of neutrons present with the 92 protons. So the symbolic presentations as follows are found; ^{235}U (92 protons plus 143 neutrons), ^{238}U (92 protons plus 146 neutrons), and other such elemental forms. Probably the three other best known nuclear isotopes are those of Hydrogen, H. Hydrogen, ^{1}H just has a single proton for its core. Deuterium, ^{2}H has a single proton and a neutron. Deuterium is naturally occurring and stable. Finally Tritium, ^{3}H has a single proton and 2 neutrons. Tritium is unstable.

Historically the overall puzzle was still not complete. With the advent of 20th century technology, people not only isolated the distinct chemical elements, they began isolating the several isotopes of each element. Now the obvious question became why 91 or so naturally occurring elements? Why not less or more? Why doesn't the PTEC go on forever? Why, what is this radioactive thing?

The first answer is philosophical. The number of elements terminates because everything in the physical universe terminates, has boundaries, or limits. Nothing physical goes on forever. Only conceptual mathematical sequences can be viewed as infinite. How in this specific case does the number of distinct elements terminate? The short simple layman's answer is; because the nucleus of the atoms gets so big that "it falls apart under its own good looks".

The long answer to these questions is the same. The nucleus of the atoms, all above the first one, hydrogen, contain an increasing number of protons "packed" in a very tight space. For example, the diameter of the nucleus of Iron is about 9.0×10^{-15} meters. Since all the protons are alike and have a positive charge, they "don't like each other very much" no more than do bull elks in the same meadow in mating season. Again repetition is found. The nuclear cores of the atoms have an organization, structure, or layers of their own, similar to the electron shells around the outside. The neutrons mixed in the nuclear core of the atoms keeps the protons somewhat insulated from each other. The whole nuclear core doesn't instantly come unglued as the protons try to get away from each other. Ultimately though with a large enough nuclear core nothing can save it. Starting with the elements 90, Thorium Th, and 92, Uranium U, the nucleus of these elements start self destructing. They effectively start downsizing just like top heavy large corporations, and the radioactive thing starts occurring.

Historically scientists didn't stop with just isolating the different isotopes of the elements. They began investigating the reactions of the different isotopes. That is, reactions that happen with the nuclear core of the elements. This is, as well as with the chemical reaction behaviors of the surface electrons. Scientists began exposing the various elements of the PTEC to the radiation, such as alpha particles that had been emitted during the natural decay of the two native radioactive isotopes of Th and U. Incidentally an alpha particle is the same as the nucleus of the Helium, He, atom. Additionally, the production of protons is reletively easy by stripping off the only surface electron from the hydrogen atom. Protons and electrons due to their electrical charge both can be accelerated and focused with electromagnets, and bombarded into selected target materials. Eventually researchers learned to make just about anything radioactive, and the origins of the words "artificial" radioactivity came into being. Each isotope of each element behaves differently after ingesting excess particles into their nuclear core. Some re-stabilized themselves by emitting electrons or positrons discussed shortly. Some spit out neutrons or just regurgitate alpha particles. Researchers during the 1900s learned to make designer elements, isotopes with nuclear cores intended for some specific purpose.

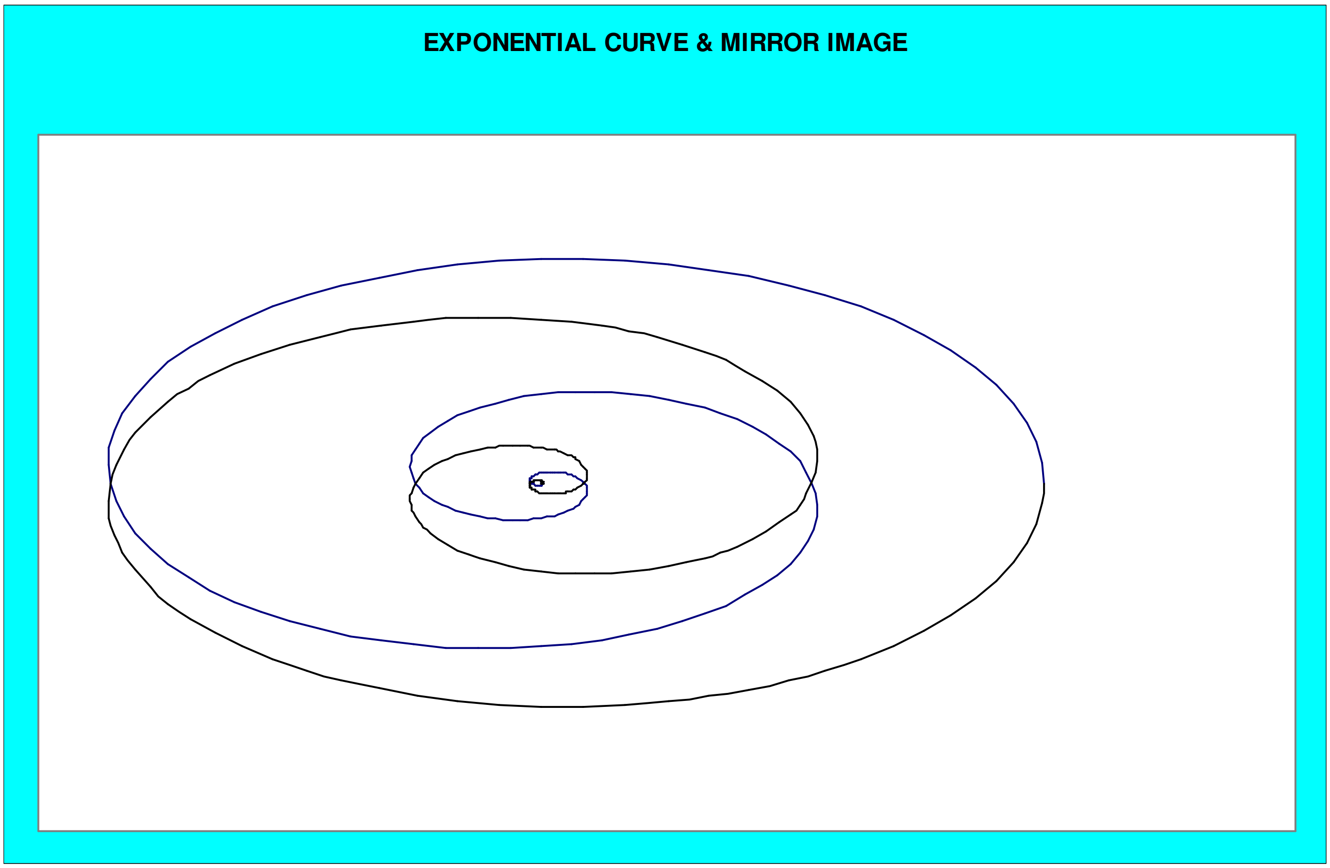
Chemists and physicists rapidly identified several modes of radioactivity. For the lower weight elements in the PTEC, wherever there are nuclear instabilities, the nucleus of the atoms try to stabilize themselves by ejecting particles. Neutrons can disintegrate or decay into protons which have slightly less mass. This reaction process ejects beta particles, fast electrons, which keep the net reaction balanced electronically. Protons can capture incoming fast electrons and turn into neutrons. Protons which pick up enough energy can turn into neutrons and spit out positron particles, fast anti-electrons, which keep the overall charge of the reaction balanced. There are several such simple reaction schemes of a single particle splitting into two major pieces plus some left over energy. Or equally, reactions can occur when two particles combine with enough excess energy to become one. In all such reactions which occurred at this subatomic or nuclear scale the net result of the whole reaction is such that the sum of the charges of the reactants plus the products always remained balanced. That is, after the reaction there was no net change in the charge of the system, or the universe, compared with before the reaction. With these as the only observed reactions, scientists automatically concluded or made up a law of conservation of charge.

At the high weight end of the PTEC, the heavy elements show a different behavior. The nuclear cores of these elements, just like top heavy modern corporations, start downsizing. These elements spit out pairs of protons and neutrons, alpha particles or helium nuclei. The two naturally occurring radioactive elements ^{232}Th and ^{235}U do exactly that. There is nothing magical, inherently sinister, or cursed about uranium. Uranium is just another metal like all the rest which have practical uses in modern societies, but one of its natural isotopes just happens to have an unstable nucleus.

Early on there was another objective to nuclear research. That was to make a self sustaining nuclear reaction. Elements which spit out excess neutrons became the working tools for this endeavor. Scientists first used the two natural radioactive elements to make other elements unstable or radioactive. This is not too difficult with the elements at the bottom of the PTEC which already have very bulky nuclei. These man made radioactive isotopes of various elements were "designed" to be neutron emitters. These neutron emitters were in turn used to overdose the ultimate objective element with neutrons until it went unstable. These neutron processes, rather than being done in accelerators as discussed above, were done in small research nuclear reactors. Researchers found that if some of the almost unstable isotopes of various elements were bombarded with enough neutrons, then dramatic things happened quickly. For example, ^{235}U went completely unstable and totally disintegrated or fissioned. Since such reactions released vast amounts of energy which had held together the organization of the nuclear cores, BOOM. Nuclear bombs or nuclear power production became achievable. Accordingly the World War II efforts of American scientists to produce a nuclear bomb via fission were successful.

Scientists also quickly reasoned how the sun produces the heat that it does. If protons and neutrons are packed together tightly enough, with cosmic scale pressures, at cosmic scale temperatures, and are held there for enough time, then 2 protons and 2 neutrons combine, fuse, to become a stable 4 body nuclear core. Effectively hydrogen is turned into helium. Since scientists could not create a controlled sun on the earth, they "cheated" in the process. They used lithium hydride LiH which is a stable and solid compound, rather than just straight gaseous hydrogen. Lithium deuteride, $^{6}Li^{2}H$ and $^{7}Li^{2}H$, both easily revert to He if they are bombarded with neutrons from another decaying source. Just as with the efforts to create self sustaining fission reactions, scientists again used materials which had been ever so briefly overdosed with neutrons to make fusion reactions. This process of upgrading nuclear cores into larger ones, fusion, as done in suns also results in large heat releases. Again BOOM, humans created hydrogen bombs. As was found, though, to produce the energy and conditions necessary to get fusion reactions to light off, the output of fission reactions was required. Accordingly the World War II efforts of German scientists to produce a nuclear bomb via fusion were not successful. People, scientists and engineers, still have not yet been able to build sustainable, meaning controllable, fusion devices for power production or any other purpose.

There are many web sites which discuss the isotopes, radioisotopes, decay modes, half lives, et cetera. Several suggested starters are: www.webelements.com
http://ie.lbl.gov/education/isotopes.htm
www.iaea.org/inisnkm/nekr/indr/subjects/index.html



4 Particle Physics – Hadrons, Baryons, Mesons, Quarks, Leptons, Neutrinos

This is about where chemistry and physics part company. Scientists found that neutrons are always "slightly" unstable and ultimately always decay. They do not have to be in nuclear cores to fall apart or self destruct. Again the question, how can one of the ultimate building blocks of the universe fall apart? Many more questions also naturally arise. How is it that protons and neutrons have almost identical masses, only different by about 1 part in 1000? Why not totally identical or widely different? Why are protons charged and neutrons neutral if they are almost identical siblings by mass? Why do the protons and electrons, which have exactly the opposite charges, have such radically difference masses? The proton is almost 1836 times more massive than the electron? Why do neutrons decay into protons and not totally fission and fall apart into a myriad of smaller subcomponents?

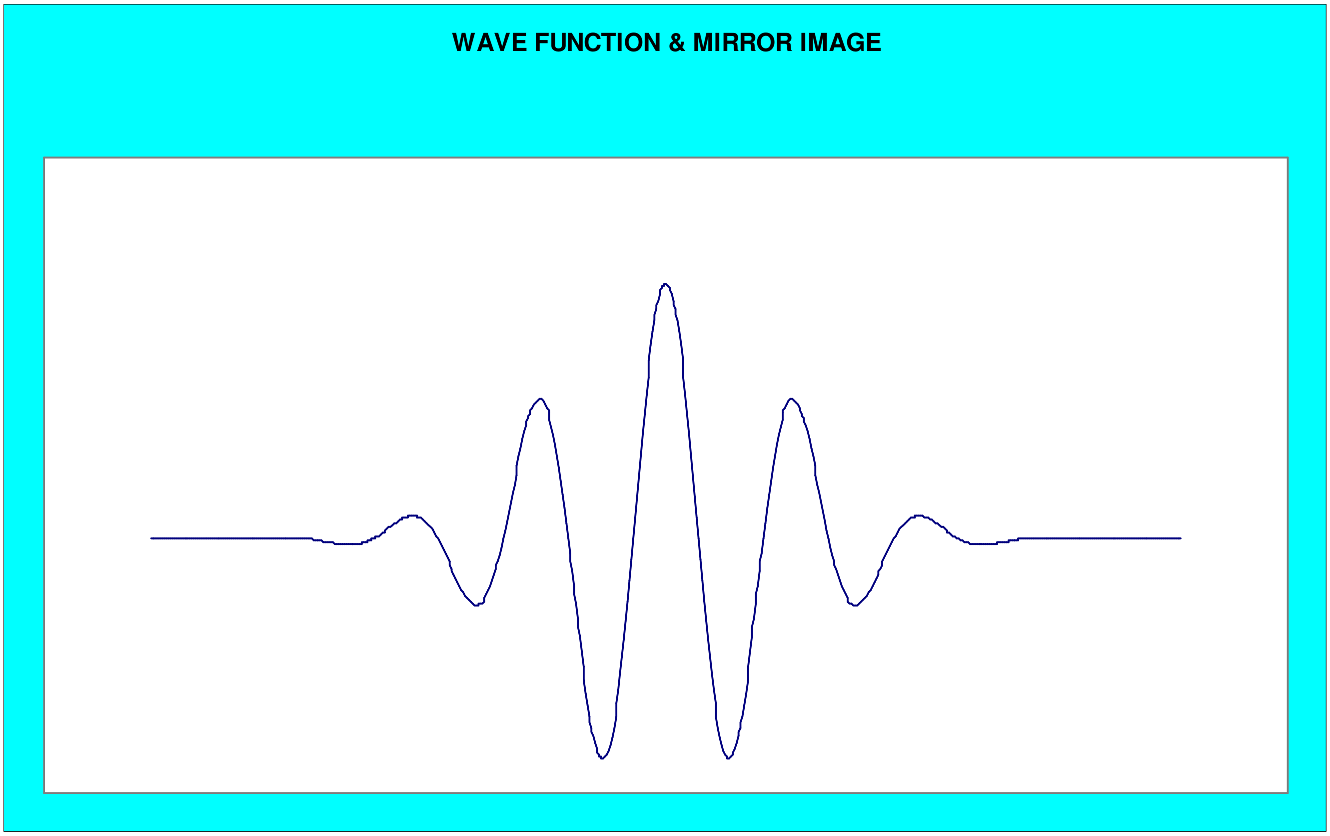
Physicists during the early part of the 1900s learned that the protons and neutrons obviously could not be ultimate building blocks of the physical universe, or at least not the only ones. With the development of particle accelerators and colliders during the mid to late 1900s, particles of smaller masses, electrons and positrons, anti-electrons, were slammed together. Some of these collisions produced slightly stable or even highly unstable but clearly distinct particles of large masses. By the late 1900's scientists had again produced a whole zoo of odd short lived particles, dozens, even hundreds. Again as with chemistry only certain collisions or reactions appeared to result in product objects, larger particles. The vast majority of the collision reactions just resulted in the destruction of the colliding particles and the production of energy. In some cases this energy was carried away in the form of distinct smaller particle-objects, neutrinos.

Again many questions arose. First of course was, how can there be this many "elementary" subatomic particles. There developed a great fascination of how different species of these basic particles could transmute into each other. Reaction rules and various conservation laws were again proposed. Also there appeared to be whole classes of particles which responded to a new or different force not seen at the size realm of chemistry. The color force was added as a basic force of nature, to the two already known gravitational and electromagnetic forces. Particles, subatomic entities, responding to or possessing this force were called hadrons. These hadrons include the proton and neutron. The leptons and neutrinos do not respond to or possess this color force.

Yet again there is size regression. The confusion of a multitude of hadrons was reduced by proposing yet even smaller basic building blocks. In 1964 Murry Gell-Mann and independently George Zweig proposed the existence of the quarks to bring some order and simplification to the world of sub-sub-atomic particles. This proposal did an excellent job of bringing order to the hundreds of hadrons. These quark particles come in two basic varieties, the up (u) and down (d). Each has several more massive family members; charm (c), strange (s), top (t), and bottom (b). The quarks are in many ways analogous to or could be lined up in two columns similar to the elements of the Periodic Table of the Elements of Chemistry (PTEC). These quarks combine in twos and threes, apparently according to certain rules. The dozens of mesons (binaries) and hundreds of baryons (ternaries) are now understood to be composites, compounds, or physics molecules of the more basic particles, the quarks. These compounds now are understandable as the physicists' analogies to the simple molecules of chemistry, such as N_2 and CO_2. Specifically the proton is composed of two u's and one d, (u_2d) just like H_2O. The neutron is composed of one u and two d's, (ud_2). The neutron is also unstable just like the counterpart HO_2, except that HO_2 is so unstable that it effectively doesn't exist at all. The upper members of the quark families and all the composite mesons and baryons, with the exception of the proton and neutron, decay very quickly. What can be called a stable particle and an unstable form or "object" is somewhat nebulous.

This success of reducing many elementary objects to a few was short lived and only partially complete. Particles of small masses were being smashed together to produce temporary high energy reaction intermediaries, such as the "weak force" particles and many others. Because of the short lives of all the reaction products, the distinction between an elementary particle, a composite or physics molecule, and a temporary high energy reaction intermediary had seriously blurred. Further some of these reaction intermediaries were decaying in manners which violated the rules. Several of the conservation laws that humans had made up were being violated.

The success of the idea of a sub-sub-atomic quark had explained the production of hadrons but had not explained itself. Now there are still too many basic or elementary particle-objects. There are at least 6 quarks (not counting anti-quarks), the 3 leptons (not counting anti-leptons), and the 3 neutrinos and their anti's. This was again getting to be irritating. The physicists basically had a small Periodic Table of the Elements of Physics (PTEP); 1 column of neutrinos, 1 column of leptons, and 2 columns of quarks. But again they had no explanation for how come there were so many of these, the most "elementary" of all building blocks of the physical universe. They had and still have no explanation for why these building blocks had the properties that they did, particularly their highly disparate masses. And that is about where progress in subatomic particle physics stopped, 30 years ago. Physicists did not have the ever more powerful machines required to do further experimentation, particle smashing. Equally they did not have any satisfactory framework, (correlative, hypothetical, or any other logical modality), to explain the information which they had already obtained.



5 The Periodic Table Of The Elements Of Physics (PTEP)

Table 2 following gives a listing of the elementary particles of physics. This table is organized to appear like the previous table which showed the elements of chemistry. The chemistry PTEC has Group O elements which have a zero "valence", or a preferred charge state of 0. The physics PTEP has those particles which have 0 charge, the neutrinos. The neutrinos only respond to gravity and presumably only encapsulate gravitational energy or only stabilize the gravitational force. The PTEC has two columns of elements, Groups I and VII, whose preferred valences are +1 and -1 respectively. The PTEP has a single column of particles, the leptons, which can be found in forms with +1charge or anti-forms with -1 charge. The leptons respond to two forces, gravitational and electromagnetic. Presumably these particles stabilize these two forces as encapsulated energies or wave patterns. The lepton report discusses exactly this, the mathematical nature of the distinct gravitational and electromagnetic waves found for the leptons. Finally the PTEC has two columns, Groups II and VI whose preferred valences are +2 and -2, and two columns, Groups III and V, whose preferred valences are +3 and -3. These have less precise analogies in the PTEP. There one column is found whose particles have forms with either +2/3 or -2/3 charge, and one column is found whose particles have forms with either +1/3 or -1/3 charge. Both columns of these fractionally charged particles, the quarks, respond to three forces; gravity, electromagnetism, and color.

Of course Table 2 is more complicated because more information is being conveyed there. But amongst all the rest of the facts, the primary purpose is similar to that of the previous table. That is to show how the leptons have increasing mass directly related to an increasing Laguerre orthogonal polynomial number. That is one of the ultimate core finding of the research presented in the lepton report of this overall work. The numerical values listed with each elementary particle are their masses (kg) or mass-energies (MeV/c). Not all the information shown in Table 2 is fixed or "set in concrete". For example, the neutrino masses listed are according to Wikipedia. According to Particle Adventure these masses are as follows; mass $\upsilon_L < 2.3 \times 10^{-37}$ kg, $1.6 \times 10^{-38} <$ mass $\upsilon_M < 2.3 \times 10^{-37}$ kg, $7.1 \times 10^{-38} <$ mass $\upsilon_H < 2.5 \times 10^{-37}$ kg.

Table 2 Periodic Table Of The Elements Of Physics (PTEP)

<table>
<tr><th colspan="5">Elementary Particles - Fermions</th></tr>
<tr><td>Particle Group</td><td>Neutrinos</td><td>Leptons</td><td colspan="2">Quarks</td></tr>
<tr><td>Charge</td><td>0</td><td>± 1</td><td>± 2/3</td><td>± 1/3</td></tr>
<tr><td>Forces "Held"</td><td>Gravity</td><td>G + E/M</td><td colspan="2">G + E/M + Color</td></tr>
<tr><td>Spatial Dim.[1]</td><td>probably 1 radial</td><td>1 radial + 1 angle</td><td colspan="2">4 Dim composed of 2 linked circles
2 radii and 2 anglular descriptions</td></tr>
<tr><td>Lives In[2]</td><td>2nd Spatial Dim?</td><td>3rd Spatial Dim</td><td colspan="2">4th Spatial Dimension</td></tr>
<tr><td>Laguerre Poly.
$L_6(r(t))$</td><td></td><td>shipa σ^- ?</td><td></td><td></td></tr>
<tr><td>Laguerre Poly.
$L_4(r(t))$</td><td>υ_τ or υ_H
m < 2.76×10^{-29} kg</td><td>tau τ^-
$3.167,88 \times 10^{-27}$ kg</td><td>top (t)
or truth
170,900 ± 1,800
MeV/c^2</td><td>bottom (b)
or beauty
4,100-4,400 MeV/c^2</td></tr>
<tr><td>Laguerre Poly.
$L_2(r(t))$</td><td>υ_μ or υ_M
mass < 3.0×10^{-31} kg</td><td>muon μ^-
$1.883,532,7 \times 10^{-28}$ kg</td><td>charm (c)
1,150-1,350
MeV/c^2</td><td>strange (s)
80-130
MeV/c^2</td></tr>
<tr><td>Laguerre Poly.
$L_0(r(t))$</td><td>υ_e or υ_L
mass < 3.9×10^{-36} kg</td><td>electron e^-
$9.109,389,7 \times 10^{-31}$ kg</td><td>up (u)
1.5-4 MeV/c^2</td><td>down (d)
4-8 MeV/c^2</td></tr>
<tr><td colspan="5">Fermions are "stationary" particles, originators & receivers of forces, Spin = 1/2
Have closed form wave patterns, are mathematically bounded in all spatial dimensions</td></tr>
<tr><td colspan="5"></td></tr>
<tr><th colspan="5">Examples Of Composites</th></tr>
<tr><td colspan="5">Compounds (Hadrons) of Colored Elementaries (Quarks)</td></tr>
<tr><td colspan="5">Binary Compounds (Mesons) – Homogenous; $\pi^0 = (u\bar{u} + d\bar{d})$ or $s\bar{s}$</td></tr>
<tr><td colspan="5">Binary Compounds – Heterogeneous; $\pi^\pm = d\bar{u}$ or $\bar{d}u$, $K^0 = d\bar{s}$ or $\bar{d}s$, $K^\pm = u\bar{s}$ or $\bar{u}s$</td></tr>
<tr><td colspan="5">Ternary Compounds (Baryons) – Stable; proton = u_2d, analogous to H_2O</td></tr>
<tr><td colspan="5">Ternary Compounds – Metastable, Unstable; neutron = ud_2, analogous to HO_2</td></tr>
<tr><td colspan="5"></td></tr>
<tr><td colspan="5">Notes: 1 Number of spatial dimensions of basic gravitational (mass) structure.</td></tr>
<tr><td colspan="5">2 Movement of the basic structural body as a "unit" which creates charge, color, etc.</td></tr>
</table>

Table 3 Periodic Table Of The Elements Of Physics (PTEP)

Elementary Particles – Bosons			
Particle Group	Gravitons?	Photons (1)	Gluons (8)
Force "Carried"	Gravity	Electromagnetism	Color
Comments	Do not have mass spin = 2?	Do not have mass Do not display charge	Do not have mass Have or display color
Bosons are "moving " particles, "carriers" of forces , Spin = 1 Have open form wave patterns, are unbounded in at least 1 spatial dimension			
Examples Of Composites			
Complex Form, temporary high energy reaction intermediary; "weak force" carriers W^+, W^-, Z^0			

Table 4 The Fundamental Forces

Force	Gravity	Electromagnetism	Color
Force Nature	Unary; G	Binary; E & M	Ternary; Blue, Green, Red
Encapsulated or Stabilized Form	mass Kilograms	charge Coulombs	color Whites (neutral, clear)
Spatial Strength	Inverse Square Decay w Distance	Both Have Inverse Square Decay	Non-Inverse Square Decay with 3D Distance
Temporal Strength	Unknown Decay Modality with Time	Unknown Decay Modality with Time	Unknown Decay Modality with Time
Nature of Spatial Dimensions of Waves	1 Dim – radial	2 Dim - planar for pair. Electrical – radial Magnetism - angular	4 Dim, 2 radial and 2 angular
Waves Lives In n-Dim	n = 1?	n = 3	n = 4

Additionally there are many other complications of the physics particle picture which were not shown. One of the key additions is the property or concept of particles and antiparticles. All the particles responding to the electromagnetic force can come in two varieties; the particle -1, +2/3, or -1/3, or the antiparticle +1, -2/3, or +1/3. The antiparticles are designated by over bars. The best known antiparticle is probably the "anti-electron" or positron with a +1 charge, exactly opposite of the electron with a -1 charge. The neutrinos also have their anti version but this only involves a matter of their spin. The individual colored particles, the quarks, can come in three colors; blue, green, or red, but these colors are never seen in public. They always hide to make white, clear, or neutral color when compounded together to make the mesons and baryons. And just as charge can come in two varieties a plus or a minus, the colors can also have their anti varieties; antiblue, antigreen, and antired.

Lastly seen in Table 3 there are particles which have no analogies in the world of chemistry. These particles, the Bosons, are the "carriers" of the forces. These could be better described as moving but never-the-less stable waveforms or patterns. The Fermions, the neutrinos, leptons, and quarks just discussed are of course not stationary, but the energy waveforms that these particles are, appear to be bounded in all spatial dimensions. They, or at least the leptons, only move in space with time. Whereas the "carrier" Bosons appear to be wave patterns that are inherently unbounded in at least one spatial dimension. That is, they can be described by mathematical forms which are open ended, rather than closed forms like the Fermions. Within this general category the Bosons, is where the photons are found. These are the compliment to the leptons of the greater Fermion category in that both leptons and photons respond to or are involved with both gravity and electromagnetism. The photon report of this overall work shows that there are probably strong reasons for this correspondence. The mathematical descriptions discussed in this report show that both classes of particles, the leptons and photons, have many underlying mathematical commonalities.

Finally Table 4 gives a very simple listing of the basic forces themselves. This is added since physicists work with and discuss the "free" forces when they are not stabilized as some particle or wave form of energy. The listings of the nature of the spatial dimensions involved with the particles in Tables 2 & 3 and with the forces in Table 4 are of course only speculative viewpoints.

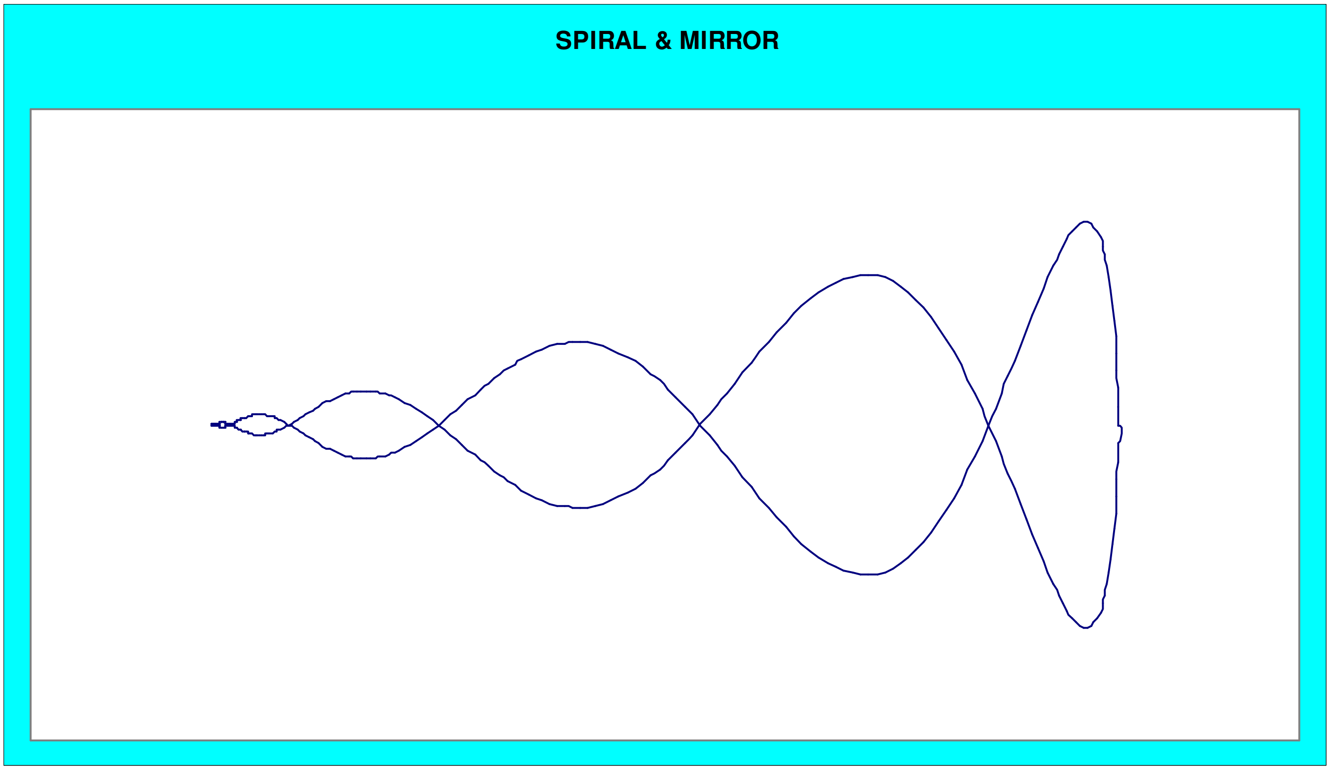
There are several more, at least, confusing items or "particles" in the world of physics. These are the "carriers" of "the weak force". These weak force particles are some sort of reaction product found in certain specific high energy collisions of other lighter particles. Like all the high energy collision or reaction products, the "weak force" bosons are extremely short lived, even by physics standards. These forms, are designated by physics as elementary particles or in this work as temporary composite "molecular" waveforms. These bosons can be viewed simply as high energy reaction intermediaries or "radicals". The words are almost a matter of semantics. The opinion expressed here is that these wave form objects are not "elementary" particles and the weak force is not a new essential or elementary force in nature.

If found at the world distance and reaction time scales of chemistry, a "weak force" particle would be viewed as some not very useful nor important temporary reaction intermediary. Analogies from chemistry are the unstable radical CH_3O- or some short lived combustion intermediary found in the burning of sugar to produce carbon dioxide and water by the ultimate reaction.

$$C_{12}H_{22}O_{11} + 12\ O_2 \rightarrow 12\ CO_2 + 11\ H_2O$$

Chemists know that the radical CH_3O-, sugar $C_{12}H_{22}O_{11}$, carbon dioxide CO_2, water H_2O, et cetera are not the source of carbon C, hydrogen H, and oxygen O. Rather C, H, and O combine in stable configurations to produce these larger molecular entities. Again the physics counterpart is rife with semantics. Are the the neutrinos, leptons, and quarks temporarily combining when smashed together under just the right conditions to produce, become the source of, the weak force particles? Or are the decaying weak particles the ultimate source of all neutrinos, leptons, and quarks? Philosophically this is a which came first argument, the chicken or the egg. In any case this discussion has no relevance to the subject matter of the reports in this work but is only presented here because the reader can find such particles in the public literature and the two web sites listed Section 2.

Likewise the recent confirmation of the existence of the "god" particle, the Higgs boson does not guarantee or even imply that this energetic form is elementary. The Higgs boson could be compared to an extremely short lived DNA molecule, again interesting and important but clearly not elementary.



6 Summary - Conclusions

Returning to the objective or purpose of this primer, this was to set a frame or context for the "objects" of discussion, leptons and photons, found in the core reports of this overall work. For this reason how these sub-sub-atomic particles fit in the broader picture of the basic building blocks of the physical world was presented. The leptons are one family amongst the several of the most elementary particles now known to humans. The report on the leptons clearly shows how these particles can be described as being some manner of standing energy waveforms. The photons have already long been known to be moving waveforms of energy.

All the discussions in this primer of the atoms of chemistry, isotopes, and subatomic entities were only given as a means for the reader to orient themselves. An in-depth knowledge or thorough understanding of this broad sweep of material is NOT necessary to read, follow, and understand the discussions in the lepton and photon reports of this overall work.

A knowledge of second semester calculus is necessary though to follow the mathematics in lepton and photon reports. But this required knowledge is very low level and limited, such as knowing what an integral symbol looks like and what it means. When these more formal reports about the leptons and photons are reached, they are found to be simple to follow, even though a bit stuffy and rigid in their form. And yet, the discoveries presented in lepton and photon reports are precise to many decimals. The material in the lepton and photon reports is an exciting totally new approach to particle physics, something the non-specialist or the lay person can read. After all, this consensus world belongs to everyone, not just the hypothetical physicists in their academic towers. So the hope is that everyone can have at least some understanding of how it is composed.

APPENDIX 2 THE NATURE OF TIME AND SPACE

1 Introduction

The vast topics of time-duration and space-distance need to be considered, at least briefly. These are such incredible broad and interesting topics that they deserve a book in their own right. Unfortunately only a short space can be devoted to them here.

Time and space are the two quantities which humans understand or at least pretend that they do. The quantities of mass, charge, and color belong to the realm of the particles. Human scientists are perpetually trying to translate these physical properties and their corresponding forces into the human conceptual world realm. Before this can be done correctly, though, humans need to more thoroughly examine and understand their own conceptual realms of time and space.

Immediately though there arises difficulties with even this task. Humans have many different conceptual views of time and space. Several of these diverse views may be of a benefit in understanding the core body of this work, the reports on the mathematical-geometric structures of the leptons and the photons. Several views on time and space can help set a broader framework or contextual field in which the mathematics and geometry of the particles can be set. The laying out of these views verbally can help show that some of the new discoveries about the particle waveforms, particularly concerning their relationships with time, are in fact in keeping with what humans intuitively hold to be true anyhow.

This appendix-report is somewhat of a compendium. Some of the many conceptual views are discussed at least briefly that humans hold about time and space and the engineering-scientific-mathematical usages these continuums.

2 The Inherent Nature Of Time And Space

The inherent nature of time and space is that they are continuums. Time and space are backgrounds. They have no self-content, are inherently empty and void. As such they have no inherent form, structure, or properties themselves. They are limitless, have no external boundaries, ends, or surfaces nor any middles, centers, or foci. Without such locations as references time and space as continuums inherently have no dimensionality. Finally time and space being continuums have no internal discreteness, subdivisions, sizings nor any smallest parts.

Upon or within time and space forms appear, exist in a place and for a period, maybe move around and affect other forms, and then go out of existence again. Forms are not continuums and are conceived as being discrete. Because of this, beings with discursive thinking abilities like to quantize or count forms and assign them properties. Forms or objects change or are impermanent internally and frequently move in reference to others externally. Forms or objects act as markers on the continuums.

The interplay of forms within the continuums is where discursive thinking beings such as humans start to make conceptual errors concerning time and space. Humans like to quantize and describe various forms. To do so, though, they also must begin to quantize the continuums. This is just a first error. Likewise self aware beings such as humans begin to reference the two continuums against each other. This is a second error.

For humans and other mobile beings this cross referencing process usually means subordinating duration-time as the lessor parameter to the perceived dominate or more important parameter of distance-space. These various thoughts lead to a few other equally curious ideas for consideration. For plants which are not mobile, is space subordinated in favor of the dominate feature of their existence time?

Realistically such a subordination of time in favor of space is quite comical in that as the implicit variable in mathematical expressions time is really the independent variable and space becomes the dependent variable. Of course the whole concept of the continuums as variables is inherently flawed and foolish, even though utterly necessary and extremely useful in the scientific and technical realms. Time does not move nor pass. It just is. Nor does space move or pass. It also just is. Time and space are just the backgrounds against which everything else (discrete) moves and passes. Individually without the presence of forms or objects the continuums cannot be referenced to themselves. Time has no duration. Space has no location. Likewise without the presence of forms or discrete markers which move or change, the two continuums cannot even be cross referenced to each other.

As an example of human mental and vocal foolishness, consider the often heard phrase "Winter is Coming". False! It isn't going anywhere. It just appears. And the land doesn't move either. Coming implies some Thing comes from some Where.

There can be two additional characteristics of time and space. Their first descriptors, already given, were that they are inherently empty and limitless. Secondly they have no awareness, maybe? Clearly the continuums do not appear to be self aware or aware of themselves. But when forms appear, the scientific communities begin great discussions as to whether time and space react or become aware of themselves or that something is there, now contained within themselves. Such concepts are central in the mathematics which propose that space reacts or curves about mass. Thirdly they have no ability to do or manifest anything, or do they? Again there are scientific debates which center on these ideas. The proposal by Dirac concerning the polarization of the vacuum inherently assumes that form, charge in this case, can spring fourth from nothing other than time and space. The questions of whether time and space are aware and whether they have manifesting potential appear to have no definitive answers.

Returning to the first "property" assigned to the continuums time and space, there can be an even greater metaphysical question or debate. Since they are said to be empty and limitless, do they even exist? Can "something" without a form exist? Do such questions of the existence of time and space even make sense? Even here in particle physics and astrophysics various somewhat contradictory views are rendered. Even the very descriptions of time and space as continuums is often negated by the assumptions of some discussions. For example some academics in these fields hold that it was the big bang itself, ie form, which created time and space or equally that time and space are formed around the particles even today. Do the continuums form particles and matter, or do the particles and matter form the continuums?

Finally there can be questions about what is not there, is missing, or at least appears not to be there. Can there be or is there in fact a third continuum of which humans are unaware or have just failed to recognize? Is there anything which prohibits there from being a third continuum, other than human consciousness? For examples; this third continuum could be awareness, consciousness, love, a metaphysical energy background or spectrum of some description.

3 Dimensionality Is Created By Forms Or References

The interaction of forms or objects with the continuums needs to further examined. When forms appear or are conceived of by discursive thinking beings, then these "objects" act as references or markers in the continuums. One object whether physically real or conceptual becomes the foci, center, or the origin within a continuum. To have the concept of here or now there needs to be at least one such object to give a sense of place or location.

When a second object appears whether physically or within a discussion, then naturally it is thought of as or stated to be away from the first reference. Since in this scenario there are only two such markers within the continuums, then naturally they are felt to have a linear relationship between them. This is, as long as time is viewed as being external to the original reference or observer's location.

When a third marker or reference appears though, then the conceptual view of the two continuums starts to split or diverge into two distinct viewpoints. Space remains an externally referenced affair. Object-markers continue to populate space outside of themselves creating references for more and more external dimensions. That is, reference objects and dimensions within space begin to subdivide the continuum.

Whereas time begins to become an internally referenced phenomenon. The initial reference or origin within time tends to somehow gets inflated conceptually so that it in effect becomes the continuum. Then by the stage when a third object-marker is conceived, it and also the second one get stuffed inside the inflated initial reference now become the continuum. This conceptual process of inflating the previous reference and placing the next new one inside it continues on unabated for time. Reference objects and dimensions within time begin to subsize the continuum.

What is seen is that the typical discussions, scientific or otherwise, about the dimensionality of time and space are less about the nature of the continuums, which inherently have no intrinsic natures, and are more about human conceptual habits. This needs to be emphasized. The true nature of time and space get confused with or subordinated how humans use or think of them. Never-the-less since scientists are also humans, they must use or abide by these habitual conventions. The following different natures of dimensionality are found.

Space is an external phenomenon. Each new dimension in space forms at right angles to, away from, all the other dimensions already in existence, or in the discussion. Space is a dimension, outside of a dimension, outside of a dimension, et cetera. An illustration of this concept of externality is seen where the corners of a room meet the ceiling.

Time is an internal phenomenon. Each new dimension forms in a parallel or co-linear fashion to, on top of, all the dimensions already in existence. Time is a dimension, inside a dimension, inside a dimension, et cetera. Dimensions of time are thought of as ever smaller, or more faster, patterns within the previously mentioned dimension. An illustration of this concept is the human voice wave pattern riding on, through, or embedded in the longer carrier radio wave.

4 Mathematical Views Of Time & Space

Again, humans have some conceptual difficulties with their uses of the two basic continuums, time and space. This time the difficulties arise in the mathematical usages of of these inherently featureless continuums which have quantized for some useful purpose.

When seeing a mathematical expression with distance raised to powers, this is commonly thought of as; d^1 a length, d^2 an area, d^3 a volume, etc. Specifically d^2 is conceptualized as a distance squared or a square distance, d^3 as a distance cubed, et cetera.

Sadly this concept of the externality of space usually is held even when d is in the denominator position of a mathematical expression. If d^n is seen in the denominator, this is still thought of as a length, area, volume, et cetera, instead of a 1st, 2nd, or 3rd derivative.

When appearances of time raised to powers are seen in a mathematical expression, t^1 is commonly thought of as simply time, a linear duration between two end points of a period. But when t^2 is seen a shift in perspective occurs. At best, the time part of the expression is ignored, that is its measurement units, and t^2 is viewed as simply a number, value, or quantity squared. The expression t^2 is just not automatically conceptualized as a time or duration squared. Alternatively the second time of the square is conceptualized as being an embedded smaller scale, often infinitesimal, time within the first time of the square.

This concept of the internality of time usually is held even when t is in the numerator position of an expression. If t^n is seen in the numerator, this is still thought of as a duration, the duration of a duration, et cetera or technically as a 1st, 2nd, or 3rd derivative, but not of a duration squared, cubed, et cetera.

The problem here with the subconscious making derivatives out of t^n in the numerator and the not making derivatives out d^n in the denominator is that the conceptual focus has shifted to the objects of discussion and has forgotten the backgrounds against which the objects are being located. Typical derivatives which involve both time and space appear as dy/dt, the rate of change of position compared with the rate of change of time. Second derivatives d^2y/dt^2 express the rate of change, of the rate of change. All of these rates of change are referring to the objects or forms, their locations, sizes, shapes, etc, and are not referring to the continuums upon which the forms are playing. The objects and the continuums have gotten mixed up and the continuums are being quantized in order to describe the objects.

To continue the investigation of time and space, a few simple mathematical examples can illustrate many of the concepts and difficulties here. Using the usual Cartesian coordinate grid system, as a way of relating the two spatial dimensions or parameters represented by the variables x and y, is by an equation such as; $y = F(x) = ax^2$. Those familiar with calculus think nothing of the ratio, $dy/dx = 2ax$.

Likewise with a usual view of only the outer most, exterior, or consensus level of time, there is no problem relating space with this concept of time by the expressions;

$R = F(t) = bt^2$ and $dR/dt = 2bt$. (01 & 02)

This of course as long as time is the independent or implicit variable.

When thinking of time within time or multiple dimensions, layers, levels, or tiers of time though humans have trouble with the expressions;

$T = F(t) = ct^2$ and $dT/dt = 2ct$. (03 & 04)

Again a practical example of this mathematical usage of time is the human voice wave pattern riding on the radio wave pattern. Or even more applicable in this context: Think of the scan lines refreshing the picture of the computer monitor at an invisibly rapid rate, at the same time the "solid" appearing figure of the screen saver appears to rotate.

The tricky part of relating multiple internal dimensions of time to each other, as in (03 & 04) is what is to be used as the independent variable. In the above ratios, derivatives, the denominator is the a-priori or independent variable. Referring to the three levels of time discussed later in Section 5, for a particle just as with a conscious creature, the level 3 experiential or relativistic time is the a-priori. This is the internal clock which determines the particle's existence.

Where both time and space appear in the same mathematical context, humans are pretty rigid conceptually. This was noted with Equations (01 & 02) above. Mathematically space takes the form of explicit functions or equations. As such space is the dependent variable. In terms of wave phenomena space is thought of as transverse waves. Mathematically time takes the form of implicit functions or equations. As such time is the independent or a-priori variable. In terms of wave phenomena time is thought of as longitudinal or compressional waves.

Again, the true nature of time and space get confused with or subordinated how humans use or think of them. Probably the cleanest and purest way to see how humans, engineers, scientists, and other technical persons use or deal with time and space, is to turn to pure classroom mathematics. Specifically the examination of calculus integrals and derivatives in their standard sterile settings is very informative. The results of integrating or taking a derivative depend on whether the quantity-parameter-variable being integrated/derived is in the numerator or denominator position. This is regardless of whether this quantity is time, space, or any other measurable quantity.

Observe what happens when the quantity-parameter-variable is in the numerator position.

Integrals add dimensions of their own kind, create more and more INTERNAL content and boundaries, head inwards.

For example; $\int (X^1) = X^2 / 2$

Derivatives subtract dimensions of their own kind, create less and less content and boundaries and head outwards towards EXTERNALS.

For example; $d(X^2) = 2X^1$

The opposite happens when the quantity-parameter-variable is in the denominator position.

Integrals subtract dimensions of their own kind, head OUTWARDS towards less references.

For example; $\int (X^{-2}) = X^{-1} /(-1)$

Derivatives add dimensions of their own kind, create more and more INTERNAL references.

For example; $d(X^{-1}) = -1X^{-2}$

In science humans habitually put distance in the numerator and time in the denominator. Distance dimensions are external, dimensions outside dimensions outside dimensions… Or distance can be considered as in n-volumes. Temporal dimensions are internal, dimensions inside dimensions, within dimensions… Or time can be considered as in layers.

Because to these typical engineering, scientific, and mathematical usages, expressions involving distance are usually set up with external multipliers or a series of factors, such as:

$R = 4/3\pi \exp^3(\theta^p)$, where $4/3\pi$ modifies R;

Or Factor 1 * Factor 2 * Factor 3

Or $\exp(-a\theta^2) \times \exp(+b\theta^1) \times$ polynomial

But time is set up with internal multipliers of the variable of a given form. This leads to a series of nested factors or a recursive appearance, such as:

$R = \exp^3(4/3\pi\theta^p)$, where $4/3\pi$ modifies the θ;

Or Factor 1 containing (Factor 2 within, containing (Factor 3 within))

Or $\exp(-\exp(+(\text{polynomial of } \theta) \times \theta^1) \times \theta^2)$

Whereas the external dimensions of space are obvious, this compacted recursive appearance of time is deceptive, and leads humans into thinking that time is not multi dimensional.

There are ways to break up these habitual thinking patterns of humans about time and space. Anyone familiar with mathematics has no trouble with the concept of distance and the distance formula in which;

distance or $ds = \sqrt{ dX^2 + dY^2 + dZ^2 + ... }$

in which there are no negative distances. Humans habitually think of time as "linear" with past to the left (negative values) and the future to the right (positive values) or the past as being behind one's self and the future in front of one's self. Instead the same radial concept used for n-dimensional space can be applied to n-dimensional time. Time can be thought of as a circular disk or a spherical solid, etc. The present is the origin, and everything else is away from that; as in radial coordinates. It doesn't matter if the "past" and the "future" are side-by-side; they are away from the present by a certain distance. In effect all time would become a positive absolute value.

Returning to the original discussion of dimensions as parameters raised to powers; d^1 a length, d^2 an area, d^3 a volume, etc. what is found in real world usages, not classroom sterility, is that these concepts are somewhat half truths or are really false. There is no such thing as a unit squared, cubed, etc

For example examine the usages of distance raised to powers. For distance in much of science and physics, the first power refers to something on the human scale (ego centric). The second refers to something about the scale of the object of discussion, usually perpendicular (external) to the first. A good example is the heat transfer coefficient found throughout engineering contexts. Heat transfer is expressed as Btu / hr x ft^2 x F / in. The surface area is in, ft^2, and the insulation thickness perpendicular to the surface in, inches. To divide (cancel) them out something is lost. The three references to distance are actually references to new extra dimensions, of the same kind; Dim 1 = length, Dim 2 = width or height, Dim 3 = depth; 3 usages of distance

Likewise for duration, there is no such thing as a square second. The first second sec^{-1} refers to the human scale. The second "second" or "second" squared, sec^{-2}, or "derivative" refers to the scale of the event occurring, usually internally, within, faster than, the external human scale.

Often, usually the human scale in time and space coincide with that of the object or process of discussion, but not always. Obvious disparities between the human scale and that of the objects of discussion are found in; astronomy, chemistry, molecular, atomic, subatomic….

What if time itself operates at different speeds? This case is quite real and was found in the reports for both the leptons and the photons. There three independent times were found; radial time, angular time, and the time of revolutions or spin of the wave forms about their centers.

What if there are 2, multiple different time scales? These considerations formed much of the bulk of the discussions in Part 3 of this work.

What if time is not fixed?

Again, a reminder is needed that none of descriptions here really apply to time and space. Mathematical expressions are human conceptual play toys and not the actual continuums. Likewise the descriptions of waveforms and objects moving across the continuums apply to the forms and not to the backgrounds.

5 Psychological Perspectives Of Time And Space

Humans are externally oriented where space dominates, and are used to thinking in terms of perpendicular dimensions. People with typical busy modern lifestyles are not used to turning inward or else they would immediately see that there are multiple simultaneous internal dimensions in time. After a brief internal examination three levels of temporal existence are found as follows.

Level 1 Time: This is the outer most level of time and applies to people's experience of the consensus physical world. This is the level of time with which people are most familiar. This is because when interacting with the consensus external world people are surrounded by time. Typically this realm of time is spoken of as linear. A short investigation shows this verbalization is inaccurate, though, because of the limitations humans have placed upon their conceptualization processes due to the nature of their symbolic languages. The past can be thought of, some event can be recalled from memory, day dreams can be indulged in, or plans can be made for some future event. But when doing so, though, the past is not automatically placed to the left and the future to the right. Likewise when thinking of time and space, the past is not oriented so as to be behind people's backs and the future so as to be in front of them. These things or events in the mind are just out there somewhere, away from the present point from where the visualizations are occurring. In the mind an event from the past which now no longer exists can just as easily be placed side-by-side with a not yet existing one from the future. These events don't have to be 180° apart from each other.

From another view time seems to flow from yesterday towards tomorrow. This may be a direction, an orientation yes, again dictated by discursive verbalizations. But this is clearly not the same thing as saying that time is linear or has a constant smooth flow. Everyone is familiar with sayings to the effect, when a person is busy, time flies. Or when a person is worried or idle, time drags. But what has just occurred? A second sense of time has been invoked to use as a yardstick to gauge the speed of passage of the first sense of time. This first sense is that which focuses on the consensus world of physical matter and energy and is gauged by a supposed agreed upon internal average human experience of the passage of events.

There are yet further problems with the original idea of the flow of time as being linear or smooth. Some cultures, their languages and thinking processes have a circular sense of time. This is particularly true of those peoples living near the equator, such as in Bali. There is one planting season after the next, with celebrations in between. The fact that X number of these occur in one cycle of the earth around the sun is completely irrelevant. Such peoples also have greater views of longer cycles within which the shorter cycles occur. Examples of such longer cycles would be those of life. Western scientists can be viewed as quite arrogant when they say that time is linear. Such statements simply show the mental bias of western science which frequently subconsciously attempts to embed physical and greater reality into the linear narratives of monotheistic religions.

At this first outer level besides distance and time, the other experiences of the consensus physical reality include a sense of matter and energy. In this realm static, stuck, or quantized energy is conceived of as matter. The words encapsulated, encompassed, enclosed, or entrapped energy could also be used to describe this "solid" fermionic material. Moving, kinetic, or bosonic material is just called energy. The mathematical-geometric descriptions of the leptons and photons show how time plays out mathematically in these matter and energy arenas.

Level 2 Time: This is the intermediate level of time that is invoked when speech such as time drags or flies is used. Such observations as these come when the awareness has been shifted to the internal sensory apparatus. There is still external awareness, even very keen awareness, that the time of the external physical world is still passing at the usual rate as measured by clocks. Somehow though the internal clocks of the current awareness are out of sync with the external world. There appears to be two different realms of time and they are not passing at the same rate.

Instead of the physical realm, this internal arena could be referred to as the feeling-emotional level of existence. In the physical world the long term "unchanging" or more "solid" material is referred to as being matter. Whereas at this level static, stuck, or long term material, is referred to as feelings. From an internal view these feelings can seem solid, real, and impenetrable. This "solidified" emotional energy sometimes can last for a lifetime and often goes totally un-noticed, just as there is only a vague awareness of solid sidewalks when moving through daily routines. The moving, flowing, rising and falling material of this level is called emotions. This emotional level energy or simply emotions usually rise and fall and pass fairly rapidly from shaping one internal scene to the next. Naturally emotions can occur and feelings exist simultaneously with physical action taking place externally.

Level 3 Time: One way to get a sense of this often hidden inner most level of time is by referring to an experience of an activity in which there had been complete and intense singleness of focus. Examples typically sited are involvement in high speed competitive athletic events or in situations which were life threatening for self or others nearby and which required immediate action of some type. In these cases the world, or the awareness of it, was narrowed down to the specific small area which was the focus of importance at the time. In these events the world may still have been very real and maybe even vividly clear. But physical reality appeared to be moving in slow motion. Likewise emotions and thoughts were out there somewhere, if there was any awareness of them at all. The sensory apparatus of the consciousness had shifted to a deep internal level. The conscious awareness had been shifted to an observer mode while the physical body and the emotions were doing whatever they were doing.

We might refer to this level as the mental-conceptual level. Here “solid” material consists of fixed ideas, opinions, beliefs, or attitudes some of which may last for many generations. The very rapidly moving short lived material consists of passing thoughts and free floating or fleeting ideas. Obviously thoughts occur while emotions are rising and dissolving away again, and while physical action is occurring.

Summarizing: Humans have no difficulty with multiple dimensions in space because they are externally visually oriented. The bodily physical sensory apparatus of humans can see or touch different places simultaneously. Humans see space is multidimensional or parametered. They preceive these dimensions to exist simultaneously and pass thru each other. These are seen as all being "external" to each other and new dimension forming at right angles to all those previously existing. Waves in space are easy as being transverse waves, like ocean waves or ripples in the toilet bowl.

Whereas only the emotional or conceptual apparatus can be used to see or touch different realms in time. In contrast to the external or objective level 1, measurable time, levels 2 and 3 are internal or subjective. Additional descriptors for these two underlying temporal levels may be experiential, personal, projected, apparent, or relativistic time. Again, looking at a year in a person's life. what is found is time is multidimensional or multi-tiered or has multi levels. Thoughts happen very rapidly at one level, emotions endure longer at another, meanwhile daily activities at work are occurring. All these temporal dimensions exist simultaneously and pass thru each other. These are seen as all being "internal" to each other or new dimension forming "parallel" to all those previously existing. Waves in time tend to be viewed as compressional or longitudinal.

Although people talk about the passage of time, this never gets stated mathematically as seen in Equations (03 & 04). While people may concede that a rabbit, a fly, and a saguaro cactus experience time differently from humans, this idea hasn't as yet been translated into scientific or mathematical terms.

These human conceptual tools of time and space get even more turned on their heads when considering the world experience of other animals. Animals with keen senses of hearing probably put time in the numerator and distance secondarily in the denomenator. Some animals are only "here" less than half of the time. Bears hibernate during winter, cats sleep 18 hours a day. Animals with sharp senses of smell probably are much more internally oriented. For example; dogs walking out into the street are just noses with 4 legs under them. They don't have a clue where they are in space, or time, or any other human dimensionality. They live in a whole other dimension, called smell. Stationary life forms, plants, saguaro cactus, grass, etc. may only experience the passage of time and know nothing of human's beloved space.

5.1 Tie-In With The Subatomic Findings Of This Work

From the brief investigation above, three simultaneous dimensions in time become evident, just as there are three simultaneous dimensions in space. While this was a psychological perspective, there is a very useful benefit of this insight to the mathematical scientific work here. What is found in the lepton and photon reports, describing both the fermion and boson classes of subatomic particles or waveforms, is that for every spatial dimension associated with a particle there is also a temporal dimension. Mathematically what was found in the subatomic realm investigated here is that for every explicit spatial variable there is an associated implicit temporal variable.

This nature of time and space as formed by the particles, surrounding the particles, as experienced by them, choose a phrase, at first appears to be in direct contradiction to "common sense". Applied to the consensus universe of humans this means the long standing scientific narrative that the world consists of three spatial dimensions and one temporal dimension is just that a narrative, a misunderstanding, misrepresentation, or misnomer. For the average person with no scientific background, multiple dimensions, layers, levels, or tiers of time as just illustrated makes perfect sense. Mary Jane and Myra Jean on the street could have told the physicists this a long time ago, but no-one in their concrete academic high rise towers bothered to ask.

This supposedly new view of the consensus world that was found in this work has been shown to fit with people's personal intimate experience of the world. The mathematics of the subatomic waveforms has described what has been known all along, that time is not one dimensional, linear. Multiple spatial happenings are occurring all the time and measured against different temporal backgrounds or references.

Specifically for the leptons and the photons three temporal dimensions were found. The first was just an unsubscripted t found in the vector expression of R(t) which led to the curvature or torsion based charge for the leptons and the lack of a charge for the photons. This variable related to the motion of the basic circular forms with their two spatial dimensions as they propagate thru the exterior third spatial dimension around themselves. This unsubscripted t could be thought of as a measure of the outer most layer or tier of time, consensus time in terms of which humans usually think.

The other two usages of time concerning the leptons and photons are related to the inherent structure of the particles or waveforms themselves, rather than to their exterior motion. The variable labeled t_θ related to the angular nature or pattern of the waveforms. This variable was embedded as an argument within the spatial or outer most mathematical expression describing the angular mass density functions. As a loose analogy, this variable could be equated to the second layer or tier of time. Conceivably there can be multiple such second level dimensions in time as other spatial angles become necessary to describe the inherent nature of more complex particles such as the quarks.

Eliminating all the more n-dimensional parameters, the most basic descriptor of all particles is a radial parameter or a radial sense of sizing. This spatial parameter has embedded within it an implicit independent variable of time. This one dimensional feature appears to be related to mass itself as a measure of entrapped energy. Even the neutrinos which only seem to respond to gravity appear to have this most basic geometric feature.

In terms of the leptons and photons with two inherent spatial dimensions, the variable labeled t_r was clearly embedded in both their expansive and contractive exponential radial functions. What was not so obvious was that within the distance function which was a part of their expansive exponentials, there appeared to be a further embedded or precursor usage of time. This hidden or "doubly embedded" usage of time was discussed in the Photon Report 1.2, Section 5.3. This precursor variable was not given its own symbols as a further subscripted t there. This was to keep the plethora of new ideas from becoming over whelming. For both the leptons and the photons this potential inner inner or doubly implicit variable had an uncanny power relation analogous to the variable in the particles' initial spatial conditions of the Fraunhofer Diffraction Function. For the leptons this inner inner concept of time and that of their initial condition were both related to t^1. For the photons this inner inner concept of time and that of their initial condition were both related to $t^{1/2}$. To complete the picture here, this doubly implicit precursor t could be equated to the third or inner most level of time.

From another viewpoint a case could be made that the neutrinos, which probably only have a 1-dimensional radial appearance, only have a singly embedded or implicit radial variable of time. Whereas as just discussed, the elementary electromagnetic particles with 2-dimensional radial appearances have doubly embedded or implicit usages of time in their radial mass equations. Finally speculation could be made that if the quarks have an inherently 3 or 4 dimensional radial appearance, then their radial mass expressions could be expected to have triply or quadruplely embedded or implicit expressions of time. Of course all of this is contingent upon the ability to make the somewhat arbitrary distinction as to where one implicit mathematical function leaves off as an argument and the next outer encompassing one as a dependent variable begins.

Finally, as was found from the mathematics in this work; the photon, and presumable all bosons, experience time differently from the leptons, and presumably all the fermions. The photon sense of time, which was described in the photon report, has a different mathematical description from the leptons' sense of time.

6 Higher Dimensions In Space

Sections 5 and 5.1 focused on what could be called higher dimensions in time. That is, these sections discussed multi-dimensional aspects of time which are not usually considered by conceptually rigid western scientists. A similar treatment is now needed for higher dimensions in space.

Mathematicians take great delight in analyzing mathematical expressions with more than 3-dimensions. When working with such expressions, though, mathematicians do not usually bother with such distinctions as to whether the objective of the modeling process is spatial, temporal, or being modeled in any other domain. This is because pure mathematical expressions are just human conceptual play toys or narratives. The application of mathematical expressions and equations to physical reality where such expressions are assigned measurement units is often left to engineers and scientists. This state of affairs is acceptable in that the application of pure mathematical models to the real physical world cannot be done until a practical need arises.

Unfortunately some scientists, such as hypothetical physicists, have also gotten themselves lost in mental-conceptual realms and likewise have never connected their work with physical reality. Hypothetical physicists take great delight in analyzing conceptual objects with 9, 10, 11, or 26 dimensions. Unfortunately though to connect such hypotheses and speculative models with the real physical world requires the immediate covering up or hiding all these extra dimensions. This is because all such dimensions are conceived of as being spatially real. Several viewpoints upon the parametrics or nature of dimensionality are discussed more fully in Report 3.1, Measurement Units and Scales, Sections 6.1 and 6.2. Specifically in the last paragraph of this Section 6.1 the issue of the hypothetical physicists equating the arisal of every variable, parameter, and measurement unit in a discussion or a mathematical expression with an actual spatial dimension is exposed.

In the Report 1.3 A Model For Determining Physical Properties IV: Charge Of the Quarks, the quarks are shown to be represented by intrinsically 4-dimensional curves. Or at least their ± 2/3 and ± 1/3 charges are exactly matched with the fixed curvature of certain curves in 4D space. The mathematical derivation upon which this report was based was for a true 4th dimensional curve traveling in 4-dimensional space. There is nothing unreasonable, new, or counter intuitive with this view. This invoking of a fourth spatial dimension for a real physical form does requires an explanation. One extra spatial dimension which appears at odds with the human experience of physical reality is no better than 6, 7, 8, or 23. What needs to be shown is that such an extra spatial dimension while at odds with the human sensual experience is at least plausible.

First from a conceptual and mathematical view nothing prohibits there from being a physically real fourth spatial dimension. Two distinct points determine a line, rather than a mere point. Three points not in a line determine a plane or 2-dimensional form in a plane. Four points or references not in a plane determine a "solid" of 3-dimensional form. Likewise all that is needed mathematically to create or define a 4-dimensional space is 5 references not in a "solid" body.

Such a mental explanation is not good enough, because it does not agree with human's visual and tactual experience of the world. There are two issues here. First the nature of the human biological apparatus needs to be examined. Second based upon the input received thru the electro-chemical sensory instrumentation of the human body, again the habitual narrative that humans are making up about the world needs to be examined.

Humans and other animals have no means of "seeing" or "feeling" a 4th dimension. All animal eyes see photons, which are 3D in nature. Likewise the skin of humans and other animals feel the heat of the sun or fires, which are again just an expression of photons. Animal ears hear sound, compressional waves in physical media. In short humans have no sensory appratus capable of experiencing a 4th dimension.

The nature of the human bio-mechanical machinery is that at the smallest practical level it, like all earthly life forms, is made up of chemistry. Chemistry is the language and communication systems of the human body. This is obvious with the blood and nervous systems. Without chemistry, electrons, and photons human consciousness would have no connection or communication with the external world nor even with its own internal world.

The realm of chemistry is restricted to the atoms of the periodic table and their joining together as composites or molecules. That is, the whole atoms with their surrounding electron shells. Radioactive, nuclear, and subatomic species and events are not relevant to the workings of chemistry. This atomic-electron shell connection is particularly true for water and organic molecules of which living forms are predominantly composed. The language of chemistry is that of the two most elementary electromagnetic waveforms, the photons and electrons. Whether these are free electrons or those claimed by elements or molecules does not matter. Electrons in effect cover, veil, and obscure the entire spatial surroundings of everything of which living forms are composed.

As presented in Part 1 of this work, the electrons are 2-dimensional forms which traverse thru 3-dimensional space, probably at a scale realm of 36 orders of magnitude smaller in distance than humans. At a scale many many orders of magnitude larger, is the Bohr atomic radius of 10^{-11} meters. In this size realm the electrons form 3-dimensional shells or wave patterns around the nuclear core of the elements of the periodic table of chemistry. The worlds or realms of chemistry are essentially 3-dimensional in their spatial scope.

Since the nature of the human body is primarily that of chemistry and all chemistry is clothed in a veil of electrons, then the human sensory apparatus are not capable of seeing the core of the atoms and molecules. Consider the eyes of living earthly creatures. They too are composed of chemicals and function off photons causing electrons to shuffle from one chemical complex to another. Except for the topics of mass, weight, density, and such, humans for all practical purposes have no awareness of the nuclear cores of everything which surrounds them in space. This is because there are no bodily electro-chemical sensory instruments capable of detecting such nuclear cores.

The importance of the nuclear cores of the elements of chemistry in this explanation is of course that they are composed of quarks and gluons. These waveforms are inherently part of the high energy realms beyond the realm of chemistry. They communicate with high energy x-rays, color forces, and other such phenomena with which organic molecules and tissues are not capable of interacting. That is, except of course for interactions which result in the destruction of the organics.

There is nothing unusual with humans or other living forms being blind or partially blind in one or many of their physical senses. Until a relatively short time ago in the history of humanoid existence, people had no awareness of radio waves, x-rays, microscopic objects, sounds outside the human hearing range, or awareness of smells by which other animals arrange their entire existence.

In Report 1.3 a plausible proposal was given for the existence of a fourth spatial dimension thru which truly 4-dimensional quarks traverse. Here in Appendix 2 a reasonable explanation has been given as to why humans cannot sense this fourth dimension. Finally being unable to sense the fourth dimension of the quarks' transitory existence, humans do as they have always done. They make up stories about, that which they have not experienced, cannot sense, or understand. These stories are all negative and say the unsensed does not exist. They project their story out onto the external world, observe their own projections, and then offer these observations as proof of the truth and validity of their stories. When offered evidence of the falseness of such stories many people, including physical scientists, frequently continue to deny the existence of such broader realms of reality, or else shun such new experiences or understandings as evil. History has shown such conceptual rigidness for what it is too many times to count.

To make a complete or more accurate discussion of the scenario here a person does need to consider the size realm or relative scale of the quarks. The quarks exist at the Stoney scale realm, 36 orders of magnitude smaller in distance than humans and 44 orders of magnitude smaller than the human invented second. There is no quarantee that their 4th dimensional existence has any reality, relevance, or correspondence at the human scale of existence.

There are some speculative scenarios in which some form of conscious awareness might be able to detect quarks and then could be capable of sensing the fourth spatial dimension. If a self conscious or "living" form were to inhabit the residual cores of burned out suns, neutron stars, black holes, or the likes, then there is a chance that sensing the world as a four spatially dimensional universe would be perfectly normal. This is because under these circumstances all the leptons would have either been driven off exposing the quarks or else would have themselves been smashed into quarks, gluons, or the likes. Additionally any such conscious form would itself be composed of quarks and hadron compounds rather than chemistry and chemical compounds.

There are other odd aspects of such speculative situations with which the astrophysicists may want to amuse themselves. Considering neutron stars, black holes and the likes, these objects appear to be highly compact or incredible dense from the exterior perspective of humans with organic bodies. Internally though these celestial objects could actually be quite "roomy". This would be because of the high predominance of quarks. If the quarks do prove to have the mathematical-geometric nature proposed in Report 1.3, then the presence of the fourth spatial dimension would allow for all this apparent compaction, while actually having plenty of extra space available. Of course there could be no more mass hidden in this fourth dimension. Newton's Laws, relativity, and other such calculational models should still account for the correct amount of mass being present.

7 The Quantization Of Time And Space

Several important aspects of the two basic human measuring sticks need to be noted. Measurements of time and distance are always locally referenced. In the simplest usages distance is always referred to the arbitrary mathematical-geometric origin of a figure in a textbook or on a blackboard, the end of a tape measure, the turn of a car odometer, etc. Likewise the duration of time is referenced to the starting of a stop watch, a fictitious mythological or historical human event, etc. In a way these two concepts can be said to be locally absolute. The concept of an absolute measurement scale as used in this work is a scale that is of relevance to the topic of discussion, immediate environment, and actually reveals some underlying structure of the subject being measured.

In fact, though, in the grander scheme of things, the universe, these two concepts can never be made absolute for all times, places, or size and duration realms. That is if the proper sense of absolute is used, as meaning a scale which reveals something about the structure of the universe of discussion. If the nature of the physical universe is the topic of discussion, then there is always a distance or duration from some greater or more important structure, which can become the reference. Or likewise with atomic physics there are always some phenomena smaller or more rapid and having enough importance to become a reference structure. This continually adjusting the scope or arbitrary starting points for these two variables is actually good. This means these two measurement units can remain applicable, relevant, have meaning, and be of use to people.

Referring to the discussion of kinds or types of units in Report 3.1, there several examples are given illustrating the conceptual difference between relative and absolute units. Thinking of units, variables, parameters in terms of objects, those scales describing or referring to content were called relative. Those descriptors which measured or described structure or organization of a system were called absolute. In terms of the discussions here referring to space, forms-objects-content can be thought of as a first or beginning tier. The broader perspective of the organization can be thought of as a second tier.

Analogously if temporally oriented words are substituted for the spatially oriented words, the content of a process in time would again represent a first or beginning tier. Content again would be described by a relative unit or parameter, just one referring to durations or situations rather than lengths of objects. Instead of spatial structure and organization, temporal words such as those referring to processes or operations can be conceived as a second temporal layer or tier. Finally from a conceptual view or framework, a top or final tier in space or in time can be referenced as the focus of the discussion or awareness. These would relate to universal or meta units.

One of the curious features about both time and space is the means which humans have used to quantize them. Even though there may be 2 or 3 spatial dimensions in a typical mathematical or engineering discussion, only one measuring stick of distance-length is used. This is of course if rectilinear coordinates are used.

This measure is the meter in scientific work. That is, if the discussion concerns objects within reasonable proportions to the human size. Astronomy tends to use its own sizing. As discussed in the entirety of Part 3 of this work the absolute Squigs units should be used with subatomic physics work. These Squigs scales are based upon the measurement units put forth by George Johnstone Stoney in 1874. Except the Squigs scales have had his assumed 2 or 3 dimensional π constants removed. There have been other historical units of measure used for distance-length which were independent from the meter. But now except in the United States, all the other distance units which were independent from the meter have been eliminated. The use of common multiples of the meter are of course still sanctioned and accepted. That is, if these common multiples are multiples of ten.

This quantization of length situation is not much better if general radial-angular coordinates are used. Then one measure of distance-length is used for the radial parameter. But only one measure of angular positioning-distance is used though, no matter how many spatial dimensions and angles are being discussed. This angular measure is the radian.

For temporal measure the situation is somewhat even worse. Since science in general and physics in particular appear to have not even recognized that there are multiple dimensions in duration-time, then only one measuring device is used. The second is used for everything, again within reasonable proportions to the human physical experience of the world. Of course, astronomy and subatomic physics should rightfully have their own absolute measurement units of duration-time. Whether time is used in a radial sense or angular sense there is only one measurement unit.

There are two other quasi measurement units for time used when referring to angular phenomena, cycles or revolutions. Really these references are not to time though. They are to the movement of objects or forms within time and space.

There are other common measures of time; the minute, hour, day, year, etc. But again these are only multiples of the base unit the second. In the political, social, and religious arenas of life, lunar senses or uses of time still exist. Except in these realms, though, all other senses of time which are independent of the base unit the second have been driven out of existence.

This whole measurement scenario for distance and duration that humans have devised is simple to use and understand. Sadly though this very simplicity makes it very limiting conceptually. The idea that there can be physical dimensions or mathematical spatial axis in which the distance measures of the various axes are independent of each other has now been excluded from arising conceptually. Likewise the conceptual possibility of other senses of time has now been eliminated.

By the very unification of scientific measures humans have now excluded themselves from understanding that there can be other realms or ways of sensing and experiencing the world. That the worldly experience is different for other people whom may have different pulses, respirations, bio-rhythms or circadian rhythms is hard to grasp for scientists bound by the conceptual rigidity of a single measure of external time. That a rabbit, a fly, or a saguaro cactus literally live in a different temporal realms cannot even be conceived or discussed, much less understood. How do physicists even pretend that they understand the world that electrons, photons, and the other particles inhabit? Again the quarks exist at the Stoney scale realm, 36 orders of magnitude smaller in distance than humans and 44 orders of magnitude smaller than the human invented second. A person can ask can physicists really act as effective translators from that realm to the human realm?

8 Motion And Relativity

No discussion of time and space would be considered complete without some mention of motion and relativity. Planck was totally enamored with the motion of objects and waves and the concept of action. Einstein became king of the physics heap by his proposals concerning relativity. How do these concepts and mathematics play out with the view here of time and space as continuums?

First repeating what was stated in Section 2, time and space do not move or pass. Only objects or waveforms move and pass across these backgrounds and in reference to each other. Only objects or waveforms have durations or locations in reference to each other. Without the presence of other external forms as references or observers what becomes important is the inherent internal senses of time and space that a form may have. Using this internal sense of "self" then objects or waveforms can look out or sense the influence of other forms upon them. The internal sense always comes first.

How does this play out with real objects moving at high speeds as viewed by external observers? First the photons in free space always move at one speed. They are waveforms which are inherently unbounded in one mathematical direction, that of their propagation. If they are fed or ingest more energy, then they just rotate faster about their center line of propagation. In effect their wave length shortens. This is not very interesting in terms of relativity.

Considering "objects" such as the leptons, though, fun things happen as they move faster and faster in a linear sense. First an electron at rest can sense the whole world around itself. As the electron begins to move and speeds up, then it cannot focus so well on other objects external to itself. Its sense of time and space have shifted to a more internal focus. Finally when an "object" such as an electron reaches a very high velocity, say that of its maximum capability, then it effectively becomes blind. It cannot see the world outside of itself except as a blur. At this state the predominant sense that an electron has is that of itself. That is of course until it runs into something.

A graphic analogy can be made here to the final scenes in the movie"2001: A Space Odyssey", directed and produced by Stanley Kubrick. As the landing vessel settles towards the "portal" everything seems fine. As the vessel accelerates and flies ever faster, then the movie depicts its surroundings as if it were flying between two colored plates or down a tube which effectively becomes just smears of colored light. The landing vessel still is what it is, but the environment outside itself has gone nuts.

What has occurred here? Effectively the internal sense of time and space of the high speed form and external sense of time and space of an observing or reference form have become very divergent. So which is the true sense of time and space? Both are true simultaneously. The internal sense of time and space are true for the high speed form. Likewise the slower somewhat external sense of time and space against which the speeding form is judged are true for the observer form. This is because it is impossible for the observer to completely drop its own internal senses or references and to take on those of the moving form.

Time and space as continuums can accommodate an infinite number of perspectives or senses about themselves because they do not really participate in the moving madness of the forms. The perspective of each form, reference, or observer applies to itself and itself only. The continuums are unaffected by such projections.

9 Another View – A Disjointed One

There are other views of time and space, especially in the mathematic and scientific communities. Just because explanatory benefit was found in the case of this specific work, for the use of this multi-dimensional or a tiered view of time, this does not exclude the equal and simultaneous validity of other views of time.

For example El Naschie and his followers in their many published work use the highly disjointed mathematical Cantor sets to model physical reality. These models have a very discontinuous view of time and space at the subatomic particle level.

Upon making an internal investigation people may find that there is some appeal to this view. From a short internal examination most people find it easy to see how the mind is constantly jumping around. The mind jumps from one sensory input to another like a wild monkey. The mind jumps from the external physical, to a passing thought picture, to a disturbing emotion, and is constantly moving around. Like a multi-tasking CPU in a computer, the mind focuses briefly on one topic after the next.

The view of a highly disjointed world as a possibility could easily be agreed upon as a working model. Such an explanation of time might serve a useful purpose in some fields of physics investigations.

The initial description of both time and space presented in this report is that they are continuums. Can the highly disjointed view used by El Naschie and his followers as a model be reconciled with this original description? What of other equally seemingly contradictory views of time and space? Yes, El Naschie's view and others which may be equally bizarre or seemingly directly opposed to the original descriptions used in this report are in fact compatible or reconcilable.

The original nature of time and space as given in this report in Section 2 is that they are continuums, backgrounds, inherently having no form, structure, dimensionality, self-content, or other such descriptions. This is neither in physical reality nor in conceptuality. As such, time and space do not really care what mathematical models or conceptual projections humans lay over top of them. All such human narratives can be seen as just limiting subsets of the ultimate reality of time and space and do not really affect their inherent nature.

While this explanation might suffice at the conceptual level, how does this play out in terms of practical mathematics? The answer is the same. The views of El Naschie and his followers, or other such groups proposing odd or limiting views of time and space, are subsets of the unlimited realms that these continuums offer. As analogies in mathematics the continuous spectrum of numbers offered by the real number realm does not care if humans only choose to use integers or rational numbers. The real number realm is unaffected by such limited models and continues to be what it is. Likewise for example the irrational number pi, π, is what it is. It has an infinite number of digits past the decimal point. The pope declaring the value of π to be 3, or the legislature of the State of Ohio declaring π to have the value 3.14, as they have in fact done in the past, do not change π at all. That is the beauty of time and space as continuums. They can accept and be compatible with any and all models proposed about them.

While these discontinuous or disjointed views of time and space may be conceptually compatible with time and space as continuums, a person can ask the very valid and obvious questions of them, "What good are they". To put it bluntly, to anyone outside the realm of academic concrete high rise towers, such bizarre proclaminations about time and space appear to be intellectual masturbation, the sole purpose of which is for the chief king professor and his graduate student minions to make a name for themselves.

10 Summary

This report discussed several general conceptual views concerning time and space. The key word here is conceptual. Humans have imposed many different narratives upon these two features of the universe. While many of these assertions made by humans appear to be contradictory, the fact is that they can all be true simultaneously. This is because the inherent nature of time and space is that they are empty continuums. The mathematical stories made up by humans about them do not affect their inherent nature.

In the lepton report several assumptions about the mathematical nature of time and space were made. The concepts assumed there proved to be beneficial and helped lead to the mass density equations which were discovered for the leptons. Specifically what was found in the lepton work concerning time at first appeared not only to be novel and different but to also be counter intuitive. This was that for every spatial variable there was underlying it an independent temporal parameter. The total of three spatial dimensions necessary to describe the form of the leptons and their motion required three underlying temporal dimensions.

What has been seen in this report is that the concept of multiple dimensions, layers, levels, or tiers of time rather than being new, novel, or counter intuitive, it is in keeping with what people already know from their own internal experiences with time.

APPENDIX 3 EXPANSION COORDINATES

1 Introduction

Expansion coordinates or generalized n-spherical angular coordinates are necessary when considering work in more than 3 dimensions. The usual declination spherical coordinates do not set a pattern, can not be generalized, and are useless in more than 3 dimensions.

The reasons are simple. In basic algebra books when students are introduced to graphing, the independent variable is called X, the first variable and axis of the discussion. The dependent variable is represented by Y and the second axis and variable of discussion. Trigonometric angles are introduced by drawing an angle θ out away from this first axis X of discussion. Polar coordinates, 2-dimensional radial-angular coordinates, follow this same convention, simply renaming the X axis to be the polar line. When 3 dimensions are introduced Z is created as the third variable and axis of discussion. When shifting to spherical coordinates, 3-dimensional radial-angular coordinates, the second necessary angle is referred to this third Z axis or variable or discussion. An angle φ is dropped down, declined, from the traditional vertical axis to become angle 2. In 3 dimensions one has the first angle referred to the first rectilinear axis and the second angle referred to the third rectilinear axis. The second axis of discussion is left out as an angular reference. This declination system had legitimate historical roots in the navigation of sailing ships, but as seen has created a long enduring mathematical anomaly.

The problem when continuing on to more dimensions immediately becomes obvious. For n-dimensional spherical coordinates, there is always a radius, the first dimension, and n-1 angles and angular dimensions. On referring back to n-dimensional rectilinear coordinates to obtain reference axis or lines, there are always one less angle than there are dimensions or reference axis-lines. The question only needs to asked, "Which dimensional axis or variable of discussion gets left out". With the 3-dimensional declination system it is the Y or number 2 axis. In 4, 5, or 6 dimensions which axis gets left out; Y number 2, Z number 3, Number 4, Number 5, or Number 6? Always the second of discussion Y? Always the next to last, in the case of 6 dimensions, Number 5? The system breaks down and cannot be generalized. There needs to be a system which can be immediately generalized as more dimensions are added to a discussion without the disruption of formulas and patterns already established.

Stated in trigonometric terms students learn that for polar-angular coordinates

$$X = r\cos(\theta) \text{ and } Y = r\sin(\theta) \qquad (01)$$

Students again simply memorize by brute force that for spherical-angular coordinates

$$X = \rho\cos(\theta)\sin(\varphi),\quad Y = \rho\sin(\theta)\sin(\varphi),\ \text{ and }\ Z = \rho\cos(\varphi) \qquad (02)$$

The question is never asked, does this create a pattern which can be followed into 4, 5, and 6 dimensions?

This appendix-report discussed a possible means of creating such a generalized coordinate system. A system of n-dimensional radial-angular coordinates which can be generalized any time that more dimensions are needed is that of expansion coordinates. That is because this system starts from the "inside" and builds outward. It starts from the already known or given axis of discussion and works toward any new ones added. This system never refers to the most recent axis added, and never caps off the discussion. By not referring to the last axis added, this system allows that maybe it won't be the last, that maybe more dimensions could be added later if needed.

2 Expansion Coordinates Construction & Mathematical Properties

Expansion Coordinates to describe the position of a point in n-space are constructed as follows, after a radius is known or given.

1. Go out the radial line, axis #1 usually the X axis, to the position of the ultimate projection of the n-dimensional point on the axis.
2. Erect (expand) a perpendicular into the 2nd dimensional space from this radial line, parallel to the axis #2, to the projection of the n-dimensional point on this newly created plane or 2-dimensional space.
3. Now angle #1 defines the position of this projected point in this 1,2-dimensional space. This is nothing more than a fancy way to say, the first angle is constructed in its usual manner.
4. Erect (expand) a perpendicular into the 3rd dimensional space from the previous perpendicular, parallel to axis #3, to the projection of the n-dimensional point in this newly created volume or 3-dimensional space.
5. Now angle #2 from the origin defines the position of this projected point in this 1,2,3-dimensional space. This is where expansion coordinates really start and differ from what is usually a declination angle definition for angle #2.
6. Continue is this fashion of erecting (expanding) perpendiculars from known, already defined, projection locations into the next new dimension of discussion.
7. Ultimately the last expansion angle, angle number n-1, is the 90^0 complement of the nth direction angle. These direction angles are defined in the usual declination sense used to refer to vectors in calculus books.

Tables 1-4 show many of the common properties, such as dS^2 and dVol, used in mathematical work for the 1st thru 6th dimensional expansion coordinate systems.

Why the extension to so many dimensions when the leptons are only 2-dimensional figures revolving into the third dimension with time. First, this simplistic nature of the leptons was not known at the start of the project. Initially 3 and 4-dimensional forms were investigated. Secondly and more importantly now, these extensions to 4 dimensions appear to be necessary to resolve the structural nature of the quarks. Who knows what next?

Table 1 Expansion Coordinates Part 1

Basic Definitions			
R	=	Ln, The radius, the total length, the last length, or the final length	
	=	$(L_1^2+L_2^2+L_3^2+L_4^2+\cdots)^{1/2}$	
L_i	=	Length (of a projection) on a rectalinear axis	
	=	$(L_i^2)^{1/2}$	$[R^2-(L_j^2+L_k^2+L_l^2+\cdots)]^{1/2}$
L_{ij}	=	Length of a projection on a 2dimensional plane	
	=	$(L_i^2+L_j^2)^{1/2}$	$[R^2-(L_k^2+L_l^2+L_m^2+\cdots)]^{1/2}$
L_{ijk}	=	Length of a projection in a 3 dimensional space	
	=	$(L_i^2+L_j^2+L_k^2)^{1/2}$	$[R^2-(L_l^2+L_m^2+L_n^2+\cdots)]^{1/2}$

Direction Angle (a declination angle from an axis)	a_i	=	$\mathrm{acos}(L_i/L_n)$
Projection Angle	a_P(i to ijk)	=	$\mathrm{acos}(L_i/L_{ijk})$
Expansion Angle	a_E(ijk...m to n)	=	$\mathrm{acos}(L_{ijk\ldots m}/L_{ijk\ldots n})$

Table 2 Expansion Coordinates Part 2

Dim	nD Rectilinear Cord = F(nD Spherical Cord)			nD Spherical Cord = G(nD Rectalinear Cord)		
1	X	=	R	R^2	=	X^2
2	X	=	R $\cos(A_1)$	R^2	=	$X^2 + Y^2$
	Y	=	R $\sin(A_1)$	A_1	=	$\text{atan}[Y/(X^2)^{1/2}]$
3	X	=	R $\cos(A_1)\cos(A_2)$	R^2	=	$X^2 + Y^2 + Z^2$
	Y	=	R $\sin(A_1)\cos(A_2)$	A_1	=	$\text{atan}[Y/(X^2)^{1/2}]$
	Z	=	R $\sin(A_2)$	A_2	=	$\text{atan}[Z/(X^2 + Y^2)^{1/2}]$
4	X	=	R $\cos(A_1)\cos(A_2)\cos(A_3)$	R^2	=	$X^2 + Y^2 + Z^2 + W^2$
	Y	=	R $\sin(A_1)\cos(A_2)\cos(A_3)$	A_1	=	$\text{atan}[Y/(X^2)^{1/2}]$
	Z	=	R $\sin(A_2)\cos(A_3)$	A_2	=	$\text{atan}[Z/(X^2 + Y^2)^{1/2}]$
	W	=	R $\sin(A_3)$	A_3	=	$\text{atan}[W/(X^2 + Y^2 + Z^2)^{1/2}]$
5	X	=	R $\cos(A_1)\cos(A_2)\cos(A_3)\cos(A_4)$	R^2	=	$X^2 + Y^2 + Z^2 + W^2 + V^2$
	Y	=	R $\sin(A_1)\cos(A_2)\cos(A_3)\cos(A_4)$	A_1	=	$\text{atan}[Y/(X^2)^{1/2}]$
	Z	=	R $\sin(A_2)\cos(A_3)\cos(A_4)$	A_2	=	$\text{atan}[Z/(X^2 + Y^2)^{1/2}]$
	W	=	R $\sin(A_3)\cos(A_4)$	A_3	=	$\text{atan}[W/(X^2 + Y^2 + Z^2)^{1/2}]$
	V	=	R $\sin(A_4)$	A_4	=	$\text{atan}[V/(X^2 + Y^2 + Z^2 + W^2)^{1/2}]$
6	X	=	R $\cos(A_1)\cos(A_2)\cos(A_3)\cos(A_4)\cos(A_5)$	R^2	=	$X^2 + Y^2 + Z^2 + W^2 + V^2 + U^2$
	Y	=	R $\sin(A_1)\cos(A_2)\cos(A_3)\cos(A_4)\cos(A_5)$	A_1	=	$\text{atan}[Y/(X^2)^{1/2}]$
	Z	=	R $\sin(A_2)\cos(A_3)\cos(A_4)\cos(A_5)$	A_2	=	$\text{atan}[Z/(X^2 + Y^2)^{1/2}]$
	W	=	R $\sin(A_3)\cos(A_4)\cos(A_5)$	A_3	=	$\text{atan}[W/(X^2 + Y^2 + Z^2)^{1/2}]$
	V	=	R $\sin(A_4)\cos(A_5)$	A_4	=	$\text{atan}[V/(X^2 + Y^2 + Z^2 + W^2)^{1/2}]$
	U	=	R $\sin(A_5)$	A_5	=	$\text{atan}[U/(X^2 + Y^2 + Z^2 + W^2 + V^2)^{1/2}]$

Table 3 Expansion Coordinates Part 3

nD Rectalinear Cord			nD Spherical Cord		
			1 dimension		
length	=	$2^1/(1)\ \pi^0 r^1$ if figure sym about origin	surface, endpoints	=	$2\pi^0 r^0$
dLength	=	dx	dLength	=	dr
length as single ∫	=	$\text{const}\int F(x)^0 dx$	length	=	$2\pi^0 \int dr$, if fig sym about origin
			2 dimensions		
area, circle	=	$1/1!\ \pi^1 r^2$	surface, perimeter	=	$2\pi^1 r^1$
dArea	=	dx dy	dArea	=	$r\ dr\ da_1$
area as single ∫ in dx	=	$\text{const}\int F(x)^1 dx$	area as single ∫in dr	=	$2\pi^1 \int F(r) r^1 dr$
		const = 2 if fig sym about origin	area as single ∫in da_1	=	$1/2\pi^0 \int F^2(a_1) da_1$
			3 dimensions		
volume, sphere	=	$2^2/(1x3)\ \pi^1 r^3$	surface, shell	=	$4\pi^1 r^2$
dVolume	=	dx dy dz	dVolume	=	$r^2 \cos(a_2)\ dr\ da_1\ da_2$
vol as single ∫ in dx	=	$\text{const}\int F(x)^2 dx$	vol as single ∫in dr	=	$4\pi^1 \int F(r) r^2 dr$
		const is for symmetry about origin	vol as single ∫in da_1	=	$2/3\pi^0 \int F^3(a_1) da_1$
			4 dimensions		
volume, "sphere"	=	$1/2!\ \pi^2 r^4$	surface, shell	=	$2\pi^2 r^3$
dV	=	dx dy dz dw	dV	=	$r^3 \cos(a_2) \cos^2(a_3)\ dr\ da_1\ da_2\ da_3$
vol as single ∫ in dx	=	$\text{const}\int F(x)^3 dx$	vol as single ∫in dr	=	$2\pi^2 \int F(r) r^3 dr$
		const is for symmetry about origin	vol as single ∫in da_1	=	$1/4\pi^1 \int F^4(a_1) da_1$
			5 dimensions		
volume, "sphere"	=	$2^3/(1x3x5)\ \pi^2 r^5$	surface, shell	=	$8/3\ \pi^2 r^4$
dV	=	dx dy dz dw dv	dV	=	$r^4 \cos(a_2) \cos^2(a_3) \cos^3(a_4)\ dr\ da_1\ da_2\ da_3\ da_4$
vol as single ∫ in dx	=	$\text{const}\int F(x)^4 dx$	vol as single ∫ in dr	=	$8/3\ \pi^2 \int F(r) r^4 dr$
		const is for symmetry about origin	vol as single ∫ in da_1	=	$4/15\pi^1 \int F^5(a_1) da_1$
			6 dimensions		
volume, "sphere"	=	$1/3!\ \pi^3 r^6$	surface, shell	=	$1\pi^3 r^5$
dV	=	dx dy dz dw dv du	dV	=	$r^5 \cos(a_2) \cos^2(a_3) \cos^3(a_4) \cos^4(a_5)\ dr\ da_1\ da_2\ da_3\ da_4\ da_5$
vol as single ∫ in dx	=	$\text{const}\int F(x)^5 dx$	vol as single ∫in dr	=	$1\pi^3 \int F(r) r^5 dr$
		const is for symmetry about origin	vol as single ∫in da_1	=	$1/2\pi^2 \int F^6(a_1) da_1$

Table 4 Expansion Coordinates Part 4

$dS^2 =$	$\nabla^2 =$
1 dimension	
$(dr)^2$	$\partial^2(F) / \partial r^2$
2 dimensions	
$(dr)^2$	$1 / r \, \partial / \partial r \, [r \, \partial(F) / \partial r]$
$+ r^2 (da_1)^2$	$+ 1 / [r^2] \; [\partial^2(F) / (\partial a_1)^2]$
3 dimensions	
$(dr)^2$	$1 / r^2 \, \partial / \partial r \, [r^2 \, \partial(F) / \partial r]$
$+ r^2 \cos^2(a_2) (da_1)^2$	$+ 1 / [r^2 \cos^2(a_2)] \; [\partial^2(F) / (\partial a_1)^2]$
$+ r^2 (da_2)^2$	$+ 1 / [r^2 \cos(a_2)] \, \partial / \partial a_2 \, [\cos(a_2) \, \partial(F) / \partial a_2]$
4 dimensions	
$(dr)^2$	$1 / r^3 \, \partial / \partial r \, [r^3 \, \partial(F) / \partial r]$
$+ r^2 \cos^2(a_2) \cos^2(a_3) (da_1)^2$	$+ 1 / [r^2 \cos^2(a_2) \cos^2(a_3)] \; [\partial^2(F) / (\partial a_1)^2]$
$+ r^2 \cos^2(a_3) (da_2)^2$	$+ 1 / [r^2 \cos(a_2) \cos^2(a_3)] \, \partial / \partial a_2 \, [\cos(a_2) \, \partial(F) / \partial a_2]$
$+ r^2 (da_3)^2$	$+ 1 / [r^2 \cos^2(a_3)] \, \partial / \partial a_3 \, [\cos^2(a_3) \, \partial(F) / \partial a_3]$
5 dimensions	
$(dr)^2$	$1 / r^4 \, \partial / \partial r \, [r^4 \, \partial(F) / \partial r]$
$+ r^2 \cos^2(a_2) \cos^2(a_3) \cos^2(a_4) (da_1)^2$	$+ 1 / [r^2 \cos^2(a_2) \cos^2(a_3) \cos^2(a_4)] \; [\partial^2(F) / (\partial a_1)^2]$
$+ r^2 \cos^2(a_3) \cos^2(a_4) (da_2)^2$	$+ 1 / [r^2 \cos(a_2) \cos^2(a_3) \cos^2(a_4)] \, \partial / \partial a_2 \, [\cos(a_2) \, \partial(F) / \partial a_2]$
$+ r^2 \cos^2(a_4) \; (da_3)^2$	$+ 1 / [r^2 \cos^2(a_3) \cos^2(a_4)] \, \partial / \partial a_3 \, [\cos^2(a_3) \, \partial(F) / \partial a_3]$
$+ r^2 (da_4)^2$	$+ 1 / [r^2 \cos^3(a_4)] \, \partial / \partial a_4 \, [\cos^3(a_4) \, \partial(F) / \partial a_4]$
6 dimensions	
$(dr)^2$	$1 / r^5 \, \partial / \partial r \, [r^5 \, \partial(F) / \partial r]$
$+ r^2 \cos^2(a_2) \cos^2(a_3) \cos^2(a_4) \cos^2(a_5) (da_1)^2$	$+ 1 / [r^2 \cos^2(a_2) \cos^2(a_3) \cos^2(a_4) \cos^2(a_5)] \; [\partial^2(F) / (\partial a_1)^2]$
$+ r^2 \cos^2(a_3) \cos^2(a_4) \cos^2(a_5) (da_2)^2$	$+ 1 / [r^2 \cos(a_2) \cos^2(a_3) \cos^2(a_4) \cos^2(a_5)] \, \partial / \partial a_2 \, [\cos(a_2) \, \partial(F) / \partial a_2]$
$+ r^2 \cos^2(a_4) \cos^2(a_5) (da_3)^2$	$+ 1 / [r^2 \cos^2(a_3) \cos^2(a_4) \cos^2(a_5)] \, \partial / \partial a_3 \, [\cos^2(a_3) \, \partial(F) / \partial a_3]$
$+ r^2 \cos^2(a_5) (da_4)^2$	$+ 1 / [r^2 \cos^3(a_4) \cos^2(a_5)] \, \partial / \partial a_4 \, [\cos^3(a_4) \, \partial(F) / \partial a_4]$
$+ r^2 (da_5)^2$	$+ 1 / [r^2 \cos^4(a_5)] \, \partial / \partial a_5 \, [\cos^4(a_5) \, \partial(F) / \partial a_5]$

APPENDIX 4 CYLINDRICAL CURVES

1 Introduction

Cylindrical curves were first discussed in Part 1 Chapter 1.1, the lepton paper. There the groundwork was laid for the application or use of some of the vector properties of these mathematical forms in explaining the origin of the value of the elementary charge of the leptons, e = 1.602,177,33 x 10^{-19}C. The purpose of this appendix is to rigorously develop some of the results used in Chapter 1.1, Sections 3 and 4.1. In this appendix-report in Sections 3 and 4, a step-by-step derivation of the two vector quantities curvature κ and torsion τ is given.

Various aspects of cylindrical curves were discussed in Chapter 1.1, Section 3 but these were only noted because of their direct relevance to the work there. A few other comments about these curves are worth noting.

The specific 3D curve first discussed below is known by many names, most notably as the cylindrical helix or the cylindrical spiral. Presentations of its vector mathematics are found in most calculus texts because the relative simplicity of these vector forms allows for straight forwards examples for students. Solid physical representations of this curve have been known since early human times. The Archimedies Screw has been the best known practical application of the solid version of this curve. It is still used today in poor countries for transferring water into irrigation ditches. A modern day outline matching this curve is the toy, the Slinky.

Much of the work in the lepton paper bases off of properties of the generalized cylindrical curve.

$$\mathbf{R}(t) = a\cos[F(t)]\,\mathbf{i} + a\sin[F(t)]\,\mathbf{j} + bG(t)\,\mathbf{k}$$

In particular the derivation of, and final mathematical formulas for, the curvature κ and the torsion τ start with this form. The importance of these two differential geometry quantities, both for the specific case of the cylindrical helix and for the generalized curve, are that;

1 Both quantities are free from or independent of the original transcendental trigonometric functions.
2 Both are independent of the original functions of F(t) and G(t).
3 Both are independent of the free standing original implicit variable t.
4 As such both are numerical constants.
5 Both by their calculational definitions are scalar quantities and not still vectors. As such they are free from the unit vectors **i**, **j**, **k**.

The function G(t) shown for the generalized cylindrical curve, ultimately assigned as being synonymous with F(t), is completely general. Further it does not need to explicitly stated anywhere before, during, or after the derivation shown, or even known for that matter. Any real physical energetic wave forms which may be described by such generalized cylindrical curve could possibly have identical curvatures or torsions but very different embedded appearances in time. These appearances could be polynomial series, exponentials, trigonometric functions, et cetera and still to an outside world of humans making measurements of the effects of their curvature, they would appear identical.

An important side note for 2-dimensional curves is the concept of the circle of curvature, defined as; the circle which is tangent to the curve at a specific chosen point and which also has the same curvature as the vector curve of discussion. This is also called the osculating circle. The importance of this concept for this work is that the radius of this circle, also called the radius of curvature, is mathematically equal to the reciprocal of the absolute value of the curvature κ for the 2-dimensional curve. While a formalized definition for the 3 dimensional analogy appears not to have been made, such an osculating sphere can be conceived. The radius of curvature for such a sphere for this generalized cylindrical curve, like the curvature κ, would be identical for many different functions of the implicit variable t.

2 General 3D Space Curve

Define a generalized 3D space curve of the implicit variables in time

X = F(t), Y = G(t), Z = H(t)

or in vector notation

R(t) = F(t) **i** + G(t) **j** + H(t) **k** 001

For such a vector the curvature and torsion are rigorously calculated by the following definitions.

curvature $\kappa = \frac{|\mathbf{R}'(t) \times \mathbf{R}''(t)|}{|R'(t)|^3}$ 002

torsion $\tau = \frac{[\mathbf{R}'(t) \times \mathbf{R}''(t) \bullet R'''(t)]}{|\mathbf{R}'(t) \times \mathbf{R}''(t)|^2}$ 003

Chosing a specific simple cylindrical spiral as an example

R(t) = + a cos(t) **i** + a sin(t) **j** + b(t) **k** 004
R'(t) = - a sin(t) **i** + a cos(t) **j** + b **k** 005
R''(t) = - a cos(t) **i** - a sin(t) **j** + 0 **k** 006
R'''(t) = + a sin(t) **i** - a cos(t) **j** + 0 **k** 007

The curvature κ and torsion τ are found to be

$\kappa = \frac{a}{a^2+b^2}$ 008

$\tau = \frac{b}{a^2+b^2}$ 009

The steps from R(t), R'(t), R''(t), and R'''(t) to get to κ and τ are found in almost all calculus texts.

3 Specific Cylindrical Curve Of Concern For This Work

3.1 Preliminaries, Derivatives Of R(t) The Trial Form

The objective here is to show that the above generic simplistic formulas for κ and τ, Equations (08-09) hold for a more generalized 3-dimensional cylindrical "spiral" or curve as below

Note that individual TERMS of the equations are numbered throughout the remainder of this presentation, NOT the complete equations. Gaps are intentionally left in the numbering for the ease of logical expansion or additions later.

R(t) = +a [2 terms} + b {1 term}
a{+1 cos(F(t)) **i** } 011
a{+1 sin(G(t)) **j** } 012
+1 b H(t) **k** 013

Here in R(t) the **i** and **j** vectors describe the generalized 2-dimensional form and the **k** vector describes the curve's motion into the 3rd spatial dimension. The important distinction between this form of **R**(t) and that of Equation (04) above is that here the simplistic linear t of (04) has now become functions of t. Further these functions are completely general and more importantly they are as yet unspecified or unknown and can remain that way.

R'(t) = +a { 2 terms} +b {1 term}
a{- 1 F'(t) sin(F(t)) **i** } 021
a{+1G'(t) cos(G(t)) **j** } 022
+1 b H'(t) **k** 023

R''(t) = +a { 4 terms} +b {1 term}
terms (031) and (032) are from (021)
a{[-1 $(F'(t))^2$ cos(F(t)) 031
-1 F''(t) sin(F(t))] **i** } end **i** vector 2 terms 032
terms (033) and (034) are from (022)
a{[-1 $(G'(t))^2$ sin(G(t)) 033
+1 G''(t) cos(G(t))] **j** } end **j** vector 2 terms 034
term (035) is from (023)
+1 b H''(t) **k** end **k** vector 1 term 035

R'''(t) = +a { 6 terms} + b {1 term}
terms (041) and (042) are from (031)
a{[-1 $(F'(t)^3$ sin(F(t)) 041
-2 F'(t) F''(t) cos(F(t)) combine with 043 042
terms (043) and (044) are from (032)
-1 F'(t) F''(t) cos(F(t)) combine with 042 043
-1 F'''(t) sin(F(t)) 044
(042) + (043) = -3 F'(t) F''(t) cos(F(t))] **i** } end **i** vector 3 terms 045
terms (046) and (047) are from (033)
a{[-1 $(G'(t))^3$ cos(G(t)) 046
-2 G'(t) G''(t) sin(G(t)) combine with 048 047
terms (048) and (049) are from (034)
-1 G'(t) G''(t) sin(G(t)) combine with 047 048
+1 F'''(t) cos(G(t)) 049
(047) + (048) = -3 G'(t) G''(t) sin(G(t))] **j** } end **j** vector 3 terms 050
term (051) is from (035)
+1b H'''(t) **k** end **k** vector 1 term 051

│**R**'(t)│= + $\{ \text{ 3 terms } \}^{1/2}$
From i^{th} component
a^2 {+1 $(F'(t))^2$ $\sin^2(F(t))$ } 061
From j^{th} component
a^2 {+1 $(G'(t))^2$ $\cos^2(G(t))$ } 062
From k^{th} component
+1 b^2 $(H'(t))^2$ 063

3.2 Reduction Of │R'(t)│

This derivation-presentation has been totally general up to this point. Now to continue, a first simplifying assumption is made.
Assumption 1: Assume G(t) = F(t)

From i^{th} and j^{th} component which went into $|\mathbf{R}'(t)|$
$(061) + (062) = a^2(F'(t))^2$ 064
This gives
$|\mathbf{R}'(t)| = [a^2 (F'(t))^2 + b^2 (H'(t))^2]^{1/2}$ 065
Finishing the final reduction of $|\mathbf{R}'(t)|$ a second assumption is needed.
Assumption 2: Assume H'(t) = F'(t)
This yields the final form of
$|\mathbf{R}'(t)| = F'(t) [a^2 + b^2]^{1/2}$ 066
Of course
$|\mathbf{R}'(t)|^3 = (F'(t))^3 [a^2 + b^2]^{3/2}$ 067

4 Determination of Curvature and Torsion of the Trial Form by Dropping the Form into a Determinant Grid

4.1 Determination Of [R'(t) X R''(t)]

At this point to continue with [**R**'(t) X **R**''(t)], $|\mathbf{R}'(t) \times \mathbf{R}''(t)|$, $|\mathbf{R}'(t) \times \mathbf{R}''(t)|^2$, and [**R**'(t) X **R**''(t) • **R**'''(t)] the first assumption G(t) = F(t) is used throughout, but the second assumption H'(t) = F'(t) is not used until the last steps where it is necessary to show the final reduced forms of the curvature κ and the torsion τ.

First reducing or respecifying more narrowly the expressions that are already available. Again the TERMS within the equations are numbered NOT the overall equations.

R(t) = +a { 2 terms} +b {1 term}
a{+1 cos(F(t)) **i** 111
+ 1 sin(F(t)) **j** } 112
+1b H(t) **k** 113

R'(t) = +a { 2 terms} +b { 1 term}
a{ -1 F'(t) sin(F(t)) **i** 121
+ 1 F'(t) cos(F(t)) **j** } 122
+1 b H'(t) **k** 123

R''(t) = +a { 4 terms} +b { 1 term}
a{[-1 $(F'(t))^2$ cos(F(t) 131
-1 F''(t) sin(F(t))] **i** 132
[-1 $(F'(t))^2$ sin(F(t)) 133
+1 F''(t) cos(F(t))] **j** } 134
+1 b H''(t) **k** 135

R'''(t) = +a { 6 terms} +b { 1 term}
a{[+1 $(F'(t))^3$ sin(F(t)) 141
-1 F'''(t) sin(F(t)) 142
-3 F'(t) F''(t) cos(F(t))] **i** 143
[-1 $(F'(t))^3$ cos(F(t) 144
+1 F'''(t) cos(F(t)) 145
-3 F'(t) F''(t) sin(F(t))] **j** } 146
+1b H'''(t) **k** 147

Given two 3-dimensional vectors **A** and **B** of the form,

$\mathbf{A} = a_1\mathbf{i} + a_2\mathbf{j} + a_3\mathbf{k}$ 150
$\mathbf{B} = b_1\mathbf{i} + b_2\mathbf{j} + b_3\mathbf{k}$ 151
then using the usual mechanistic procedure specified in calculus and linear algebra texts the following expressions result for the operation defined as a 3-dimensional vector cross product ×.
$\mathbf{A} \times \mathbf{B} =$
$+[+a_2b_3 - a_3b_2]\ \mathbf{i}$ 152
$- [+ a_1b_3 - a_3b_1]\ \mathbf{j} = + [- a_1b_3 + a_3b_1]\ \mathbf{j}$ 153
$+[+ a_1b_2 - a_2b_1]\ \mathbf{k}$ 154

Using this procedural definition,
[**R**'(t) X **R**''(t)] **i** component = ab x all the following terms
From + [**R**'(t) **j** component x **R**''(t) **k** component]
[+ 1 F'(t) cos(F(t))
x [+1 H''(t)]
From – [**R**''(t) **j** component x **R**'(t) **k** component]
- [-1 $(F'(t))^2$ sin(F(t))
+1 F''(t) cos(F(t))]
x [+1 H'(t)]
These give
[+1 F'(t) H''(t) cos(F(t)] 201
[+1 $(F'(t)^2$ H'(t) sin(F(t) 202
-1 F''(t) H'(t) cos(F(t))] 203

[**R**'(t) X **R**''(t)] **j** component = ab x all the following terms
From - [**R**'(t) **i** component x **R**''(t) **k** component]
- [-1 F'(t) sin(F(t))]
x [+1 H''(t)]
From + [**R**''(t) **i** component x **R**'(t) **k** component]
[-1 $(F'(t))^2$ cos(F(t)
-1 F''(t) sin(F(t))]
x [+1 H'(t)]
These give
[+ 1 F'(t) H''(t) sin(F(t))] 211
[-1 $(F'(t))^2$ H'(t) cos(F(t)) 212
-1 F''(t) H'(t) sin(F(t))] 213

[**R**'(t) X **R**''(t)] **k** component = a^2 x all the following terms
From + [**R**'(t) **i** component x **R**''(t) **j** component]
[-1 F'(t) sin(F(t))]
x [-1 $(F'(t))^2$ sin(F(t))
+1 F''(t) cos(F(t))]
From – [**R**''(t) **i** component x **R**'(t) **j** component]
- [[-1 $(F'(t))^2$ cos(F(t)
-1 F''(t) sin(F(t))]
x [+ 1 F'(t) cos(F(t))]
These give
[+1 $(F'(t))^3$ $\sin^2$(F(t) combine with 223 221
-1 F'(t) F''(t) sin((F(t) cos(F(t))] null with 224 222

$[\ +1\ (F'(t))^3 \cos^2(F(t)$	combine with 221	223
$+1\ F'(t)\ F''(t) \sin((F(t) \cos(F(t))\ ]$	null with 222	224
$(221) + (223) = +1\ (F'(t))^3$		225

Final Result is

$[\mathbf{R}'(t) \times \mathbf{R}''(t)]$ **i** component = ab x 3 terms

$[\mathbf{R}'(t) \times \mathbf{R}''(t)]$ **j** component = ab x 3 terms

$[\mathbf{R}'(t) \times \mathbf{R}''(t)]$ **k** component = a^2 x 1 term

4.2 Determination Of $|\mathbf{R}'(t) \times \mathbf{R}''(t)|$ And Curvature κ

Opening of $|\mathbf{R}'(t) \times \mathbf{R}''(t)| = \{\ 13 \text{ terms}\ \}^{1/2}$

From **i** component, squared terms, a^2b^2 x following

$+1\ (F'(t))^2\ (H''(t))^2 \cos^2(F(t))$	combine with 311	301
$+1\ (F'(t))^4\ (H'(t))^2 \sin^2(F(t))$	combine with 312	302
$+1\ (F''(t))^2\ (H'(t))^2 \cos^2(F(t))$	combine with 313	303

From **i** component, cross product terms, a^2b^2 x following

$+2\ (F'(t))^3\ H'(t)\ H''(t) \sin(F(t)) \cos(F(t))$	null with 314	304
$-2\ F'(t)\ F''(t)\ H'(t)\ H''(t) \cos^2(F(t))$	combine with 315	305
$-2\ (F'(t))^2\ F''(t)\ (H'(t))^2 \sin(F(t)) \cos(F(t))$	null with 316	306

From **j** component, squared terms, a^2b^2 x following

$+1\ (F'(t))^2\ (H''(t))^2 \sin^2(F(t))$	combine with 301	311
$+1\ (F'(t))^4\ (H'(t))^2 \cos^2(F(t))$	combine with 302	312
$+1\ (F''(t))^2\ (H'(t))^2 \sin^2(F(t))$	combine with 303	313

From **j** component, cross product terms, a^2b^2 x following

$-2\ (F'(t))^3\ H'(t)\ H''(t) \sin(F(t)) \cos(F(t))$	null with 304	314
$-2\ F'(t)\ F''(t)\ H'(t)\ H''(t) \sin^2(F(t))$	combine with 305	315
$+2\ (F'(t))^2\ F''(t)\ (H'(t))^2 \sin(F(t)) \cos(F(t))$	null with 306	316

From **k** component, squared terms

$+1\ a^4(F'(t))^6$		321

From **k** component, cross product terms, none

Initial simplification of $|\mathbf{R}'(t) \times \mathbf{R}''(t)| = \{\ 13 \text{ terms}\}^{1/2}$ by combining like terms and using the identity $\sin^2(F(t)) + \cos^2(F(t)) = 1$ gives;

$(301) + (311) =$	$+1\ a^2b^2\ (F'(t))^2\ (H''(t))^2$	331
$(302) + (312) =$	$+1\ a^2b^2\ (F'(t))^4\ (H'(t))^2$	332
$(303) + (313) =$	$+1\ a^2b^2\ (F''(t))^2\ (H'(t))^2$	333
$(305) + (315) =$	$-2\ a^2b^2\ F'(t)\ F''(t)\ H'(t)\ H''(t)$	334

and carrying down

$+1\ a^4(F'(t))^6$		335

Removing $a^2(F'(t))^4$ from all terms to outside, in front, of the $\{$ remaining 5 terms$\}^{1/2}$ gives

$|\mathbf{R}'(t) \times \mathbf{R}''(t)| = a^2(F'(t))^2\ \{\ \text{remaining 5 terms}\}^{1/2}$

The remaining 5 terms, now modified, become

$(331) \rightarrow\ +1\ b^2\ (H'(t))^2 / (F'(t))^2$	341
$(332) \rightarrow\ +1\ b^2\ (H'(t))^2$	342
$(333) \rightarrow\ +1\ b^2\ (F''(t))^2\ (H'(t))^2 / (F'(t))^4$	343
$(334) \rightarrow\ -2\ b^2\ F''(t)\ H'(t)\ H''(t) / (F'(t))^3$	344
$(335) \rightarrow +1\ a^2(F'(t))^2$	345

Grouping the terms still remaining in the {} braces there are

$(345) + (342) \rightarrow [\, +1\, a^2(F'(t))^2 +1\, b^2\, (H'(t))^2\,]$ 350

$(341) + (344) + (343) \rightarrow b^2\, [\, +1\, (H'(t))^2 / (F'(t))^2$ 351

$-2\, F''(t)\, H'(t)\, H''(t) / (F'(t))^3$

$+1\, (F''(t))^2\, (H'(t))^2 / (F'(t))^4\,]$

The collective (351) can now be seen as either

$b^2\, [\, (H''(t))^1 / (F'(t))^1 - (F''(t))^1\, (H'(t))^1 / (F'(t))^2\,]^2$ 352

or equally

$b^2\, [\, - (H''(t))^1 / (F'(t))^1 + (F''(t))^1\, (H'(t))^1 / (F'(t))^2\,]^2$ 353

This composite term (351) expressed as either (352) or (353) is obviously irreducible, and contains undesirable mixes of the derivatives of both F(t) and H(t); F'(t), F''(t), H'(t), H''(t)

To finish the final reduction of $|\mathbf{R}'(t) \times \mathbf{R}''(t)|$ the second assumption is now needed.

Assumption 2: Assume H'(t) = F'(t)

Immediately then H''(t) = F''(t)

This then means the composite term (351) or (352) or (353) vanishes to 0, and the 3 terms (341), (343), and (344) from which these arose likewise disappear.

The other composite term (350) now reduces to

$(350) \rightarrow \{(F'(t))^2\, [\, a^2 + b^2]\}$ 354

So the final net resulting form of $|\mathbf{R}'(t) \times \mathbf{R}''(t)| = \{a^2(F'(t))^4 \times (354)\}^{1/2}$ is

$|\mathbf{R}'(t) \times \mathbf{R}''(t)| = a^1(F'(t))^3\, [a^2 + b^2]^{1/2}$ 355

Therefore

$|\mathbf{R}'(t) \times \mathbf{R}''(t)|^2 = a^2(F'(t))^6\, [a^2 + b^2]^1$ 356

The final resulting form of $|\mathbf{R}'(t)|$ is Equation (066), repeated here.

$|\mathbf{R}'(t)| = F'(t)\, [\, a^2 + b^2]^{1/2}$ 066

Therefore

$|\mathbf{R}'(t)|^3 = (F'(t))^3\, [\, a^2 + b^2]^{3/2}$ 067

Returning to the first objective, which is the determination of the curvature κ. Repeating Equation 002.

$$\text{curvature } \kappa = \frac{|\mathbf{R}'(t) \times \mathbf{R}''(t)|}{|\mathrm{R}'(t)|^3} \qquad 002$$

The final result is now seen,

$$\frac{|\mathbf{R}'(t) \times \mathbf{R}''(t)|}{|\mathbf{R}'(t)|^3} = \frac{a^1(F'(t_1))^3[a^2+b^2]^{1/2}}{(F'(t_1))^3[a^2+b^2\,]^{3/2}} = \frac{a}{[a^2+b^2]} \qquad 360$$

4.3 Determination Of $|\mathbf{R}'(t) \times \mathbf{R}''(t)|^2$, $[\mathbf{R}'(t) \times \mathbf{R}''(t) \bullet \mathbf{R}'''(t)]$ And Torsion τ

Both assumption1, G(t) = F(t) and assumption 2, H'(t) = F'(t) are retained in force to make the algebra simple.

R'''(t) has the seven terms shown in (141) thru (147)

$a\{[\, +1\, (F'(t))^3 \sin(F(t))$ 141

$-1\, F'''(t) \sin(F(t))$ 142

$-3\, F'(t)\, F''(t) \cos(F(t))\,]\, \mathbf{i}$ 143

$[-1 (F'(t))^3 \cos(F(t)$ 144
$+1 F'''(t) \cos(F(t))$ 145
$-3 F'(t) F''(t) \sin(F(t))]$ **j** } 146
$+1b H'''(t)$ **k** 147

[**R**'(t) X **R**''(t)] has the seven terms shown in (201) thru (225)
$+ab [+1 F'(t) H''(t) \cos(F(t)$ 201
$+1 (F'(t)^2 H'(t) \sin(F(t)$ 202
$-1 F''(t) H'(t) \cos(F(t))]$ **i** 203
$+ab [+ 1 F'(t) H''(t) \sin(F(t))$ 211
$-1 (F'(t))^2 H'(t) \cos(F(t))$ 212
$-1 F''(t) H'(t) \sin(F(t))]$ **j** 213
$+a^2 [+1 (F'(t))^3]$ **k** 225

[**R**'(t) X **R**''(t) • **R**'''(t)] determination, 19 terms
From [**R**'(t) X **R**''(t)] **i** component x **R**'''(t) **i** component, 9 terms

$+a^2b^1[+1(F'(t))^4 H''(t) \sin(F(t)) \cos(F(t))$	null with 411	401
$-1 F'(t) F'''(t) H''(t) \sin(F(t)) \cos(F(t))$	null with 412	402
$-3 (F(t))^2 F''(t) H''(t) \cos^2(F(t))$	combine with 413	403
$+1 (F'(t))^5 H'(t) \sin^2(F(t))$	combine with 414	404
$-1 (F'(t))^2 F'''(t) H'(t) \sin^2(F(t))$	combine with 415	405
$-3 (F'(t))^3 F''(t) H'(t) \sin(F(t)) \cos(F(t))$	null with 416	406
$-1 (F'(t))^3 F''(t) H'(t) \sin(F(t)) \cos(F(t))$	null with 417	407
$+1 F''(t) F'''(t) H'(t) \sin(F(t)) \cos(F(t))$	null with 418	408
$+3 F'(t) (F''(t))^2 H'(t) \cos^2(F(t))$	combine with 419	409

From [**R**'(t) X **R**''(t)] **j** component x **R**'''(t) **j** component, 9 terms

$+a^2b^1[-1(F'(t))^4 H''(t) \sin(F(t)) \cos(F(t))$	null with 401	411
$+1 F'(t) F'''(t) H''(t) \sin(F(t)) \cos(F(t))$	null with 402	412
$-3 (F(t))^2 F''(t) H''(t) \sin^2(F(t))$	combine with 403	413
$+1 (F'(t))^5 H'(t) \cos^2(F(t))$	combine with 404	414
$-1 (F'(t))^2 F'''(t) H'(t) \cos^2(F(t))$	combine with 405	415
$+3 (F'(t))^3 F''(t) H'(t) \sin(F(t)) \cos(F(t))$	null with 406	416
$+1 (F'(t))^3 F''(t) H'(t) \sin(F(t)) \cos(F(t))$	null with 407	417
$-1 F''(t) F'''(t) H'(t) \sin(F(t)) \cos(F(t))$	null with 408	418
$+3 F'(t) (F''(t))^2 H'(t) \sin^2(F(t))$	combine with 409	419

From [**R**'(t) X **R**''(t)] **k** component x **R**'''(t) **k** component, 1 term
$+a^2b^1[(F'(t))^3 H'''(t)]$ 421

Initial simplifications of [**R**'(t) X **R**''(t) • **R**'''(t)] by combining like terms and using the identity $\sin^2(F(t)) + \cos^2(F(t)) = 1$ gives;
$(403) + (413) = +a^2b^1[-3 (F(t))^2 F''(t) H''(t)]$ 431
$(404) + (414) = +a^2b^1[+1 (F'(t))^5 H'(t)]$ 432
$(405) + (415) = +a^2b^1[-1 (F'(t))^2 F'''(t) H'(t)]$ 433
$(409) + (419) = +a^2b^1[+3 F'(t) (F''(t))^2 H'(t)]$ 434

The remaining terms of [**R**'(t) X **R**''(t) • **R**'''(t)] are a^2b^1 x the following 5 terms

$+(F'(t))^3\ H'''(t)$ 421
$-3\ (F(t))^2\ F''(t)\ H''(t)$ 431
$+1\ (F'(t))^5\ H'(t)$ 432
$-1\ (F'(t))^2\ F'''(t)\ H'(t)$ 433
$+3\ F'(t)\ (F''(t))^2\ H'(t)$ 434

Using assumption 2, $H'(t) = F'(t)$ gives a^2b^1 x the following 5 terms

(421) → $+(F'(t))^3\ F'''(t)$	null with 444	441
(431) → $-3\ (F(t))^2\ (F''(t))^2$	null with 445	442
(432) → $+1\ (F'(t))^6$		443
(433) → $-1\ (F'(t))^3\ F'''(t)$	null with 441	444
(434) → $+3\ (F'(t))^2\ (F''(t))^2$	null with 442	445

Second simplifications of [**R**'(t) X **R**''(t) • **R**'''(t)] gives
(443) → $+1\ a^2b^1\ (F'(t))^6$

The final resulting form of $|\mathbf{R}'(t) \times \mathbf{R}''(t)|$ is Equation (355), repeated here.

$$|\mathbf{R}'(t) \times \mathbf{R}''(t)| = a^1(F'(t))^3\ [a^2 + b^2]^{1/2} \quad 355$$

Therefore

$$|\mathbf{R}'(t) \times \mathbf{R}''(t)|^2 = a^2(F'(t))^6\ [a^2 + b^2]^1 \quad 356$$

Returning to the second objective, which is the determination of the torsion τ. Repeating Equation (003)

$$\text{torsion } \tau = \frac{[\mathbf{R}'(t)\times\mathbf{R}''(t)\bullet \mathrm{R}'''(t)]}{|\mathbf{R}'(t) \times \mathbf{R}''(t)|^2} \quad 003$$

the final result is now seen,

$$\frac{[\mathbf{R}'(t)\times\mathbf{R}''(t)\bullet \mathrm{R}'''(t)]}{|\mathbf{R}'(t) \times \mathbf{R}''(t)|^2} = \frac{a^2b^1(F'(t_1))^6}{a^2(F'(t_1))^6[a^2+b^2]} = \frac{b}{[a^2+b^2]} \quad 450$$

APPENDIX 5 SEPARATION OF VARIABLES

1 Introduction

High school and college chemistry books typically introduce the periodic chart, by discussing the electron shells around the atomic nucleus. Usually simplistic derivations are given there and much more detailed ones given in university quantum mechanic texts covering the same topic. All these derivations start with a grand sweeping equation, a wave equation, Schrodinger's equation, a Hamiltonian operator, or some other such presentation. These equations typically involve a mix of a radial and several angular variables. The objective of much work in such derivations is to separate these variables, so that ultimately other isolated equations can be developed which explain or model the nature of each variable.

This appendix-report presents the step-by-step routine for the separation of the radial and all the angular variables from each other, those variables as might be found in a multi dimensional wave equation modeling a system of a known geometry.

To separate the mathematical variables a grand sweeping assumption that they are independent is made from the get-go. The forces that these variables are modeling are likewise assumed to also be independent. Of course it is inherently assumed that physical reality goes along with this narriative whose sole purpose is to make the work of human mathematicians and physicists easier. Naturally a whole raft of other assumptions are made to go along with, bolster, or justify this first self serving assumption. Reading thru one quantum mechanic text, a list 5 pages long of the assumptions was made leading up to the final presentation of the hydrogenic electron shells. Many of these assumptions were hidden between the lines, implicit, and not stated explicitly. The problem with all these presentations and assumptions is that the math is force fitted to the nature of an already known physical system, or to one that is at least pretty well verified or else assumed. Examples of the math being taylored (fudged) for the human benefit are; that the imaginary solutions to equations are typically summarily thrown out in favor of real solutions. Also any mathematical equations which go to infinity at values of either zero or infinity of their independent variables are given some sort of hand waving treatment as to why they don't apply at the boundaries and something simpler substituted for them there. The problem with this approach for this work was that *the nature of the physical system was not known* in advance, since physicists do not even admit that the leptons are structural systems.

Never-the-less it is instructive to go thru the separation of variables, in that the final results have some applicability to this work. Also one will find the origin of a key major difference from the equations developed in this work and those for the atomic electron shells.

First some general comments and assumptions.

1. The independence assumption: The variables of the initial grand energy equation are assumed to be independent. That is cross products and transcendentals are prohibited, both mathematically and physically. Implicit variables or spatial variables being functions of one another, dependent, are prohibited.
2. The second derivative assumption: Humans typically associate mechanistic energy with acceleration, the second derivative of position. Usually equations dealing with mechanistic energy quantities and flows have the appearance of second order differential equations. In vector representations these differential equations result in ∇^2. This ∇^2 in tern results in the appearance of $1 / r^2$ in all the angular terms. There is the need for some mathematically justifiable means to separate the variables, at a minimum the radial away from the angular.

As a side note mechanistic physics rarely deals with the 3rd derivative or third order differential equations. The official expression sometimes used is "jerk". The total 3rd derivative involves three terms, instead of merely the two terms of potential and kinetic, and in vector notation appears as ∇^3. That is for models which involve third order or third power mathematics or in some other way represent ternary systems. This ∇^3 in tern results in $1 / r^3$ in all the angular terms. Why the side note? Because this idea may be necessary when investigating the color forces of the quarks.

3. The two term – two force assumption: In the grand energy equation describing a system, a particle, classical physics assumed that there are only two forces involved; a kinetic term representing the unary force gravity and a potential term representing the binary force pair electro-magnetic. The necessity for a third term to include the ternary force triplet red-green-blue has not arisen. That is as long as physicists assume that the basic particles, including the quarks, have no structure. In static, steady state, or stable systems these two terms represent two balanced forces. These two opposing forces often appear as an acceleration term, potential energy, and a velocity squared term, kinetic energy. Stated differently the potential term represents something static balancing the kinetic term representing something moving or dynamic.

4. The kinetic term form assumption: The kinetic expression is assumed to be invoked from ∇^2, and assumed to be involved in all factors, the radial and all the angular factors, of the initial grand equation. The kinetic multiplier, ultimately scaling the mathematic expressions to the consensus world of physics, is assumed to be concentrated, localized, or to apply to a point, uniform interior fill, or volume. This interior focused kinetic term is totally self referential and focused where time dominates.

5. The potential term form assumption: The potential expression is assumed to be invoked from ∇^1, and is involved only with the radial nature of the initial grand system structure. Mathematically, the potential force is rigidly excluded from being involved with the angular expressions. The potential multiplier, ultimately scaling the mathematic expressions to the consensus world of physics, is assumed to be diffuse, distributed, delocalized, or to appear as a shell, surface, or boundary quantity. Typically the potential multiplier involves $4\pi\ r^2$ for 3 dimensional systems. This exterior focused potential term is totally referenced on its environment and its elements, and focused where space and position dominate.

6. The one form of each force assumption: Typically physicists assume that there are only one kinetic force-term and only one potential force-term. If more than one of either, it is assumed that they can be combined as a linear combination.

As already seen there are a plethora of constraints put upon the physical system and the mathematical expression of it before the separation of variables even begins. This separation is seen in Tables 2 and 3. Again as with the description of the generalized n-dimensional spherical angular coordinate system, many more dimensions are used here than necessary. Again two reasons. First, many dimensions are necessary to see patterns. With simply two dimensions it is virtually impossible to really discern what is occurring. Secondly investigation of the quarks and gluons might require several dimensions past the third.

TABLE 2.1 SEPARATION OF VARIABLES

Starting Point – Assume – An energy Equation Of The Form				
Total energy Of The System E	=	H(system)		
	=	$H_1(r) * H_2(a_1) * H_3(a_2) * H_4(a_3) * H_5(a_4) * H_6(a_5) * ...$		
Total energy Of The System E	=	∑ (KE, a system kinetic energy term & PE, a system potential energy term)		
where KE	=	km, a kinetic multiplier * $\nabla^2[F_1(r), F_2(a_1), F_3(a_2), F_4(a_3), F_5(a_4), F_6(a_5), ...]$		
and PE	=	pm, a potential multiplier * $\nabla[G_1(r)$ alone]		
Therefore H(system)	=	km * $(1/r^{dim-1}\ \partial/\partial r[r^{dim-1} * \partial H_1/\partial r]) + 1/r^2 * \boldsymbol{F}_1[\partial H_2/\partial a_1] + 1/r^2 * \boldsymbol{F}_2[\partial H_3/\partial a_2]$		
		$+ 1/r^2 * \boldsymbol{F}_3[\partial H_4/\partial a_3] + ... + 1/r^2 * \boldsymbol{F}_{dim-1}[\partial H_{dim}/\partial a_{dim-1}] + pm * \nabla[G_1(r)] - E = 0$		
where $\boldsymbol{F}_1[\partial H_2/\partial a_1]$	=	$1 / [\cos^2(a_2) * \cos^2(a_3) * \cos^2(a_4) * \cos^2(a_5)] * \partial^2[H_2] / (\partial a_1)^2$		
$\boldsymbol{F}_2[\partial H_3/\partial a_2]$	=	$1 / [\cos(a_2) * \cos^2(a_3) * \cos^2(a_4) * \cos^2(a_5)] * \partial / \partial a_2 [\cos(a_2) * \partial(H_3) / \partial a_2]$		
$\boldsymbol{F}_3[\partial H_4/\partial a_3]$	=	$1 / [\cos^2(a_3) * \cos^2(a_4) * \cos^2(a_5)] * \partial / \partial a_3 [\cos^2(a_3) * \partial(H_4) / \partial a_3]$		
$\boldsymbol{F}_4[\partial H_5/\partial a_4]$	=	$1 / [\cos^3(a_4) * \cos^2(a_5)] * \partial / \partial a_4 [\cos^3(a_4) * \partial(H_5) / \partial a_4]$		
$\boldsymbol{F}_5[\partial H_6/\partial a_5]$	=	$1 / [\cos^4(a_5)] * \partial / \partial a_5 [\cos^4(a_5) * \partial(H_6) / \partial a_5]$		
Algebraic Manipulations				
1. Multiply H(system) by r^n, where n = 2 the order of ∇ in the kinetic term. 2. Divide by km.				
3. Take the required derivatives. 4. Divide the result by H(system).				
This yields				
$r^{(-dim+3)} / H_1 * \partial/\partial r [r^{(dim-1)} * \partial H_1/\partial r] + 1/H_2 * \boldsymbol{F}_1[\partial H_2/\partial a_1] + 1/H_3 * \boldsymbol{F}_2[\partial H_3/\partial a_2] + 1/H_4 * \boldsymbol{F}_3[\partial H_4/\partial a_3]$				
$+ ... + 1/H_{dim} * \boldsymbol{F}_{dim-1}[\partial H_{dim}/\partial a_{dim-1}] + km/pm * \nabla[G_1(r)] + r^2/km * (-E)$			=	0
Separating the radial terms from the combined angular terms yields				
Eq 1		$r^{(-dim+3)} / H_1 * \partial/\partial r [r^{(dim-1)} * \partial H_1/\partial r] + km/pm * \nabla[G_1(r)] + r^2/km * (-E)$	=	qn_1
Eq 1a		$1/H_2 * \boldsymbol{F}_1[\partial H_2/\partial a_1] + 1/H_3 * \boldsymbol{F}_2[\partial H_3/\partial a_2] + 1/H_4 * \boldsymbol{F}_3[\partial H_4/\partial a_3] + ... + 1/H_{dim} * \boldsymbol{F}_{dim-1}[\partial H_{dim}/\partial a_{dim-1}]$	=	$-qn_1$
Finish cleaning up radial equation				
1. Multiply Eq 1 by H_1. Divide by r^n				
This yields the final separated radial equation Eq radial				
Eq rad		$1/r^{dim-1} * \partial/\partial r[r^{(dim-1)} * \partial H_1/\partial r] + [km/pm * \nabla[G_1(r)] - E / km * F_1(r) - qn_1 / r^2] * H_1(r)$	=	0
Obviously the three terms multiplying $H_1(r)$ must match in units. This is typically accomplished by ASSUMING				
$\nabla[G_1(r)]$ is proportional to r^{-1}. $F_1(r)$ is proportional to r^0. and $H_1(r) = J(r) / r^{(dim-1)/2}$ for all values of dim.				

TABLE 2.2 SEPARATION OF ANGULAR VARIABLES

	Continuing on from Eq 1a gives		
	$- 1/[\cos^2(a_2) * \cos^2(a_3) * \cos^2(a_4) * \cos^2(a_5)] * 1/H_2 * \partial^2[H_2] / (\partial a_1)^2$	=	$qn_1 + 1/H_3 * \boldsymbol{F}_2[\partial H_3/\partial a_2]$
			$+ 1/H_4 * \boldsymbol{F}_3[\partial H_4/\partial a_3] + ... +1/H_{dim} * \boldsymbol{F}_{dim-1}[\partial H_{dim}/\partial a_{dim-1}]$
	Multiplying by $[\cos^2(a_2) * \cos^2(a_3) * \cos^2(a_4) * \cos^2(a_5)]$ gives		
Eq 2	$-1/H_2* \partial^2[H_2] / (\partial a_1)^2$	=	qn_2
Eq 2a	$[\cos^2(a_2) * \cos^2(a_3) * \cos^2(a_4) * \cos^2(a_5)] * \{qn_1 + 1/H_3 * \boldsymbol{F}_2[\partial H_3/\partial a_2] + 1/H_4 * \boldsymbol{F}_3[\partial H_4/\partial a_3]$		
	$+ ... +1/H_{dim} * \boldsymbol{F}_{dim-1}[\partial H_{dim}/\partial a_{dim-1}] \}$	=	qn_2
	Rearranging Eq 2 gives the final angular 1 equation		
Eq ang$_1$	$\partial^2[H_2] / (\partial a_1)^2 + qn_2 * H_2(a_1)$	=	0
	Continuing on from Eq 2a gives		
	$\cos(a_2)/H_3 * \partial/\partial a_2[\cos(a_2) * \partial(H_3)/\partial a_2] + \cos^2(a_2)/H4 * \partial/\partial a_3[\cos^2(a_3) * \partial(H_4)/\partial a_3] + [\cos^2(a_2) * \cos^2(a_3)] /\cos(a_4)/H_5 * \partial /\partial a_4[\cos^3(a_4) * \partial(H_5)/\partial a_4] + [\cos^2(a_2) * \cos^2(a_3) * \cos^2(a_4)] /\cos^2(a_5)/H_6 * \partial/\partial a_5[\cos^4(a_5) * \partial(H_6)/\partial a_5] +...+[\cos^2(a_2) * \cos^2(a_3) * \cos^2(a_4) * \cos^2(a_5) *...] * qn_1$	=	qn_2
	Dividing by $\cos^2(a_2)$ gives		
Eq 3	$-1/\cos(a_2)/H_3 * \partial/\partial a_2[\cos(a_2) * \partial(H_3)/\partial a_2] + 1/\cos^2(a_2) * qn_2$	=	qn_3
Eq 3a	$1/H4 * \partial/\partial a_3[\cos^2(a_3) * \partial(H_4)/\partial a_3] + \cos^2(a_3)/\cos(a_4)/H_5 * \partial/\partial a_4[\cos^3(a_4) * \partial(H_5)/\partial a_4] + [\cos^2(a_3) * \cos^2(a_4)] /\cos^2(a_5)/H_6 * \partial/\partial a_5[\cos^4(a_5) * \partial(H_6)/\partial a_5] +...+[\cos^2(a_3) * \cos^2(a_4) * \cos^2(a_5) *...] * qn_1$	=	qn_3
	Rearranging Eq 3 gives the final angular 2 equation		
Eq ang$_2$	$\partial / \partial a_2[\cos^1(a_2) * \partial[H_3] / \partial a_2] - [qn_2 * \cos^{-1}(a_2) - qn_3 * \cos^1(a_2)] * H_3(a_2)$	=	0
	Continuing on from Eq 3a		
	Dividing by $\cos^2(a_3)$ gives		
Eq 4	$-1/\cos(a_3)/H_4 * \partial/\partial a_3[\cos^2(a_3) * \partial(H_4)/\partial a_3] + 1/\cos^2(a_3) * qn_3$	=	qn_4
Eq 4a	$1/\cos(a_4)/H_5 * \partial/\partial a_4[\cos^3(a_4) * \partial(H_5)/\partial a_4] + \cos^2(a_4)/\cos^2(a_5)/H_6 * \partial/\partial a_5[\cos^4(a_5) * \partial(H_6)/\partial a_5] +...+[\cos^2(a_4) * \cos^2(a_5) *...] * qn_1$	=	qn_4
	Rearranging Eq 4 gives the final angular 3 equation		
Eq ang$_3$	$\partial / \partial a_3[\cos^2(a_3) * \partial[H_4] / \partial a_3] - [qn_3 * \cos^0(a_3) - qn_4 * \cos^2(a_3)] * H_4(a_3)$	=	0
	Continuing on from Eq 4a		
	Dividing by $\cos^2(a_4)$ gives		
Eq 5	$-1/\cos(a_4)/H_5 * \partial/\partial a_4[\cos^3(a_4) * \partial(H_5)/\partial a_4] + 1/\cos^2(a_4) * qn_4$	=	qn_5
Eq 5a	$1/\cos^2(a_5)/H_6 * \partial/\partial a_5[\cos^4(a_5) * \partial(H_6)/\partial a_5] +...+ [\cos^2(a_5) *...] * qn_1$	=	qn_5
	Rearranging Eq 5 gives the final angular 4 equation		
Eq ang$_4$	$\partial / \partial a_4[\cos^3(a_4) * \partial[H_5] / \partial a_4] - [qn_4 * \cos^1(a_4) - qn_5 * \cos^3(a_4)] * H_5(a_4)$	=	0
	Assuming that there are only 6 dimensions and that angle 5 is the last variable then Eq 5a gives		
Eq 6	$1/\cos^2(a_5)/H_6 * \partial/\partial a_5[\cos^4(a_5) * \partial(H_6)/\partial a_5] + \cos^2(a_5) * qn_1$	=	qn_5
	Multiplying by $\cos^2(a_5)$ and H_6 and rearranging gives		
Eq ang$_5$	$\partial / \partial a_5[\cos^4(a_5) * \partial[H_6] / \partial a_5] - [qn_5 * \cos^2(a_5) - qn_1 * \cos^4(a_5)] * H_6(a_5)$	=	0

TABLE 3 SEPARATED WAVE EQUATION SUMMARY

Equation	Equations As Separated		
Radial	$1/ r^{(dim-1)} * \partial / \partial r [r^{(dim-1)} * \partial H_1(r)/ \partial r]$ $+ [pm / km * \nabla[G_1(r)] - E / km * F_1(r) – qn_1 / r^2] * H_1(r)$	=	0
Angle 1	$\partial^2[H_2] / (\partial a_1)^2 + qn_2 * H_2(a_1)$	=	0
Angle 2	$\partial / \partial a_2[\cos^1(a_2) * \partial[H_3] / \partial a_2] - [qn_2 * \cos^{-1}(a_2) - qn_3 * \cos^1(a_2)] * H_3(a_2)$	=	0
Angle 3	$\partial / \partial a_3[\cos^2(a_3) * \partial[H_4] / \partial a_3] - [qn_3 * \cos^0(a_3) - qn_4 * \cos^2(a_3)] * H_4(a_3)$		
Angle 4	$\partial / \partial a_4[\cos^3(a_4) * \partial[H_5] / \partial a_4] - [qn_4 * \cos^1(a_4) - qn_5 * \cos^3(a_4)] * H_5(a_4)$	=	0
Angle 5	$\partial / \partial a_5[\cos^4(a_5) * \partial[H_6] / \partial a_5] - [qn_5 * \cos^2(a_5) - qn_1 * \cos^4(a_5)] * H_6(a_5)$	=	0

Equation	Rearranged Equations		
Angle 1	$1 / \cos^0(a_1) * \partial / \partial a_1[\cos^0(a_1) * \partial[H_2] / \partial a_1] - [- qn_2] * H_2(a_1)$	=	0
Angle 2	$1 / \cos^1(a_2) * \partial / \partial a_2[\cos^1(a_2) * \partial[H_3] / \partial a_2] - [qn_2 / \cos^2(a_2) - qn_3] * H_3(a_2)$	=	0
Angle 3	$1 / \cos^2(a_3) * \partial / \partial a_3[\cos^2(a_3) * \partial[H_4] / \partial a_3] - [qn_3 / \cos^2(a_3) - qn_4] * H_4(a_3)$	=	0
Angle 4	$1 / \cos^3(a_4) * \partial / \partial a_4[\cos^3(a_4) * \partial[H_5] / \partial a_4] - [qn_4 / \cos^2(a_4) - qn_5] * H_5(a_4)$	=	0
Angle 5	$1 / \cos^4(a_5) * \partial / \partial a_5[\cos^4(a_5) * \partial[H_6] / \partial a_5] - [qn_5 / \cos^2(a_5) - qn_1] * H_6(a_5)$	=	0

Substitution Of The Separated Equations

Assume some function	$F(x)$	=	$H_{n-1}(a_n)$
where	x	=	$f(a_n)$
		=	$\sin(a_n)$
Therefore	$dF(x) / da_n$	=	$dF(x) / dx * dx / da_n$
		=	$dF(x) / dx * \cos(a_n)$
and	$d^2F(x) / (da_n)^2$	=	$d^2F(x) / (dx)^2 * (dx / da_n)^2 + dF(x) / dx * d^2x / (da_n)^2$
		=	$d^2F(x) / (dx)^2 * \cos^2(a_n) + dF(x) / dx * (-\sin(a_n))$

Making these substitutions into the above rearranged separated angular equations yields

Equation	Substituted First Term Appearances	Orthog Poly[1]
Angle 1	$d^2F(x) / (dx)^2 * (1 – x^2) + dF(x) / dx * (-1x)$	Chebyshev, T(n,x)
Angle 2	$d^2F(x) / (dx)^2 * (1 – x^2) + dF(x) / dx * (-2x)$	Jacobi, P(a,b,n,x) a=b=0
Angle 3	$d^2F(x) / (dx)^2 * (1 – x^2) + dF(x) / dx * (-3x)$	Ultraspherical, C(a,n,x) a=1
Angle 4	$d^2F(x) / (dx)^2 * (1 – x^2) + dF(x) / dx * (-4x)$	Jacobi, P(a,b,n,x) a=b=1
Angle 5	$d^2F(x) / (dx)^2 * (1 – x^2) + dF(x) / dx * (-5x)$	Ultraspherical, C(a,n,x) a=2
Angle 6	$d^2F(x) / (dx)^2 * (1 – x^2) + dF(x) / dx * (-6x)$	Jacobi, P(a,b,n,x) a=b=2

Notes: 1 Corresponding orthogonal polynomial appearances

TABLE 4 DIFFERENTIAL EQUATIONS FOR GENERAL ORTHOGONAL POLYNOMIALS				
Definition: $g_2(x) * Fn''(x) + g_1(x) * Fn'(x) + g_0(x) * Fn(x) = 0$				
Function, F(x)	$g_2(x)$	$g_1(x)$	$g_0(x)$	Dim[1]
Laguerre, L(a,n,x)	x	-x+a+1	n	1
Chebyshev 1st Kind, T(n,x)	$1-x^2$	-1x	n(n+0)	2
" T(n, cos(x))	1	0	n^2	2
Legendre, P(a,n,x) or equally Jacobi, P(a,b,n,x) where a = b = 0	$1-x^2$	-2x	n(n+1)	3
Chebyshev 2nd Kind, U(n,x) or equally Ultraspherical, C(a,n,x) where a = 1	$1-x^2$	-3x	n(n+2)	4
Jacobi, P(a,b,n,x) where a = b = 1	$1-x^2$	-4x	n(n+3)	5
Ultraspherical, C(a,n,x) where a = 2	$1-x^2$	-5x	n(n+4)	6
Jacobi, P(a,b,n,x) where a = b = 2	$1-x^2$	-6x	n(n+5)	7
Ultraspherical, C(a,n,x) where a = 3	$1-x^2$	-7x	n(n+6)	8
Notes: 1 Corresponding number of dimensions				

APPENDIX 6 GENERAL MATHEMATICAL PROPERTIES OF NEGATIVE EXPONENTIAL FORMS

1 Introduction

This appendix-report is a compendium of mathematical information related to the properties of the negative exponentials, the distance function, and various squared and cubed forms.

Chapter 2.3 discusses possible possible mathematical approaches towards finding the mathematical descriptions for the masses of the quarks. There in Section 3 the probable appearance of the as yet unknown Radial Contractive Spatial Factor, R_{csf}, as a negative second order exponential form is detailed. In this appendix more information is given as a reference for future such work for many of the negative exponential forms and other related mathematical functions.

2 Information Concerning Negative Exponential Forms $\mathbf{R_p(t) = e^{(-at^p)}}$

$\mathbf{R_p(t) = e^{(-at^p)}}$

$d^1R_p(t)/dt^1 = e^{(-at^p)}\{-p^1a^1t^{(p-1)}\}$

$d^2R_p(t)/dt^2 = e^{(-at^p)}\{(p^1-p^2)at^{(p-2)}$
$\quad +p^2a^2t^{(2p-2)}\}$

$d^3R_p(t)/dt^3 = e^{(-at^p)}\{(-2p^1+3p^2-p^3)at^{(p-3)}$
$\quad +(-3p^2+3p^3)a^2t^{(2p-3)}$
$\quad -p^3a^3t^{(3p-3)}\}$

$d^4R_p(t)/dt^4 = e^{(-at^p)}\{(6p^1-11p^2+6p^3-p^4)at^{(p-4)}$
$\quad +(11p^2-18p^3+7p^4)a^2t^{(2p-4)}$
$\quad +(6p^3-6p^4)a^3t^{(3p-4)}$
$\quad +p^4a^4t^{(4p-4)}\}$

$d^5R_p(t)/dt^5 = e^{(-at^p)}\{(-24p^1+50p^2-35p^3+7p^4-p^5)at^{(p-5)}$
$\quad +(-50p^2+105p^3-70p^4+15p^5)a^2t^{(2p-5)}$
$\quad +(-35p^3+60p^4-25p^5)a^3t^{(3p-5)}$
$\quad +(-10p^4+10p^5)a^4t^{(4p-5)}$
$\quad -p^5a^5t^{(5p-5)}\}$

$d^6R_p(t)/dt^6 = e^{(-at^p)}\{(120p^1-274p^2+225p^3-70p^4+12p^5-p^6)at^{(p-6)}$
$\quad +(274p^2-675p^3+595p^4-222p^5+31p^6)a^2t^{(2p-6)}$
$\quad +(225p^3-510p^4+375p^5-90p^6)a^3t^{(3p-6)}$
$\quad +(85p^4-150p^5+65p^6)a^4t^{(4p-6)}$
$\quad +(15p^5-15p^6)a^5t^{(5p-6)}$
$\quad +p^6a^6t^{(6p-6)}\}$

Defining the following quantities involving mass M, length L, and time, T
Momentum = M x Velocity = M (L/T)
Viscosity = Momentum / L^2 = M / (LT)
Force = M x Acceleration = M (L/T^2)
Force / L = M / T^2
Pressure = Force / L^2 = M / (LT^2)
Energy, Work, Heat, Pressure x Volume, all equally = M L^2T^{-2}
Potential Energy = M x L x Acceleration = M L (L/T^2)

Kinetic Energy = M x Velocity^2 = M (L/T)^2
Energy x Time = M L (L/T)
Power = Energy / Time = M / T (L/T)^2

Further defining distance or length L as $R_p(t) = e^{(-at^p)}$ and then,
Multiplying $R_p(t) = e^{(-at^p)}$ or its derivatives by mass (m) when appropriate gives the following.

Position of $R_p(t) = e^{(-at^p)}$
Velocity or Momentum of $R_p(t) = d^1R_p(t)/dt^1 = m^1e^{(-at^p)}\{-p^1a^1t^{(p-1)}\}$
Viscosity of $R_p(t) = d^1R_p(t)/dt^1 / (R_p(t))^2 = m^1e^{(+at^P)}\{-p^1a^1t^{(p-1)}\}$
Acceleration or Force of $R_p(t) = d^2R_p(t)/dt^2 = m^1e^{(-at^p)}\{(p^1-p^2)at^{(p-2)} + p^2a^2t^{(2p-2)}\}$
Force / Distance of $R_p(t) = d^2R_p(t)/dt^2 / R_p(t) = m^1e^{(+at^0)}\{(p^1-p^2)at^{(p-2)} + p^2a^2t^{(2p-2)}\}$
Pressure or Force / Distance^2 = $d^2R_p(t)/dt^2 / (R_p(t))^2 = m^1e^{(+at^p)}\{(p^1-p^2)at^{(p-2)} + p^2a^2t^{(2p-2)}\}$
Velocity^2 or Kinetic Energy of $R_p(t) = [d^1R_p(t)/dt^1]^2 = m^1e^{(-2at^p)}\{+p^2a^2t^{(2p-2)}\}$
Kinetic Energy x Time of $R_p(t) = [d^1R_p(t)/dt^1]^2t = m^1e^{(-2at^p)}\{+p^2a^2t^{(2p-1)}\}$
Power or Kinetic Energy / Time of $R_p(t) = [d^1R_p(t)/dt^1]^2/t = m^1e^{(-2at^p)}\{+p^2a^2t^{(2p-3)}\}$
Potential Energy of $R_p(t) = R_p(t)\text{x}\, d^2R_p(t)/dt^2 = m^1e^{(-2at^p)}\{(p^1-p^2)at^{(p-2)} + p^2a^2t^{(2p-2)}\}$
PE + KE = $m^1e^{(-2at^p)}\{(p^1-p^2)at^{(p-2)} + 2p^2a^2t^{(2p-2)}\}$

$\mathbf{R_1(t) = e^{(-at^1)}}$
$d^1R_1(t)/dt^1 = e^{(-at^1)}\{-1a^1t^0\}$
$d^2R_1(t)/dt^2 = e^{(-at^1)}\{+1a^2t^0\}$
$d^3R_1(t)/dt^3 = e^{(-at^1)}\{-1a^3t^0\}$
$d^4R_1(t)/dt^4 = e^{(-at^1)}\{+1a^4t^0\}$
$d^5R_1(t)/dt^5 = e^{(-at^1)}\{-1a^5t^0\}$
$d^6R_1(t)/dt^6 = e^{(-at^1)}\{+1a^6t^0\}$

Multiplying $R_1(t) = e^{(-at^1)}$ or its derivatives by mass (m) when appropriate gives the following.
Velocity or Momentum of $R_1(t) = m^1e^{(-at^1)}\{-1a^1t^0\}$
Acceleration of Force of $R_1(t) = d^2R_1(t)/dt^2 = m^1e^{(-at^1)}\{+1a^2t^0\}$
Velocity^2 or Kinetic Energy of $R_1(t) = m^1e^{(-2at^1)}\{+1a^2t^0\}$
Potential Energy of $R_1(t) = m^1e^{(-2at^1)}\{+1a^2t^0\}$
PE + KE = $m^1\, e^{(-2at^1)}\{+2a^2t^0\}$

$\mathbf{R_2(t) = e^{(-at^2)}}$
$d^1R_2(t)/dt^1 = e^{(-at^2)}\{-2a^1t^1\}$
$d^2R_2(t)/dt^2 = e^{(-at^2)}\{-2a^1t^0 + 4a^2t^2\}$
$d^3R_2(t)/dt^3 = e^{(-at^2)}\{+12a^2t^1 - 8a^3t^3\}$
$d^4R_2(t)/dt^4 = e^{(-at^2)}\{+12a^2t^0 - 48a^3t^2 + 16a^4t^4\}$
$d^5R_2(t)/dt^5 = e^{(-at^2)}\{-120a^3t^1 + 160a^4t^3 - 32a^5t^5\}$
$d^6R_2(t)/dt^6 = e^{(-at^2)}\{-120a^3t^0 + 720a^4t^2 - 480a^5t^4 + 64a^6t^6\}$

$d^7R_2(t)/dt^7 = e^{(-at^2)}\{+1680a^4t^1 - 3360a^5t^3 + 1344a^6t^5 - 128a^7t^7\}$

Multiplying $R_2(t) = e^{(-at^2)}$ or its derivatives by mass (m) when appropriate gives the following.
Velocity or Momentum of $R_2(t) = m^1e^{(-at^2)}\{-2a^1t^1\}$
Acceleration or Force of $R_2(t) = d^2R_2(t)/dt^2 = m^1e^{(-at^2)}\{-2a^1t^0 + 4a^2t^2\}$
Velocity^2 or Kinetic Energy of $R_2(t) = m^1e^{(-2at^2)}\{+4a^2t^2\}$
Potential Energy of $R_2(t) = m^1e^{(-2at^2)}\{-2at^0 + 4a^2t^2\}$
PE + KE = $m^1e^{(-2at^2)}\{-2at^0 + 8a^2t^2\}$

$\mathbf{R_3(t) = e^{(-at^3)}}$
$d^1R_3(t)/dt^1 = e^{(-at^3)}\{-3a^1t^2\}$
$d^2R_3(t)/dt^2 = e^{(-at^3)}\{-6a^1t^1 + 9a^2t^4\}$
$d^3R_3(t)/dt^3 = e^{(-at^3)}\{-6a^2t^0 + 54a^3t^3 - 27a^4t^6\}$
$d^4R_3(t)/dt^4 = e^{(-at^3)}\{+180a^2t^2 - 324a^3t^5 + 81a^4t^8\}$
$d^53(t)/dt^5 = e^{(-at^3)}\{+360a^3t^1 - 2160a^4t^4 + 1620a^5t^7 - 243a^6t^{10}\}$
$d^6R_3(t)/dt^6 = e^{(-at^3)}\{+1089a^2t^0 - 9720a^3t^3 + 17820a^4t^6 - 7290a^5t^9 + 729a^6t^{12}\}$

Multiplying $R_3(t) = e^{(-at^3)}$ or its derivatives by mass(m) when appropriate gives the following.
Velocity or Momentum of $R_3(t) = m^1e^{(-at^3)}\{-3a^1t^2\}$
Acceleration of Force of $R_3(t) = d^2R_3(t)/dt^2 = m^1e^{(-at^3)}\{-6a^1t^1 + 9a^2t^4\}$
Velocity^2 or Kinetic Energy of $R_3(t) = m^1e^{(-2at^3)}\{+9a^2t^4\}$
Potential Energy of $R_3(t) = m^1e^{(-2at^3)}\{-6at^1 + 9a^2t^4\}$
PE + KE = $m^1e^{(-2at^3)}\{-6at^1 + 18a^2t^4\}$

$\mathbf{R_4(t) = e^{(-at^4)}}$
$d^1R_4(t)/dt^1 = e^{(-at^4)}\{-4a^1t^3\}$
$d^2R_4(t)/dt^2 = e^{(-at^4)}\{-12a^1t^2 + 16a^2t^6\}$
$d^3R_4(t)/dt^3 = e^{(-at^4)}\{-24a^2t^1 + 144a^3t^5 - 64a^3t^9\}$
$d^4R_4(t)/dt^4 = e^{(-at^4)}\{-24a^2t^0 + 816a^3t^4 - 1152a^4t^8 + 256a^5t^{12}\}$
$d^5R_4(t)/dt^5 = e^{(-at^4)}\{+3360a^3t^3 - 12480a^4t^7 + 7680a^5t^{11} - 1024a^6t^{15}\}$
$d^6R_4(t)/dt^6 = e^{(-at^4)}\{+13152a^2t^2 - 100800a^3t^6 + 134400a^4t^{10} - 46080a^5t^{14} + 4096a^6t^{18}\}$

Multiplying $R_4(t) = e^{(-at^4)}$ or its derivatives by mass (m) when appropriate gives the following.
Velocity or Momentum of $R_4(t) = m^1e^{(-at^4)}\{-4a^1t^3\}$
Acceleration or Force of $R_4(t) = d^2R_4(t)/dt^2 = m^1e^{(-at^4)}\{-12a^1t^2 + 16a^2t^6\}$
Velocity^2 or Kinetic Energy of $R_4(t) = m^1e^{(-2at^4)}\{+16a^2t^6\}$
Potential Energy of $R_4(t) = m^1e^{(-2at^4)}\{-12at^2 + 16a^2t^6\}$
PE + KE = $m^1e^{(-2at^4)}\{-12at^2 + 32a^2t^6\}$

3 Definite Integrals of $\mathbf{R_p(t) = e^{(-at^p)}}$

To determine the definite integrals for some of these expressions of $R_p(t) = e^{(-at^p)}$ the listing of several standard mathematical formulas is convenient.

$\int_0^\infty e^{(-ax^p)}[x^n]\,dx = \frac{\Gamma(k)}{pa^k}$ for a>0, n>-1, P>0; and $k = \frac{n+1}{p}$

$\Gamma(n) = (n-1)!$ for n integer

$\Gamma(1/2) = \sqrt{\pi}$

$\Gamma(n + 1/2) = \frac{1\cdot 3\cdot 5\ldots(2n-1)}{2^n}\sqrt{\pi}$ n = 1, 2, 3, ...

Assuming $\Gamma(4/3) = \Gamma(1 + 1/3) = 1/3\,\Gamma(1/3)$ = 1/3 (2.678,938…)

Assuming $\Gamma(5/3) = \Gamma(1 + 2/3) = 1/3\,\Gamma(2/3)$ = 1/3 (1.354,117…)

Assuming $\Gamma(5/4) = \Gamma(1 + 1/4) = 1/4\,\Gamma(1/4)$ = 1/4 (3.625,609…)

Assuming $\Gamma(7/4) = \Gamma(1 + 3/4) = 1/4\,\Gamma(3/4)$ = 1/4 (1.225,416…)

Table 1 Values Related to $\int_0^\infty e^{(-at^p)}[t^n]\,dt = \frac{\Gamma(k)}{pa^k}$

		k = (n+1)/p						
n	$\Gamma(n + 1/2)$	p = 1	p = 2	p = 3	p = 4	p = 5	p = 6	$\int_0^\infty e^{(-at^2)}[t^n]\,dt =$
0	$\sqrt{\pi}$	1	1/2	1/3	1/4	1/5	1/6	$1/2\, a^{-1/2}\sqrt{\pi}$
1	$1/2\sqrt{\pi}$	2	1	2/3	1/2	2/5	1/3	$1/2\, a^{-1}$
2	$3/4\sqrt{\pi}$	3	3/2	1	3/4	3/5	1/2	$1/4\, a^{-3/2}\sqrt{\pi}$
3	$15/8\sqrt{\pi}$	4	2	4/3	1	4/5	2/3	$1/2\, a^{-2}$
4	$105/16\sqrt{\pi}$	5	5/2	5/3	5/4	1	5/6	$3/8\, a^{-5/2}\sqrt{\pi}$
5	$945/32\sqrt{\pi}$	6	3	2	3/2	6/5	1	$1\, a^{-3}$
6	$10395/64\sqrt{\pi}$	7	7/2	7/3	7/4	7/5	7/6	$15/16\, a^{-7/2}\sqrt{\pi}$

Table 2.1 Values Related to Derivatives of $\mathbf{R_2(t) = e^{(-at^2)}}$

$d^nR_2(t)/dt^n$ n =	Expression of $R_2(t)= e^{(-at^2)}$ {below}	Value @ t = 0	$\int_0^\infty \{d^nR_2(t)/dt^n\}\,dt$
0	$\{1a^0t^0\}$	$1a^0$	$1/2\, a^{-1/2}\sqrt{\pi}$
1	$\{-2a^1t^1\}$	0	$-1!/0!\, a^0$
2	$\{-2a^1t^0 + 4a^2t^2\}$	$-2a^1$	$0\, a^{+1/2}\sqrt{\pi} = 0$
3	$\{+12a^2t^1 - 8a^3t^3\}$	0	$+2!/1!\, a^1$
4	$\{+12a^2t^0 - 48a^3t^2 + 16a^4t^4\}$	$+12a^2$	$0\, a^{+3/2}\sqrt{\pi} = 0$
5	$\{-120a^3t^1 + 160a^4t^3 - 32a^5t^5\}$	0	$-4!/2!\, a^2$
6	$\{-120a^3t^0 + 720a^4t^2 - 480a^5t^4 + 64a^6t^6\}$	$-120a^3$	$0\, a^{+5/2}\sqrt{\pi} = 0$
7	$\{+1680a^4t^1 - 3360a^5t^3 + 1344a^6t^5 - 128a^7t^7\}$	0	$+6!/3!\, a^3$

Table 2.2 Values Related to Physical Interpertations of $R_2(t) = e^{(-at^2)}$

Function of $R_2(t)$	Expression of $R_2(t)$= $e^{(-at^2)}$\{below\}	Value @ t = 0	$\int_0^\infty$\{Expression\} dt
Position	$R_2(t)$	$1a^0$	$1/2\ a^{-1/2}\ \sqrt{\pi}$
Velocity or Momentum	$(D^1)R_2(t)$	0	$-1!/0!\ a^0 = -1$
Viscosity	$(D^1)R_2(t) / [R_2(t)]^2$	0	$-\infty$
Acceleration or Force	$(D^2)R_2(t)$	$-2a^1$	$0\ a^{+1/2}\ \sqrt{\pi} = 0$
Force / Distance	$(D^2)R_2 / R_2(t)$	$-2a^1$	$+\infty$
Pressure or Force / Dist^2	$(D^2)R_2(t) / [R_2(t)]^2$	$-2a^1$	$+\infty$
Velocity^2 or Kinetic Energy	$[(D^1)R_2(t)]^2$	0	$2^{-3/2}\ a^{+1/2}\sqrt{\pi}$
Kinetic Energy x Time	$[(D^1)R_2(t)]^2$ x t	0	$1/2a^0$
Power or Kinetic Energy / Time	$[(D^1)R_2(t)]^2$ / t	0	$1\ a^1$
Potential Energy	$(D^2)R_2(t)$ x $R_2(t)$	$-2a^1$	$-2^{-3/2}\ a^{+1/2}\sqrt{\pi}$
Σ (PE, KE)	$[R_2(t)]^2\{-2at^0 + 8a^2t^2\}$	$-2a^1$	0
Δ (PE, KE)	$[R_2(t)]^2\{\pm 2at^0\}$	$\pm 2a^1$	$\pm\ 2^{-1/2}\ a^{+1/2}\sqrt{\pi}$
Norm, with wt(t) = 1	$\int wt(t)\ [R_2(t)]^2\ dt$	$1a^0$	$2^{-3/2}\ a^{-1/2}\sqrt{\pi}$
Norm Factor	$1 / \text{Norm}^{1/2}$	$1a^0$	$2^{3/4}\ a^{1/4}\pi^{-1/4}$
Notes: The appearance of mass (m) is surpressed			

Table 2.3 Values Related to Rotational Interpertations of $R_2(t) = e^{(-at^2)}$

Function of $R_2(t)$	Expressions of $R_2(t)$	$[\int_0^\infty R_2(t)t^n dt / \int_0^\infty R_2(t)dt\ ]^{1/n}$
Average Radius	$[\ \int R_2(t)\ t^1\ dt / \int R_2(t)\ dt\ ]^1$	$1\ a^{-1/2}\ \pi^{-1/2}$
Radius of Gyration	$[\ \int R_2(t)\ t^2\ dt / \int R_2(t)\ dt\ ]^{1/2}$	$(1/2)^{1/2}\ a^{-1/2}$
3rd Order Radius	$[\ \int R_2(t)\ t^3\ dt / \int R_2(t)\ dt\ ]^{1/3}$	$1\ a^{-1/2}\ \pi^{-1/6}$
4th Order Radius	$[\ \int R_2(t)\ t^4\ dt / \int R_2(t)\ dt\ ]^{1/4}$	$(3/4)^{1/4}a^{-1/2}$
5th Order Radius	$[\ \int R_2(t)\ t^5\ dt / \int R_2(t)\ dt\ ]^{1/5}$	$2^{1/5}a^{-1/2}\pi^{-1/10}$
6th Order Radius	$[\ \int R_2(t)\ t^6\ dt / \int R_2(t)\ dt\ ]^{1/6}$	$(15/8)^{1/6}a^{-1/2}$

Table 3 Values Related to $R_p(t) = e^{(-at^p)}$

p	$d^2R_p(t)/dt^2 = e^{(-at^p)}$\{below\}	$\int_0^\infty R''_p(t)\ dt$	PE + KE = $m^1e^{(-2at^p)}$\{below\}	$\frac{1}{m^1}\int_0^\infty(\text{PE} + \text{KE})\ dt$
1	$1a^2t^0$	$1/a^2 \neq 0$	$+2a^2t^0$	$1/2a^2 \neq 0$
2	$-2a^1t^0 + 4a^2t^2$	0	$-2at^0 + 8a^2t^2$	0
3	$-6a^1t^1 + 9a^2t^4$	$-1a^{1/3}\Gamma(2/3) \neq 0$	$-6at^1 + 18a^2t^4$	$-1a^{1/3}\Gamma(2/3)/2^{2/3} \neq 0$
4	$-12a^1t^2 + 16a^2t^6$	$-2a^{1/4}\Gamma(3/4) \neq 0$	$-12at^2 + 32a^2t^6$	$-2a^{1/4}\Gamma(3/4)/2^{3/4} \neq 0$
5	$-20a^1t^3 + 25a^2t^8$	$\neq 0$	$-20at^3 + 50a^2t^8$	$\neq 0$
6	$-30a^1t^4 + 36a^2t^{10}$	$\neq 0$	$-30at^4 + 72a^2t^{10}$	$\neq 0$

4 Information Concerning Distance Function Appearances

Since usages of the distance function are involved in the Radial Expansive Spatial Factor for the leptons and photons and logically for the quarks examining it for the same derivative properties as with the negative exponential forms could prove to be beneficial.

$$R_{ds}(t) = [1 + a^1t^p]^{(1/2)}$$
$$d^1R_{ds}(t)/dt^1 = +1/2[1 + at^p]^{(-1/2)}\{p^1a^1t^{(p-1)}\}$$
$$d^2R_{ds}(t)/dt^2 = -1/4[1 + at^p]^{(-3/2)}\{p^2a^2t^{(2p-2)}\}$$
$$+1/2[1 + at^p]^{(-1/2)}\{(-p^1 + p^2)a^1t^{(p-2)}\}$$

$d^3R_{ds}(t)/dt^3 = +3/8[1+at^p]^{(-5/2)}\{p^3a^3t^{(3p-3)}\}$
$\quad -1/2[1+at^p]^{(-3/2)}\{(-3p^2+3p^3)a^2t^{(2p-3)}\}$
$\quad +1/2[1+at^p]^{(-1/2)}\{(+2p^1-3p^2+p^3)a^1t^{(p-3)}\}$
$d^4R_{ds}(t)/dt^4 = -15/16[1+at^p]^{(-7/2)}\{p^4a^4t^{(4p-4)}\}$
$\quad +9/8[1+at^p]^{(-5/2)}\{(-5p^3+6p^4)a^3t^{(3p-4)}\}$
$\quad -3/4[1+at^p]^{(-3/2)}\{(+11p^2-18p^3+7p^4)a^2t^{(2p-4)}\}$
$\quad +1/2[1+at^p]^{(-1/2)}\{(-6p^1+11p^2-6p^3+p^4)a^1t^{(p-4)}\}$

Multiplying $R_{ds}(t) = [1+a^1t^p]^{(1/2)}$ or its derivatives by mass (m) when appropriate gives the following.
Velocity or Momentum of $R_{ds}(t) = +1/2m^1[1+at^p]^{(0)}\{p^1a^1t^{(p-1)}\}$
Acceleration or Force of $R_{ds}(t) = d^2R_{ds}(t)/dt^2 = -1/4[1+at^p]^{(-3/2)}\{p^2a^2t^{(2p-2)}\}$
$\quad +1/2[1+at^p]^{(-1/2)}\{(-p^1+p^2)a^1t^{(p-2)}\}$
Velocity^2 or Kinetic Energy of $R_{ds}(t) = +1/4m^1[1+at^p]^{(-1)}\{p^2a^2t^{(2p-2)}\}$
Potential Energy of $R_{ds}(t) = -1/4m^1[1+at^p]^{(-1)}\{p^2a^2t^{(2p-2)}\}$
$\quad +1/2m^1[1+at^p]^{(0)}\{(-p^1+p^2)a^1t^{(p-2)}\}$
PE + KE = $m^1\{+1/2(-p^1+p^2)a^1t^{(p-2)}\}$

Table 4.1 Value of Various Distance Function Derivatives

$R_{ds}(t) = [1+at^p]^{1/2}$		
P	[PE+KE]($R_{ds}(t)$)	$\frac{1}{m^1}\int(\text{PE}+\text{KE})dt$
1	0	0
2	$1at^0$	$1at^1$
3	$3at^1$	$3/2at^2$
4	$6at^2$	$2at^3$
5	$10at^3$	$5/2at^4$
6	$15at^4$	$3at^5$
Notes: The appearance of mass (m) is surpressed		

Table 4.2 Value of Various Distance Function Derivatives

Y = F(t)	dY/dt = F ' (t)	$\frac{ds}{dt} = \left[1+\left(\frac{dY}{dt}\right)^2\right]^{1/2}$	$\text{PE}\left(\frac{ds}{dt}\right)+\text{KE}\left(\frac{ds}{dt}\right)$	$\frac{1}{m^1}\int(\text{PE}+\text{KE})dt$
$a^1t^{(1/2)}$	$-1/2a^1t^{(-1/2)}$	$[1+1/4a^2t^{-1}]^{(1/2)}$	$1/4a^2t^{(-3)}$	$1/8a^2t^{(-2)}$
$a^1t^{(1)}$	$1a^1t^{(0)}$	$[1+1a^2t^0]^{(1/2)}$	0	0
$a^1t^{(3/2)}$	$3/2a^1t^{(1/2)}$	$[1+9/4a^2t^1]^{(1/2)}$	0	0
$a^1t^{(2)}$	$2a^1t^{(1)}$	$[1+4a^2t^2]^{(1/2)}$	$4a^2t^{(0)}$	$4a^2t^{(1)}$
$a^1t^{(5/2)}$	$5/2a^1t^{(3/2)}$	$[1+25/4a^2t^3]^{(1/2)}$	$75/4a^2t^{(1)}$	$75/8a^2t^{(2)}$
$a^1t^{(3)}$	$3a^1t^{(2)}$	$[1+9a^2t^4]^{(1/2)}$	$54a^2t^{(2)}$	$18a^2t^{(3)}$
$a^1t^{(7/2)}$	$7/2a^1t^{(5/2)}$	$[1+49/4a^2t^5]^{(1/2)}$	$245/2a^2t^{(3)}$	$245/8a^2t^{(4)}$
$a^1t^{(4)}$	$4a^1t^{(3)}$	$[1+16a^2t^6]^{(1/2)}$	$240a^2t^{(4)}$	$48a^2t^{(5)}$
Substituting the appearance of $r^{(n)}$ for $t^{(n)}$ in the expressions above				
$2/1\pi^0r^{(1)}$	$2\pi^0r^{(0)}$	$[1+4\pi^0r^0]^{(1/2)}$	0	0
$1/1\pi^1r^{(2)}$	$2\pi^1r^{(1)}$	$[1+4\pi^2r^2]^{(1/2)}$	$4\pi^2r^{(0)}$	$4\pi^2r^{(1)}$
$4/3\pi^1r^{(3)}$	$4\pi^1r^{(2)}$	$[1+16\pi^2r^4]^{(1/2)}$	$96\pi^2r^{(2)}$	$32\pi^2r^{(3)}$
$1/2\pi^2r^{(4)}$	$2\pi^2r^{(3)}$	$[1+4\pi^4r^6]^{(1/2)}$	$60\pi^4r^{(4)}$	$12\pi^4r^{(5)}$
Notes: The appearance of mass (m) is surpressed in the expressions for PE and KE				

Analyzing or listing several general functional forms for these same derivative properties could also be useful or educational for the work with the quarks.

5 Information Concerning Linear, Quadratic, and Cubic Forms
General Linear Form

$F(r_1) = [R(t)]^1$
$d^1F(r_1)/dr_1^1 = \{d^1R(t)/dt^1\}$
$d^2F(r_1)/dr_1^2 = \{d^2R(t)/dt^2\}$
$d^3F(r_1)/dr_1^3 = \{d^3R(t)/dt^3\}$
$d^4F(r_1)/dr_1^4 = \{d^4R(t)/dt^4\}$
$d^5F(r_1)/dr_1^5 = \{d^5R(t)/dt^5\}$
$d^6F(r_1)/dr_1^6 = \{d^6R(t)/dt^6\}$

Multiplying $F(r_1) = [R(t)]^1$ or its derivatives by mass (m) when appropriate
Velocity or Momentum of $F(r_1) = d^1F(r_1)/dr_1^1 = m^1d^1R(t)/dt^1$
Acceleration or Force of $F(r_1) = d^2F(r_1)/dr_1^2 = m^1\{d^2R(t)/dt^2\}$
Velocity^2 or Kinetic Energy of $F(r_1) = [d^1F(r_1)/dr_1^1]^2 = m^1[d^1R(t)/dt^1]^2$
Potential Energy of $F(r_1) = F(r_1)x\, d^2F(r_1)/dr_1^2 = m^1R(t)x\, d^2R(t)/dt^2$
PE + KE = $m^1\{R(t)x[d^2R(t)/dt^2]^1 + [d^1R(t)/dt^1]^2\}$

General Quadratic Form

$G(r_2) = [R(t)]^2$
$d^1G(r_2)/dr_2^1 = 2\{R(t)\ x\ d^1R(t)/dt^1\}$
$d^2G(r_2)/dr_2^2 = 2\{R(t)\ x\ d^2R(t)/dt^2 + [d^1R(t)/dt^1]^2\}$
$d^3G(r_2)/dr_2^3 = 2\{R(t)\ x\ d^3R(t)/dt^3 + 3d^1R(t)/dt^1\ x\ d^2R(t)/dt^2\}$
$d^4G(r_2)/dr_2^4 = 2\{R(t)\ x\ d^4R(t)/dt^4 + 4d^1R(t)/dt^1\ x\ d^3R(t)/dt^3 + 3[d^2R(t)/dt^2]^2\}$
$d^5G(r_2)/dr_2^5 = 2\{R(t)\ x\ d^5R(t)/dt^5 + 5d^1R(t)/dt^1\ x\ d^4R(t)/dt^4 + 10d^2R(t)/dt^2\ x\ d^3R(t)/dt^3\}$
$d^6G(r_2)/dr_2^6 = 2\{R(t)\ x\ d^6R(t)/dt^6 + 6d^1R(t)/dt^1\ x\ d^5R(t)/dt^5 + 15d^2R(t)/dt^2\ x\ d^4R(t)/dt^4$
$+10[d^3R(t)/dt^3]^2\}$

Multiplying $G(r_2) = [R(t)]^2$ or its derivatives by mass (m) when appropriate
Velocity or Momentum of $G(r_2) = d^1G(r_2)/dr_2^1 = 2m^1R(t)^1x\, d^1R(t)/dt^1$
Acceleration or Force of $G(r_2) = d^2G(r_2)/dr_2^2 = 2m^1\{R(t)\ x\ d^2R(t)/dt^2 + [d^1R(t)/dt^1]^2\}$
Velocity^2 or Kinetic Energy of $G(r_2) = [d^1G(r_2)/dr_2^1]^2 = 4m^1[R(t)x\, d^1R(t)/dt^1]^2$
Potential Energy of $G(r_2) = G(r_2)x\, d^2G(r_2)/dr_2^2 =$
$2m^1\{R(t)^3\ x\ d^2R(t)/dt^2 + R(t)^2[\ d^1R(t)/dt^1]^2\}$
PE + KE = $2m^1\{R(t)^3\ x\ d^2R(t)/dt^2 + 3R(t)^2[\ d^1R(t)/dt^1]^2\}$

General Cubic Form

$H(r_3) = [R(t)]^3$
$d^1H(r_3)/dr_3^1 = 3\{R(t)^2\ x\ d^1R(t)/dt^1\}$
$d^2H(r_3)/dr_3^2 = 3\{R(t)^2\ x\ d^2R(t)/dt^2 + 2R(t)[d^1R(t)/dt^1]^2\}$
$d^3H(r_3)/dr_3^3 = 3\{R(t)^2\ x\ d^3R(t)/dt^3 + 6R(t)\ x\ d^1R(t)/dt^1\ x\ d^2R(t)/dt^2 + 2[d^1R(t)/dt^1]^3\}$
$d^4H(r_3)/dr_3^4 = 3\{R(t)^2\ x\ d^4R(t)/dt^4 + 8R(t)\ x\ d^1R(t)/dt^1\ x\ d^3R(t)/dt^3$
$+12[d^1R(t)/dt^1]^2\ x\ d^2R(t)/dt^2 + 6R(t)[d^2R(t)/dt^2]^2\}$

Multiplying $H(r_3) = [R(t)]^3$ or its derivatives by mass (m) when appropriate
Velocity or Momentum of $H(r_3) = d^1H(r_3)/dr_3^1 = 3m^1R(t)^2 x\, d^1R(t)/dt^1$
Acceleration or Force of $H(r^3) = d^2H(r_3)/dr_3^2 = 3m^1\{R(t)^2 \text{ x } d^2R(t)/dt^2 + 2R(t)[d^1R(t)/dt^1]^2\}$
Velocity^2 or Kinetic Energy of $H(r_3) = [d^1H(r_3)/dr_3^1]^2 = 9m^1[R(t)^2 x\, d^1R(t)/dt^1]^2$
Potential Energy of $F(r_3) = H(r_3) x\, d^2H(r_3)/dr_3^2 =$

$$3m^1\{R(t)^5 \text{ x } d^2R(t)/dt^2 + 2R(t)^4[\, d^1R(t)/dt^1]^2\}$$

PE + KE = $3m^1\{R(t)^5 \text{ x } d^2R(t)/dt^2 + 5R(t)^4[\, d^1R(t)/dt^1]^2\}$